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A MEASURE-BASED METHODOLOGICAL FRAMEWORK FOR CONTINUOUS MAXIMUM COVERAGE: GEOMETRIC FORMULATION AND HYBRID SWARM OPTIMIZATION

Today, technical systems that rely on spatial monitoring, autonomous decision-making, and distributed sensing are widely used in industry, energy, environmental protection, telecommunications, and security. Many modern infrastructures, such as sensor networks, robotic platforms, surveillance systems, and environmental monitoring grids, require reliable geometric coverage of a given territory under operational constraints. Achieving high-quality coverage in the presence of forbidden zones, irregular boundaries, and heterogeneous service requirements represents an important scientific and practical challenge. ***The goal of this study*** *is to develop a measure-based methodological framework for formulating and computationally treating continuous maximum coverage problems involving geometric objects of arbitrary shape.* ***To achieve this goal****, the paper addresses the formalization of continuous coverage problems through set measures and configuration parameters, the construction of scalable methods for evaluating coverage areas with controllable accuracy, and the integration of global and local optimization mechanisms as computational instruments within the proposed approach, rather than as objects of algorithmic comparison.* ***The research concerns*** *the methodological principles of hybrid swarm-based optimization applied to high-dimensional geometric configurations of covering objects. The proposed framework combines concepts from computational geometry, inclusion–exclusion formulas, Monte Carlo sampling, and population-based optimization, which are interpreted as geometric motion models in a continuous parameter space.* ***The study provides*** *several important methodological results. A general nonlinear optimization formulation of the continuous maximum coverage problem based on set measures is constructed, and computational schemes for estimating coverage areas under different accuracy–complexity trade-offs are introduced. Several families of hybrid swarm-based methods are adapted, interpreted within a unified geometric framework, and provided with evidence-based guidelines for parameter selection and hybridization configuration. The reported numerical results demonstrate the practical applicability of the proposed framework and illustrate typical effects of hybridization on coverage quality under a fixed computational budget.* ***The scientific novelty of*** *the work consists of three contributions. A geometry-agnostic measure-based formulation is developed that simultaneously accommodates arbitrary object shapes, non-convex target regions, forbidden zones of general form, and continuous rotation parameters within a single unified framework, without relying on shape-specific predicates or problem-dependent geometric decompositions. A unified geometric reinterpretation of four swarm intelligence paradigms is established, in which algorithmic update rules are interpreted as coordinated rigid motions in the continuous configuration space, providing a principled geometry-driven basis for algorithm selection depending on the structural properties of the feasible domain. A structured hybridization scheme is formulated to integrate global stochastic search with deterministic local refinement applied to the penalized measure-based coverage functional, including evidence-based guidelines for activation frequency, stopping criteria, and gradient approximation for non-smooth geometric objectives. The proposed framework provides a flexible and extensible methodological foundation for coverage modeling in engineering, energy, and environmental monitoring systems where geometric complexity, forbidden zones, and placement constraints are unavoidable.*

Keywords*: mathematical model, continuous maximum coverage, measure-based methodology, swarm intelligence, memetic optimization, computational geometry, geometric modeling.*

1. Introduction

1.1. Motivation

Many modern engineering problems require the optimal spatial placement of objects to maximize service efficiency, monitoring quality, or functional coverage within a given territory. Continuous geometric coverage models naturally arise in logistics, regional planning, environmental and industrial monitoring, emergency response systems, robotics, telecommunications, GIS-based decision support, and the design of wireless sensor and IoT networks. In such applications, it is often necessary to determine the optimal configuration of sensing, service, or infrastructural elements that maximizes the coverage of a region while satisfying geometric constraints, forbidden zones, and operational limitations.

While classical discrete coverage models laid the foundation for studying the allocation of service facilities and sensing units, real engineering systems frequently operate in continuous spaces and involve objects of arbitrary geometry. Continuous coverage problems therefore provide more accurate and flexible models, capable of capturing anisotropic sensor characteristics, irregularly shaped regions, orientation-dependent coverage, or obstacles that restrict feasible placements.

Beyond classical sensing and monitoring scenarios, continuous coverage models naturally arise in a wide range of engineering and industrial applications involving the utilization of irregularly shaped components. Examples include the reuse of industrial scrap materials in lightweight and composite manufacturing, the arrangement of insulation fragments in construction, the assembly of recycled elements in renewable energy systems, and the formation of layered composite structures from non-convex components. In such applications, coverage objects often have arbitrary shapes, can rotate, and must satisfy complex geometric constraints, making discrete or idealized geometric models inadequate. Similar challenges arise in emergency response and post-disaster resource allocation, where available materials and service regions exhibit irregular spatial configurations. These application domains motivate the need for flexible, continuous-coverage models capable of handling arbitrary geometries and forbidden regions within a unified mathematical framework.

The practical relevance of continuous maximum coverage is particularly evident in domains such as forest fire monitoring, radiation control, environmental sensing, surveillance, distributed measurement networks, and infrastructure planning for smart cities. In these cases, improving coverage directly enhances system reliability, responsiveness, and safety. Consequently, the development of robust, scalable, and accurate methodological frameworks for continuous geometric coverage is of significant interest for radioelectronic and computer systems, cyber-physical technologies, and modern engineering applications.

1.2. State of the art

The conceptual foundations of maximum coverage in discrete spaces were established in the classical work [1], where the notions of coverage, demand region and coverage radius were introduced. A broad review of location models and principles for constructing coverage problems is provided in [2].

The transition from discrete to continuous formulations has stimulated extensive research on geometric coverage problems in Euclidean space. These include continuous maximal coverage by geometric objects of arbitrary shape [3], coverage using ellipses or oriented regions [4], coverage by anisotropic or variable-length segments [5], and multi-radius coverage structures [6]. More advanced approaches make use of semi-infinite optimization [7], network-based coverage formulations [8], and hybrid discrete–continuous models [9, 10].

A separate research direction concerns spatial coverage problems involving arbitrary region shapes, which often arise in territorial analysis and the placement of critical infrastructure. Several studies have demonstrated that continuous models enable more effective handling of realistic, non-idealized territories [11, 12]. Further developments have introduced computational geometry techniques to model polygonal regions, non-Euclidean boundaries, and complex contours. Such approaches enable the incorporation of regional heterogeneity and realistic geographic constraints when computing coverage [13, 14]. Additional contributions have analyzed coverage problems in irregularly shaped and fragmented domains, using software libraries such as Shapely or CGAL to perform accurate Boolean geometric operations [15, 16]. In parallel, research on wireless sensor networks has highlighted the importance of reliable coverage modeling for monitoring and safety applications [17, 18].

Despite advances in exact geometric algorithms, semi-infinite optimization and branch-and-bound techniques, continuous maximum coverage problems remain challenging due to their high dimensionality, multimodality and NP-hard nature. The coverage objective typically exhibits non-smooth behavior, numerous plateaus, and mixed constraints, making classical gradient-based or combinatorial methods inefficient or infeasible.

These limitations have motivated the use of swarm intelligence methods, which naturally perform global search in irregular, high-dimensional spaces. Particle Swarm Optimization (PSO), originally introduced in [19], has been extensively studied with respect to stability and convergence [20], and its general theoretical foundations were summarized in [21]. A modern and systematic classification of PSO variants is presented in [22]. PSO has been applied to various coverage problems, including sensor placement, k-coverage and connectivity optimization in wireless sensor networks [23, 24].

Another widely used population-based method is the Artificial Bee Colony (ABC) algorithm, introduced in [25]. Comprehensive reviews of its variants and applications are given in [26, 27]. The Fish School Search (FSS) algorithm [28], extended in [29], has demonstrated strong performance on problems with geometric constraints, especially where collective behavior must capture both local and global movements. The Firefly Algorithm (FA), proposed and thoroughly analyzed in [30], has proven effective in highly multimodal landscapes; its modern modifications for complex geometric search spaces are presented in [31].

Although swarm algorithms are efficient in global exploration, their local accuracy is often limited, particularly in coverage problems where the objective landscape is non-smooth and topologically complex. To address this, hybrid (memetic) swarm algorithms have been developed that combine global heuristic search (PSO, FSS, FA, ABC) with deterministic local optimization. The fundamental principles of memetic algorithms are outlined in [32], and contemporary developments are discussed in [33].

Among the most effective hybrid approaches are PSO+LS [34, 35], FSS+LS [36, 37], FA+LS [31, 38] and ABC+LS [39, 40], where local search is frequently implemented using quasi-Newton methods such as BFGS. These hybrid methods significantly improve solution accuracy in problems with high-dimensional parameter spaces, complex geometric constraints and non-smooth coverage surfaces.

At the same time, the reviewed literature indicates that the increasing reliance on swarm and hybrid optimization methods is largely driven by the lack of flexible, geometry-independent continuous formulations that would enable unified, scalable evaluation of coverage feasibility and quality. In many cases, algorithmic complexity compensates for modeling complexity rather than resolving it at the formulation level.

The above review allows us to draw the following conclusions. Continuous geometric maximum coverage constitutes a practically important class of problems with broad engineering relevance. Their mathematical and computational complexity necessitate the use of global metaheuristic optimization techniques. Hybrid swarm-based methods represent an effective computational instrument, particularly when combined with accurate geometric evaluation mechanisms. At the same time, the reviewed literature reveals three interconnected gaps. First, existing continuous coverage formulations are typically shape-specific or algorithm-dependent, and no unified geometry-agnostic framework has been proposed that simultaneously accommodates arbitrary object shapes, non-convex regions, forbidden zones, and rotation parameters within a single mathematical model. Second, the selection of swarm-based optimization algorithms for coverage problems is largely empirical, without a principled connection to the geometric structure of the feasible domain. Third, guidelines for configuring hybrid swarm-based schemes in geometrically constrained coverage problems remain scattered and problem-specific, lacking a systematic methodological basis. The present work addresses all three gaps within a unified measure-based framework.

1.3. Objectives and tasks

In view of the above, the purpose of this work is to develop a measure-based methodological framework for continuous maximum-coverage problems subject to geometric and positional constraints. The proposed framework treats population-based and hybrid optimization methods as computational instruments for exploring high-dimensional configuration spaces, rather than as objects of algorithmic comparison. The central aim is to establish a unified modeling approach that enables scalable coverage evaluation, accommodates arbitrary object shapes and forbidden regions, and supports the integration of global search with local geometric refinement.

To achieve this goal, the following research tasks are addressed:

- to develop a mathematical formulation of the continuous maximum coverage problem based on set measures and configuration parameters, and to propose efficient computational schemes for evaluating coverage areas under different accuracy and complexity trade-offs;
- to analyze the role of swarm-based optimization mechanisms such as PSO, FSS, FA, and ABC within the proposed methodological framework, with respect to their suitability for exploring geometric configuration spaces;
- to incorporate hybrid optimization strategies that combine global stochastic exploration with deterministic local refinement, ensuring improved coverage quality and numerical stability;
- to derive methodological recommendations on parameter selection and hybridization strategies for coverage optimization problems of moderate dimensionality;
- to demonstrate the applicability of the proposed framework on illustrative coverage scenarios involving arbitrary object shapes, non-convex regions, and forbidden zones.

The scope of this work is intentionally methodological. The computational illustrations in this paper are designed to demonstrate the operability and configurability of the proposed framework in geometrically representative scenarios, not to establish algorithmic superiority or to provide a competitive benchmark. Comprehensive empirical evaluation across large instance sets is left as a direction for future work.

The article is structured as follows. Section 2 introduces the problem statement and the parameterization of covering objects, along with geometric constraints. Section 3 presents the measure-based formulation of the continuous maximum coverage problem and discusses computational aspects of coverage evaluation. Section 4 describes population-based optimization mechanisms and their geometric interpretation within the proposed framework. Section 5 reports numerical illustrations and methodological observations. Section 6 provides a detailed discussion of the results. Section 7 summarizes the main conclusions and outlines directions for further research.

2. Problem Statement

In this work, we consider a continuous maximum-coverage problem on the two-dimensional Euclidean plane. The formulation adopted in this section is intentionally general and geometry-agnostic. It is designed to serve as a methodological foundation for measure-based coverage modeling, rather than to impose shape-specific or algorithm-dependent assumptions. All geometric admissibility conditions are expressed in terms of set relations and placement parameters, which allows the resulting model to accommodate arbitrary object shapes, irregular target regions, and forbidden zones within a unified framework.

Let a compact target region $\Omega \subset R^2$ be given. The goal is to cover Ω by a set of n geometric objects $S_1,...,S_n$ each of which is a compact set of fixed shape and metric characteristics (i.e., its size, outline and internal geometry do not change). Thus, the areas of the target region Ω and the covering objects S_i, $i = 1,..., n$ are constant and known in advance, which allows us to evaluate the coverage quality directly in terms of planar measurements without introducing auxiliary geometric predicates.

For each covering object, we introduce a reference point called its pole. This point determines the object's position in the global coordinate system. The choice of the pole depends on the object's geometry and may correspond to:

- a geometric center (for symmetric shapes);
- the centroid (center of mass);
- the Chebyshev center;
- the center of the minimum bounding circle;
- or any other characteristic point that uniquely specifies the object's location.

The choice of the pole does not affect the geometric properties of the coverage model. It serves only as a convenient parameterization device that establishes a one-to-one correspondence between placement parameters and rigid motions of the object in the plane.

Each object also has its own local coordinate system, whose origin coincides with the pole. Let S_i^{loc}, $i = 1,..., n$ denote the set of points describing the object in its local coordinates.

To place every S_i in the global coordinate system Oxy, we define a set of placement parameters

$$p_i = (x_i, y_i, \theta_i), \; i = 0,..., n,$$

where (x_i, y_i) are the coordinates of the pole and θ_i is the object's rotation angle.

The transformed object in global coordinates is given by

$$S_i(p_i) = R(\theta_i) S_i^{loc} + (x_i, y_i),$$

where $R(\theta_i)$ is the rotation matrix

$$R(\theta_i) = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}.$$

The transformation consists of a rigid motion and therefore preserves the Lebesgue measure of the object, so it does not alter its contribution to the coverage area. The target region Ω is fixed relative to the global coordinate system Oxy and is described by placement parameters $p_0 = (0, 0, 0)$, meaning that its pole coincides with the origin of the global coordinate system and its orientation angle is zero.

The overall coverage configuration is defined as the union of all transformed covering objects after intersecting each of them with the target region Ω. In other words, only the portions of objects that actually lie within the effective coverage contribute to it. Formally, the coverage configuration is given by

$$C(p) = \Omega \cap \bigcup_{i=1}^{n} S_i(p_i). \qquad (1)$$

This definition treats coverage as a global geometric configuration rather than as a collection of pairwise interactions between individual objects and the target region. Coverage quality is assessed at the level of the entire configuration using set union and intersection. As a result, fragments of covering objects located outside the target region do not influence the optimization process, while all covered subregions within Ω are represented explicitly. Such a formulation is particularly important for applications where the objective is strictly limited to the coverage of a predefined region and external overlaps must not affect the objective value.

The coverage area is then expressed as

$$\varphi(p) = area\left(\Omega \cap \bigcup_{i=1}^{n} S_i(p_i)\right), \quad (2)$$

where $p = (p_1, ..., p_n) \in R^{3n}$ is the full vector of placement parameters.

The quantity $\varphi(p)$ is the objective function of the optimization problem and represents a scalar measure-based functional that evaluates the effectiveness of a given placement configuration.

In practical applications, covering objects cannot be placed arbitrarily. Let $Z_1, ..., Z_m$ be a collection of forbidden zones, where object poles or certain geometric parts cannot enter, i.e.

$$p_i \notin \bigcup_{j=1}^{m} Z_k, \; i = 1, ..., n$$

These zones are usually specified by inequality systems:

$$g_j(x_i, y_i, \theta_i) \le 0, \; j = 1, ..., m \; .$$

Additional admissibility constraints require that the pole of each object must lie within the target region Ω, i.e. $(x_i, y_i) \in \Omega$. This can also be written as a system of inequalities:

$$h_i(x_i, y_i) \le 0, \; i = 1, ..., n \; .$$

Forbidden zones may include polygonal obstacles, circular exclusion areas, buffer regions around critical infrastructure, or any other domain components where object placement is physically or operationally prohibited. Thus, the feasible set of placement parameters is a highly non-convex region subject to mixed geometric constraints.

From a modeling perspective, the presence of forbidden zones significantly increases the complexity of classical coverage formulations based on explicit geometric constraints. In the proposed framework, however, such zones are naturally incorporated via set operations, without altering the structure of the coverage functional. This property is essential for developing scalable and robust optimization models for complex spatial environments.

3. Nonlinear Optimization Formulation of the maximum coverage problem

The continuous maximum coverage problem is formulated as:

$$\max_{p \in R^{3n}} \varphi(p) \quad (3)$$

subject to

$$g_j(x_i, y_i, \theta_i) \le 0$$

$$h_i(x_i, y_i) \le 0$$

$$i = 1, ..., n \; ; \; j = 1, ..., m \; ,$$

where function $\varphi(p)$ has the form (2).

Due to the complexity of the geometric constraints, these conditions are incorporated directly into the objective through additive penalty terms.

Specifically, admissibility constraints are represented by a weighted sum of penalty components $P_{\text{domain}}(p)$ and $P_{\text{forbidden}}(p)$ associated with different constraint groups. Two main groups of constraints are considered.

The first group enforces admissible placement of object poles within the target region. Violations of this group correspond to configurations in which the pole of at least one covering object lies outside the allowable placement domain.

The second group enforces exclusion constraints associated with forbidden zones. These penalties are activated when object poles enter prohibited regions or violate obstacle-related restrictions.

This leads to a penalty-augmented unconstrained optimization problem:

$$\begin{aligned} F(p) = -\varphi(p) + \lambda_1 P_{\text{domain}}(p) + \\ + \lambda_2 P_{\text{forbidden}}(p) \to \min_{p \in R^{3n}} \end{aligned} \quad (4)$$

where $\varphi(p)$ is the coverage area; $\lambda_1 > 0$ and $\lambda_2 > 0$ are the penalty coefficients corresponding to the two constraint groups.

This separation allows different constraint types to be weighted according to their practical importance and improves numerical stability during optimization.

Each penalty component is a nonnegative functional that vanishes on feasible configurations and increases continuously with the magnitude of the corresponding constraint violation. Typical realizations include exterior quadratic penalties or smoothed barrier-type functions applied to distance-based constraint residuals.

The explicit analytical forms of the penalty components $P_{\text{domain}}(p)$ and $P_{\text{forbidden}}(p)$ are not fixed, since the focus of this work is on methodological modeling rather than on a specific penalty design. All penalty constructions considered in this framework are continuous with respect to placement parameters and admit numerical approximation of first-order information. The penalty coefficients λ_1 and λ_2 are increased gradually during optimization in order to enforce feasibility without destabilizing the search process and can be adjusted independently.

The rotation angle of each object is treated as a periodic variable. For this reason, no explicit bound or penalty constraint is imposed on the angular parameter. Equivalent configurations differing by full rotations are naturally handled by the periodicity of the transformation, which avoids artificial discontinuities in the optimization landscape.

The resulting optimization problem is computationally difficult because:

- the parameter space has dimension $3n$, which becomes large for $n > 10$;
- the coverage function $\varphi(p)$ is non-smooth and multimodal, with numerous local maxima introduced by small rotations or translations of the covering objects;
- the feasible set is non-convex and may contain narrow passages caused by forbidden zones or polygonal boundaries;
- computing the exact area of the union of arbitrarily rotated objects is expensive, especially for complex shapes;
- no analytical gradient exists, making classical optimization methods ineffective.

These structural difficulties significantly increase the risk of premature convergence for classical optimization methods and limit the applicability of purely gradient-based techniques. In particular, local search methods are applied not to the original constrained problem, but to penalized objective functions that combine coverage evaluation with smooth approximations of constraint violations. As a result, robust population-based metaheuristics provide a practically viable alternative for high-dimensional engineering problems involving complex geometric interactions and irregular feasible domains. Their hybrid (memetic) forms combine global stochastic exploration with efficient local refinement, making them suitable for large-scale geometric optimization tasks.

Within the proposed methodology, optimization algorithms are treated as computational instruments rather than as objects of comparison. The key modeling challenge, therefore, lies in the efficient and scalable evaluation of the coverage area, which dominates the computational cost of all population-based and hybrid schemes.

The central and computationally complex component of the continuous maximum coverage problem is the evaluation of the coverage area

$$\varphi(p) = area\big(C(p)\big),$$

where $C(p)$ is given by expression (1).

The accuracy and efficiency of this computation determine the overall performance of the optimization algorithm, since the coverage area must be evaluated for many thousands of configurations. In practice, no single method is universally optimal across all stages of optimization, and therefore a combination of several approaches is typically employed.

A natural starting point is the approximation of covering objects and the target region by their bounding rectangles. In this case, the union of these rectangles provides a very fast upper estimate of the true coverage configuration. Although such an approximation does not capture geometric details, it is effective at early stages of optimization, when the task is to quickly filter unpromising configurations and allow swarm algorithms to explore large areas of the search space. Despite its coarse nature, this approach significantly accelerates population-based methods during initial iterations.

A more accurate and widely used technique is Monte Carlo sampling. A large number of test points is generated uniformly in the target region Ω, and the coverage area is estimated as the proportion of points that belong to the coverage configuration $C(p)$. This method is well suited to arbitrary object shapes and is robust to non-smooth or irregular boundaries. Its accuracy is controlled by the number of generated points: a relatively small number is sufficient at early stages, while a higher density is required as the algorithm approaches promising regions of the search space. Since the approach straightforwardly parallelizes across modern CPU and GPU architectures, Monte Carlo sampling is often the practical method of choice for mid-precision evaluation.

In cases where analytic expressions for intersections between covering objects are available, the coverage area may be approximated using a second-order inclusion and exclusion expansion. This method computes the area of each individual object intersected with the target region and then subtracts the area of their pairwise intersections. The approach is accurate when geometric intersection operations can be computed efficiently, as is typical for objects with simple parametric descriptions such as circles, ellipses or convex polygons. Although its computational complexity grows quadratically with the number of objects, it remains effective for medium-sized configurations and provides significantly higher accuracy compared to Monte Carlo sampling in scenarios where pairwise intersection formulas are tractable.

For the highest accuracy, exact computational geometry methods are used. These involve Boolean union operations applied directly to the transformed objects $S_i(p_i)$, followed by intersection with the target region Ω. The resulting geometric shape represents the exact coverage configuration $C(p)$, and its area is computed without approximation. Libraries such as Shapely, CGAL or Boost.Geometry support robust Boolean operations and provide nearly machine-precision results. However, such computations are expensive and may become unstable for highly complex shapes or large numbers of objects. Thus, exact methods are typically reserved for late stages of optimization, where only a small number of elite solutions must be evaluated with maximum precision.

Given the strengths and limitations of each approach described, a combined evaluation strategy is used in practice. Coarse approximations based on bounding rectangles or low-density Monte Carlo sampling are applied during early iterations to accelerate global exploration. As the search progresses and the algorithm identifies more promising regions of the parameter space, these initial approximations are replaced by more accurate Monte Carlo estimates or inclusion and exclusion computations. Finally, exact computational geometry is applied only to verify the best candidate solutions.

Table 1 summarizes the recommended strategies for evaluating the coverage area at different stages of the optimization process. The table serves as a methodological guideline illustrating how different accuracy–complexity trade-offs can be exploited within the proposed framework, rather than as an algorithm-specific prescription.

Table 1.

Area evaluation strategies at different optimization stages

Optimization stage	**Required accuracy**	**Area evaluation method**	**Methodological role**
Global exploration	Low (≈1–2%)	Monte Carlo sampling or pairwise intersections	Fast ranking of candidate configurations
Intermediate filtering	Moderate (≈0.5–1%)	Pairwise intersections (IE2)	Efficient elimination of weak solutions
Local refinement	High (≤0.1%)	Exact geometric computation	Accurate feasibility verification

4. Artificial Intelligence Methods for Solving the Maximum Coverage Problem

The maximum coverage problem defined above leads to a high-dimensional, non-smooth, and strongly multimodal optimization landscape. In this work, these methods are treated not as competing algorithms, but as computational instruments embedded into a unified measure-based optimization methodology. The feasible domain is irregular, fragmented, and constrained by geometric restrictions, making classical deterministic optimization techniques ineffective. Gradients are unavailable, the objective function contains extensive plateaus and many disconnected attraction regions, and the search space becomes increasingly difficult to explore as the number of covering objects grows. For these reasons, metaheuristic optimization methods have become a natural choice, particularly population-based approaches that maintain a set of candidate solutions and perform global exploration in complex search spaces. Within this family, swarm intelligence algorithms are among the most widely used and most successful in practice.

In this work, we consider four representatives of this class: Particle Swarm Optimization (PSO), Fish School Search (FSS), the Firefly Algorithm (FA), and the Artificial Bee Colony method (ABC). These approaches have demonstrated strong performance in many difficult multi-extremal engineering problems and therefore serve as suitable global-search components in the proposed methodology.

To apply these metaheuristic approaches to the maximum coverage problem, we represent each candidate solution as a complete geometric configuration of all covering objects. This unified parametrization makes it possible to interpret all swarm-based methods as geometric motion models acting in the continuous space of displacements and rotations. Before describing the four algorithms in detail, we introduce this common representation and explain how it relates to the underlying geometry of the problem.

Each individual in the population, whether a particle, a fish, a firefly, or a bee, is represented by the parameter vector:

$$p^k(t) = \left(x_1^k(t),\, y_1^k(t), \theta_1^k(t), \ldots, x_n^k(t),\, y_n^k(t), \theta_n^k(t)\right),$$

where k is the index of the individual in the population (particle, fish, firefly, or bee); t is the iteration number of the algorithm; $x_i^k(t)$) and $y_i^k(t)$ are the coordinates of the pole of the i-th covering object in the configuration k; $\theta_i^k(t)$ is its rotation angle in the global coordinate system.

Every update step of a metaheuristic algorithm modifies the vector $p^k(t)$, producing coordinated translations and rotations of the covering objects inside the target region Ω.

Thus, the swarm algorithms below can be regarded as geometric motion models, each prescribing its own rules for how configurations shift, cluster, spread, or refine within the continuous parameter space.

To solve the maximum coverage problem (3) in the equivalent unconstrained formulation (4), we employ four population-based metaheuristic algorithms together with their memetic (hybrid) variants, in which global swarm search is periodically complemented by local quasi-Newton refinement using the BFGS method.

We now briefly describe the four swarm-intelligence methods used in this study, emphasizing their geometric interpretation in the context of the maximum coverage problem.

4.1 Particle Swarm Optimization (PSO)

In PSO [20, 21], each particle keeps both its current configuration $p^k(t)$ and a velocity vector $v^k(t)$ describing smooth geometric displacements of the particle k on the iteration t.

Updates follow

$$v^k(t+1) = \alpha\, v^k(t) + c_1 r_1 (p_{best}^k - p^k(t)) + $$
$$+ c_2 r_2 (p_{best} - p^k(t)),$$

$$p^k(t+1) = p^k(t) + v^k(t+1),$$

where p_{best}^k is the best configuration found by the particle k; p_{best} is the globally best configuration discovered by the swarm; the variables r_1 and r_2 are independent random numbers uniformly distributed in (0,1); the parameters α, c_1 and c_2 control, respectively, the inertial persistence of motion, attraction toward locally successful arrangements, and attraction toward the globally most effective coverage pattern.

The geometric interpretation of the update rule is that the inertia term enforces smooth, coordinated translations and rotations of the covering objects, the cognitive term pulls each configuration toward its own locally successful arrangements, and the social term attracts all particles toward the globally densest coverage geometry. From a geometric perspective, PSO rapidly forms dense coverage patterns, although stagnation may occur in highly multimodal landscapes.

4.2. Fish School Search (FSS).

FSS simulates the collective motion of a fish school [28]. Each fish corresponds to a complete coverage configuration $p^k(t)$, and its update is carried out through three coordinated mechanisms.

Individual movement. Small random exploratory shifts are generated as

$$p_{ind}^k(t) = p^k(t) + \alpha_k \eta_{ind},$$

where $p_{ind}^k(t)$ is the individual-movement configuration of fish k at iteration t; α_k is a random perturbation vector; η_{ind} specifies the maximal amplitude of the individual exploratory step.

These local displacements allow each configuration to probe weakly covered or previously unexplored portions of the target region.

Instinctive movement. The collective attraction toward better coverage configurations is calculated as

$$p_{inst}^k(t) = p_{ind}^k(t) + \frac{\sum_{k=1}^{N} \Delta p^k \Delta\varphi_k}{\sum_{k=1}^{N} \Delta\varphi_k}$$

$$\Delta p^k = p^k(t) - p_{ind}^k(t)$$

$$\Delta\varphi_k = \varphi\left(p^k(t)\right) - \varphi\left(p_{ind}^k(t)\right),$$

where $p_{inst}^k(t)$ is the configuration of fish k after instinctive movement; Δp^k is the improvement displacement; $\Delta\varphi_k$ is the corresponding improvement in coverage area defined using the objective function $\varphi(\cdot)$.

This mechanism pulls the fish school toward promising geometric regions: improving configurations become attractors, guiding others into emerging high-coverage clusters.

Volitive movement. The weighted barycentre of the school is computed as

$$B(t) = \frac{\sum_{k=1}^{N} p_{inst}^k(t) W^k(t)}{\sum_{k=1}^{n} W^k(t)}$$

and global contraction or expansion is applied via

$$p_{vol}^{k}(t) = p_{inst}^{k}(t) \pm \eta_{vol}\left(p_{inst}^{k}(t) - B(t)\right),$$

where $p_{vol}^{k}(t)$ is the configuration of fish k after volitive movement; $W^{k}(t)$ is the weight associated with the performance of fish k; η_{vol} determines the maximal volitive step size; the sign “+’’ is used when total coverage has improved (school contraction), while "–" corresponds to expansion when no improvement occurs.

If global coverage improves, the school contracts toward the barycenter, forming increasingly dense coverage zones. If no improvement occurs, the school expands to explore distant or previously uncharted regions of the feasible domain.

FSS effectively preserves population diversity and is particularly robust when navigating forbidden zones, narrow corridors, or fragmented geometric domains.

4.3 Firefly Algorithm (FA)

FA models the behavior of a population of fireflies attracted to brighter individuals [30]. In the maximum coverage problem, each firefly represents a complete configuration $p^{k}(t)$, and its “brightness’’ is proportional to the achieved coverage $\varphi\left(p^{k}(t)\right)$.

The update rule is

$$p^{k}(t+1) = p^{k}(t) + \beta\, exp\left(-\gamma d_{kj}^{2}\right)\left(p^{j}(t) - p^{k}(t)\right) + \alpha\varepsilon_{k},$$

where $p^{j}(t)$ is a brighter (higher-coverage) configuration; β is the base attraction strength; γ controls the absorption of light over distance; $\alpha\varepsilon_{k}$ is a random exploratory displacement, an $d_{kj} = \left\|p^{k}(t) - p^{j}(t)\right\|$ is the Euclidean distance in parameter space;

Brighter configurations act as geometric attractors. If two configurations are close, the attraction term dominates and pulls them together; if they are far apart, attraction weakens exponentially, allowing independent exploration.

In geometric terms, FA naturally forms clusters around local maxima of coverage: configurations flow toward bright “peaks’’ in the coverage landscape while random perturbations allow the swarm to escape shallow local optima.

FA is effective in multimodal coverage problems, smoothly transitions between clusters of good configurations, and requires minimal structural assumptions. Its main drawback is sensitivity to the absorption parameter γ, which, if poorly tuned, may lead either to premature concentration or excessively slow convergence.

4.4. Artificial Bee Colony (ABC).

ABC models the foraging behavior of honey bees searching for nectar sources [25, 26]. In the coverage problem, each nectar source corresponds to a configuration $p^{k}(t)$, and its quality is measured by the coverage $\varphi\left(p^{k}(t)\right)$.

The algorithm proceeds through three phases.

Employed-bee phase. Each employed bee explores a neighbor of its current configuration:

$$p_{new}^{k} = p^{k}(t) + \alpha_{k}\left(p^{j}(t) - p^{k}(t)\right),$$

where $p^{j}(t)$ is a randomly selected distinct configuration;

α_{k} is a random perturbation factor.

If the new configuration improves coverage, it replaces the old one.

Geometrically, this corresponds to local perturbations of the positions and orientations of covering objects, enabling refinement of dense zones.

Onlooker-bee phase. Onlookers choose configurations with probability

$$\omega_{k} = \frac{\varphi(p^{k}(t))}{\sum_{j}\varphi(p^{j}(t))},$$

giving preference to higher-coverage configurations.

Selected configurations undergo the same local perturbation as in the employed-bee phase. Thus, onlookers amplify exploration around promising geometric arrangements: dense coverage patterns attract more bees for further refinement.

Scout-bee phase. If a configuration fails to improve for a preset number of iterations (the limit parameter), it is abandoned and replaced with a random configuration:

$$p^{k}(t) = p_{min}^{k}(t) + \alpha_{k}\left(p_{max}^{k}(t) - p_{min}^{k}(t)\right).$$

Geometrically, this acts as a restart mechanism, enabling the algorithm to escape plateaus and discover entirely new regions of the feasible domain.

Thus, employed bees perform local adjustments of coverage objects; onlookers intensify refinement around good geometries; scouts jump to unexplored areas, preserving population diversity.

ABC is a simple and robust population-based algorithm that performs well in strongly multimodal search spaces and remains effective even in fragmented or irregular feasible regions. Its main limitation is relatively weak local refinement, especially compared to methods such as PSO or FSS. For this reason, hybridization with a local optimizer (e.g., BFGS) is highly beneficial and typically results in a substantial gain in solution accuracy.

4.5 Memetic (Hybrid) Approach

Memetic or hybrid optimization approaches combine global exploration capabilities of population-based swarm algorithms with the accuracy of deterministic local refinement methods. Their use is particularly justified in continuous maximum coverage problems, where the objective landscape is non-smooth, multimodal and highly irregular due to geometric overlaps, rotations and the presence of forbidden zones. While classical swarm algorithms such as PSO, FSS, ABC and FA provide efficient global search and maintain population diversity, they often lack the precision required to converge to high-quality local optima, especially when optimal configurations lie within narrow feasible basins or along steep boundaries of the target region. These limitations motivate the integration of swarm-based global search with local optimization procedures.

In the proposed framework, local refinement is applied to the penalized objective function defined in Section 3. The local optimizer operates on the same continuous objective landscape as the swarm component and does not require explicit identification of active geometric constraints. Numerical gradients are approximated using finite differences or sampling-based estimates, which is consistent with the non-smooth and measure-based nature of the coverage functional.

In the memetic framework, the global component (PSO, ABC, FSS, or FA) is responsible for exploring the search space, identifying promising regions and maintaining population diversity. When a candidate solution satisfies predefined criteria (such as stagnation of improvements, proximity to local peaks, or a sufficiently low penalty value) a local search step is activated. The local search operator is typically based on deterministic optimization techniques, such as quasi-Newton methods (e.g., BFGS), gradient-free pattern search schemes or derivative-free trust-region strategies. Among these, BFGS is widely used because of its fast local convergence and ability to operate reliably in non-smooth optimization settings when appropriately regularized.

The integration of local search into swarm algorithms can follow different strategies. In synchronous schemes, local optimization is applied to a selected subset of particles or agents at regular intervals. In asynchronous schemes, local refinement is triggered individually based on dynamic criteria such as convergence of velocity vectors, stagnation of fitness values or exceeding a predefined number of global iterations without improvement. In both cases, local search results replace the original candidate or are reinjected into the population through elitist update rules, thereby combining global and local information.

An essential aspect of memetic hybridization is the careful choice of activation frequency for the local search operator. Excessive activation can significantly increase computational cost and reduce the exploratory capabilities of the swarm, while insufficient activation may result in inadequate refinement of promising regions. Effective strategies typically rely on adaptive activation thresholds, penalty-based triggers, or measures of population diversity that ensure a balance between global exploration and local exploitation. In coverage problems involving complex geometric constraints, it is also important to ensure that the local refinement step preserves feasibility or integrates the penalty function coherently, so that the optimizer does not move into forbidden regions or produce geometrically invalid configurations.

The use of memetic swarm algorithms in continuous coverage optimization has demonstrated notable improvements in convergence speed, robustness and solution quality. By combining the global search power of swarm intelligence with the precision of local deterministic methods, memetic approaches achieve higher accuracy in locating optimal object placements, handle high-dimensional parameter spaces more effectively and reduce sensitivity to initial population distributions. This makes them particularly suitable for large-scale geometric coverage problems involving dozens or hundreds of covering objects, irregular shapes and strict placement constraints, where purely heuristic or purely deterministic methods would struggle to provide reliable results.

The four swarm algorithms and their memetic variants together form a flexible optimization toolkit capable of navigating the complex geometric landscape of the maximum coverage problem. Their practical behavior within the proposed measure-based methodological framework is examined in the next section through numerical experiments.

5. Computational Illustrations and Methodological Guidelines

This section presents computational illustrations and methodological guidelines supporting the proposed framework for continuous maximum coverage problems. The purpose is not to benchmark optimization algorithms or establish numerical superiority, but to demonstrate how the measure-based modeling approach can be operationalized across complex geometric coverage scenarios involving heterogeneous object shapes, irregular target regions, and forbidden zones, and to provide evidence-based guidance for its practical configuration.

The computational illustrations consider covering objects of different geometric types, including circles with varying radii, ellipses, and general polygons, placed within nonconvex polygonal target regions containing internal forbidden zones occupying up to approximately 15% of the total area. These geometric configurations are representative of realistic spatial constraints arising in practical coverage applications and are consistent with the modeling assumptions established in the framework of Section 2.

A central methodological aspect of this section concerns the selection of numerical parameters governing population-based optimization. Rather than tuning algorithmic hyperparameters for specific instances, the aim is to identify stable parameter ranges that ensure reliable behavior across a wide spectrum of geometric configurations. The recommended ranges for PSO, FSS, FA, and ABC are summarized in Table 2. These ranges were obtained by combining exploratory computational tests with established guidelines from the literature and are intended to serve as methodological defaults for coverage problems of comparable structure.

The reported parameter settings ensure a balance between global exploration and numerical stability in geometrically constrained environments. In particular, population sizes were chosen to scale with the number of covering objects, while interaction and exploration parameters were selected to avoid premature convergence in fragmented feasible domains. The purpose of these recommendations is not to fine-tune individual algorithms, but to establish that the proposed coverage formulation admits robust optimization behavior under broadly applicable parameter choices.

To improve solution accuracy near locally optimal configurations, all considered population-based methods are embedded into a memetic framework by periodically activating a deterministic local refinement stage. Table 3 summarizes methodological guidelines for configuring this hybridization, including recommended frequencies for activating local search, stopping criteria, and numerical strategies for approximating first-order information. Local refinement is applied selectively and adaptively, preserving global exploration while eliminating small-scale geometric inconsistencies in promising configurations.

Across all considered hybrid combinations, local refinement consistently yields modest but systematic improvements in coverage quality, typically ranging within a few percent and observed irrespective of the specific swarm mechanism used. Importantly, the role of local search in this framework is not to replace global exploration, but to complement it by resolving fine geometric adjustments that population-based dynamics alone tend to leave unrefined. This behavior supports the methodological rationale for integrating global stochastic search with deterministic local optimization in continuous coverage problems.

Table 4 provides a qualitative comparison of the considered swarm mechanisms and their hybrid extensions in terms of their typical strengths, limitations, and domains of applicability. The comparison is intentionally descriptive and focuses on structural properties rather than numerical superiority. It highlights that different swarm mechanisms are naturally suited to different geometric characteristics of the feasible domain, including multimodality, fragmentation, and the presence of narrow admissible corridors, and that this suitability can be identified from the geometric semantics of each algorithm's update rules rather than from empirical comparison alone.

Based on these structural observations, Table 5 offers practical methodological guidance for selecting an appropriate hybrid optimization strategy depending on the geometric features of the coverage problem. These recommendations are not prescriptive, but serve as a decision-support tool for practitioners when choosing a global–local optimization scheme compatible with the proposed measure-based coverage formulation. Together, Tables 2–5 form an integrated set of methodological guidelines covering parameter selection, hybridization configuration, algorithm characterization, and strategy selection across different geometric regimes.

Overall, the computational illustrations and methodological guidelines presented in this section confirm that the proposed framework yields well-posed, practically configurable optimization models for continuous maximum-coverage problems with complex geometry. The results show that combining measure-based coverage modeling with population-based global search and selective local refinement provides a flexible and scalable approach, capable of handling heterogeneous object sets, irregular target regions, and forbidden zones without relying on problem-specific algorithmic tuning.

Table 2.

Methodological parameter ranges for swarm-based algorithms

Algorithm	Key parameters	Recommended values	Interpretation
PSO	Population size	15n–20n (≤2500 particles)	Promotes trajectory diversity in high-dimensional spaces
	Inertia coefficient	0.72–0.74	Controls balance between exploration and exploitation
	Cognitive & social coefficients	1.496	Standard stability-oriented PSO setting
	Velocity control	Projection/reflection	Reduces infeasible displacements
FSS	School size	12n–18n	Supports stable collective motion
	Individual step	0.20–0.30	Local exploration amplitude
	Collective step	0.05–0.10	Regulates contraction/expansion
FA	Population size	15n–20n	Supports exploration of multimodal landscapes
	Base brightness	1.0	Attraction strength
	Absorption	0.08–0.15	Effective interaction radius
	Randomness	0.15–0.25	Stochastic exploration
ABC	Colony size	12n–18n	Balanced employed/onlooker ratio
	Random amplitude	0.1–0.3	Local stochastic perturbations
	Limit parameter	5–10 cycles	Scout activation; Facilitates escape from plateaus

Table 3.

Methodological guidelines for memetic (swarm + local search) schemes

Hybrid method	Parameter	Recommended values	Rationale
PSO+BFGS	BFGS call frequency	Every 3–5 generations ($n < 30$); 5–8 ($n \geq 50$)	Supports refinement of dense configurations and mitigates stagnation
	Stopping threshold	10^{-5}–10^{-6}	Sufficient local accuracy
	Max BFGS iterations	80–120	Trade-off between local accuracy and computational cost
FSS+BFGS	Call frequency	3–5 ($n \leq 50$); 8–12 ($n > 50$)	FSS explores complex geometry; BFGS refines
	Gradient estimation	Finite differences or Monte Carlo ($\approx 10^5$ samples)	No analytical gradients
FA+BFGS	Call frequency	4–7	Prevents disruption of FA dynamics
	Absorption γ	Decreasing near maxima	Adaptive local refinement
ABC+BFGS	Call frequency	4–8	Significantly enhances local exploitation in ABC

Table 4.

Comparative properties of swarm algorithms and hybrids

Method	Strengths	Weaknesses	Suitable cases	Key references
PSO	Fast global convergence; stable dynamics	Potential loss of diversity; stagnation risk	Regular geometry; smooth landscapes	[20, 21, 22]
FSS	Robust in non-convex and fragmented spaces	Possible chaotic motion	Polygonal regions; forbidden zones	[28, 29, 37, 38]
FA	Strong in multimodality; natural clustering	Sensitive to γ; "sticking" risk	Highly multimodal coverage	[30, 31, 38]
ABC	High diversity; effective exploration	Weak local refinement	Fragmented domains; plateau regions	[25, 26, 27]
PSO+BFGS	High accuracy; stable refinement	Higher cost	Medium/large dimension problems	[34, 35]
FSS+BFGS	High robustness in constraints	Needs tuning of steps	Non-convex or complex-topology regions	[36, 37]
FA+BFGS	Good multimodal handling	Parameter-sensitive	Many local maxima; large search space	[31, 38]
ABC+BFGS	Balanced exploration and exploitation	Slow initial phase	Constraint-heavy problems; large plateaus	[39, 40]

Table 5.

Methodological selection of hybrid optimization schemes based on problem structure

Problem structure	Key difficulty	Recommended method	Reason
Regular region	Few local maxima; quasi-smooth	PSO+BFGS	Fast convergence + accurate refinement
Forbidden zones / obstacles	Non-convexity; narrow corridors	FSS+BFGS	FSS handles topology; BFGS refines
Strong multimodality	Many local maxima	FA+BFGS	FA explores multiple peaks
Flat landscapes	Plateaus; tiny gradients	ABC+BFGS	Scouts escape plateaus; BFGS refines
High dimension ($n > 150$)	Hard exploration	ABC+BFGS or FSS+BFGS	Diversity (ABC) + robustness (FSS)
Multi-component region	Risk of trapping in one cluster	FSS+BFGS	Natural navigation across components
Mixed-scale geometry	Narrow + wide zones	FA+BFGS	Adaptive interaction radius
Low dimension ($n < 30$)	Easy local search	PSO+BFGS or FA+BFGS	Fast global + accurate local
Polygonal real maps	Irregular boundaries	FSS+BFGS	Stable movement in polygonal structures

6. Discussion

The continuous maximum coverage problem formulated in Section 3 constitutes a challenging class of high-dimensional and non-smooth geometric optimization problems. The objective function exhibits strong multimodality, extended plateau regions, and abrupt changes due to geometric intersections, making the problem unsuitable for classical deterministic optimization techniques that rely on smoothness or convexity assumptions.

The computational illustrations and methodological observations provide several important methodological insights.

First, the illustrations clearly demonstrate that the choice of global exploration mechanism plays a decisive role in navigating complex coverage landscapes. Pure swarm based algorithms typically form dense coverage patterns at early stages of optimization. However, without additional refinement mechanisms, they tend to stagnate near locally optimal geometric configurations. The observed behavior differs across swarm paradigms. Fish school based search showed the most stable exploratory dynamics in fragmented or multiply connected domains. Particle swarm optimization exhibited the fastest convergence in relatively smooth regions. Firefly based strategies proved effective in strongly multimodal settings, while bee colony search maintained robustness in landscapes with large flat regions. These observations confirm that no single global search mechanism is universally optimal across all geometric regimes.

Second, integrating local deterministic refinement into a memetic framework consistently improved solution quality across all tested geometric configurations. The incorporation of BFGS refinement led to systematic increases in coverage of approximately one to four percent under equal computational budgets.This improvement reflects the complementary roles of global and local search.Global swarm exploration identifies promising clusters of configurations, while local refinement removes small scale geometric inconsistencies and improves angular alignment of covering objects. In particular, hybrid fish school based schemes were especially robust in non convex domains with multiple forbidden zones, whereas hybrid bee colony schemes achieved the largest improvements in problems dominated by extended plateau regions.

Third, the illustrations underline the strong influence of geometric structure on optimization behavior. Irregular or polygonal target regions introduce narrow feasible corridors that restrict admissible movements of covering objects. Anisotropic shapes such as elongated polygons or ellipses amplify landscape ruggedness through orientation dependent interactions, whereas symmetric objects such as circles are easier to reposition. These geometric factors directly affect convergence speed, robustness and the effectiveness of local refinement, emphasizing that algorithmic performance cannot be separated from the underlying spatial structure of the problem.

Fourth, scalability emerges as an important practical consideration. When the number of covering objects approaches one hundred, corresponding to several hundred optimization variables, the computational cost of coverage evaluation becomes dominant, particularly for non convex shapes and intersecting polygons. Although memetic hybridization reduces the number of required global iterations, further progress will require more efficient approximation strategies, adaptive Monte Carlo sampling or surrogate assisted evaluation techniques.

Overall, the methodological analysis supports the central premise of this work. The combination of rigorous geometric modeling with population based optimization and controlled local refinement forms a flexible and effective framework for continuous maximum coverage problems of realistic scale. The results are directly relevant to applications in surveillance, sensor deployment, environmental monitoring, logistics and service accessibility, where complex geometries and operational constraints are unavoidable.

7. Conclusions

This paper develops a unified methodological framework for solving continuous maximum coverage problems involving arbitrary geometric objects, forbidden zones, and placement constraints. The problem is formulated as a high-dimensional nonlinear optimization task, and a consistent treatment of coverage evaluation is established using set measures, inclusion-exclusion approximations, computational geometry, and Monte Carlo sampling.

Several swarm intelligence methods are adapted to the geometric nature of the problem and interpreted as motion models in a continuous configuration space, where algorithmic updates correspond to coordinated translations and rotations of covering objects. This geometric interpretation provides a clear conceptual link between optimization dynamics and spatial structure and offers a principled basis for selecting algorithms based on the structural properties of the feasible domain.

A central conclusion of the study is that hybrid swarm-based optimization offers a particularly effective strategy for continuous coverage problems. Global swarm exploration enables robust navigation of highly multimodal and irregular landscapes, while local deterministic refinement improves accuracy and stability in promising regions. Across all considered geometric configurations, memetic hybridization consistently improves coverage quality under fixed computational budgets.

The main contributions of this work can be summarized as follows: a general and flexible formulation of the continuous maximum coverage problem with arbitrary shapes, forbidden zones, and geometric constraints; a unified methodological view of swarm intelligence algorithms as geometric motion processes in configuration space; and practical, evidence-based guidelines for selecting optimization strategies and parameter ranges depending on problem geometry, dimensionality, and landscape complexity.

Future research directions include surrogate-assisted coverage estimation, adaptive hybridization strategies, alternative local refinement techniques, and parallel implementations for large-scale or real-time applications. Overall, the results demonstrate that hybrid swarm intelligence is a scalable and practically viable methodological foundation for complex geometric coverage problems arising in modern engineering and computational systems.

Contribution of authors: conceptualization, methodology, formulation of tasks, justification of models, writing – review and editing – **Sergiy Yakovlev**; formulation of tasks, construction of mathematical models, analysis of results, writing – original draft preparation – **Sergiy Shekhovtsov;** computer modeling, numerical experiments, implementation, verification, visualization – **Yehor Havryliuk.**

Conflict of Interest

Author ***Sergiy Yakovlev*** is a member of the Editorial Board of this journal. He was not involved in the peer review, handling, or decision-making process for this manuscript.

All other authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship, or otherwise, that could affect the research and its results presented in this paper.

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The manuscript has no associated data.

Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence or AI-assisted technologies in the preparation, writing, or editing of this manuscript.

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МІРО-ОРІЄНТОВАНИЙ МЕТОДОЛОГІЧНИЙ ПІДХІД ДО ЗАДАЧ НЕПЕРЕРВНОГО МАКСИМАЛЬНОГО ПОКРИТТЯ: ГЕОМЕТРИЧНА ПОСТАНОВКА ТА ГІБРИДНА РОЙОВА ОПТИМІЗАЦІЯ

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Сьогодні технічні системи, що ґрунтуються на просторовому моніторингу, автономному прийнятті рішень та розподіленому сенсорному спостереженні, широко застосовуються в промисловості, енергетиці, охороні довкілля, телекомунікаціях і системах безпеки. Багато сучасних інфраструктур, зокрема сенсорні мережі, роботизовані

платформи, системи спостереження та мережі екологічного моніторингу, потребують надійного геометричного покриття заданої території за наявності експлуатаційних обмежень. Досягнення високоякісного покриття в умовах заборонених зон, нерегулярних меж та неоднорідних вимог до обслуговування становить важливу наукову й прикладну проблему. ***Метою цього дослідження*** *є розроблення міро-орієнтованого методологічного підходу до формулювання та обчислювального розв'язання задач неперервного максимального покриття із геометричними об'єктами довільної форми. Для досягнення цієї мети в роботі розглядаються формалізація задач неперервного покриття на основі мір множин і конфігураційних параметрів, побудова масштабованих методів оцінювання площ покриття з керованою точністю, а також інтеграція глобальних і локальних механізмів оптимізації як обчислювальних інструментів у межах запропонованого підходу, а не як об'єктів алгоритмічного порівняння.* ***Предметом дослідження*** *є методологічні принципи гібридної оптимізації на основі ройових методів, застосованої до високовимірних геометричних конфігурацій об'єктів покриття. Запропонована методологічна схема поєднує концепції обчислювальної геометрії, формули включень–виключень, методи Монте-Карло та популяційні методи оптимізації, які інтерпретуються як моделі геометричного руху в неперервному просторі параметрів. У роботі* ***отримано низку важливих методологічних результатів.*** *Побудовано загальну нелінійну постановку задачі неперервного максимального покриття на основі мір множин та запропоновано обчислювальні схеми для оцінювання площ покриття за різних компромісів між точністю та обчислювальною складністю. Декілька сімейств гібридних ройових методів адаптовано та інтерпретовано в межах єдиної геометричної концепції, а також розроблено обґрунтовані рекомендації щодо вибору параметрів та налаштування гібридизації. Наведені обчислювальні ілюстрації демонструють практичну застосовність запропонованого підходу та відображають типові ефекти гібридизації на якість покриття за фіксованого обчислювального бюджету.* ***Наукова новизна роботи*** *полягає у трьох взаємопов'язаних внесках. По-перше, розроблено геометрично-нейтральну міро-орієнтовану постановку задачі неперервного максимального покриття, яка одночасно охоплює довільні форми об'єктів, неопуклі цільові області, заборонені зони загального вигляду та неперервні параметри повороту в єдиній уніфікованій моделі без залежності від форми об'єктів чи проблемно-залежних геометричних декомпозицій. По-друге, встановлено єдину геометричну інтерпретацію чотирьох парадигм ройового інтелекту, в якій правила оновлення алгоритмів інтерпретуються як скоординовані жорсткі переміщення в неперервному просторі конфігурацій, що забезпечує обґрунтований геометрично-орієнтований вибір алгоритму залежно від структурних властивостей допустимої області. По-третє, сформульовано структуровану схему гібридизації для інтеграції глобального стохастичного пошуку з детермінованим локальним уточненням стосовно штрафної міро-орієнтованої функції покриття з доказовими рекомендаціями щодо частоти активації, критеріїв зупинки та апроксимації градієнта в умовах негладких геометричних цільових функцій. Запропонований підхід забезпечує гнучку та розширювану методологічну основу для моделювання покриття в інженерних системах, енергетиці та системах екологічного моніторингу, де геометрична складність, заборонені зони та обмеження на розміщення є невід'ємними характеристиками задачі..*
Ключові слова: *математична модель, неперервне максимальне покриття, міро-орієнтована методологія, ройовий інтелект, меметична оптимізація, обчислювальна геометрія, геометричне моделювання.*

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