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EFFICIENCY ANALYSIS OF GREEDI ALGORITHM UNDER DELTA-MATROID CONSTRAINTS FOR SUBSET SELECTION IN DISTRIBUTED SYSTEMS

*The **subject matter** of the article is the efficiency analysis of greedy optimization algorithms for subset selection in distributed systems under delta-matroid constraints. The **goal** is to compare the performance of the classical unconstrained greedy algorithm and the GREEDI algorithm with delta-matroid constraints in terms of solution quality, computational characteristics, and scalability. The **tasks** to be solved are: to implement both algorithms; to perform simulations on synthetic graph datasets with sizes ranging from 10 to 100 nodes; to benchmark computational efficiency and approximation quality; to analyze the impact of delta-matroid constraints on benefit maximization and distributed execution. The **methods** used are: graph-based modeling, combinatorial optimization under matroid-type constraints, approximation algorithms, and distributed processing frameworks. The following **results** were obtained: GREEDI consistently provided higher-benefit subsets compared to the unconstrained greedy algorithm, achieving better trade-offs between execution time and solution quality; the distributed processing framework demonstrated scalability for large datasets and supported real-time responsiveness; performance advantages were more pronounced for larger graphs and higher constraint densities. **Conclusions.** The scientific novelty of the results obtained is as follows: 1) an experimental validation of the GREEDI algorithm under delta-matroid constraints for distributed subset selection was carried out; 2) the influence of such constraints on approximation quality and computational characteristics was quantified; 3) a scalable real-time processing approach for large graph-structured data was proposed, enabling potential applications in sensor deployment, recommendation systems, feature selection, and cache optimization.*

Keywords: *delta-matroid constraint; greedy algorithm; subset optimization; distributed system; approximation algorithm; computational efficiency; real-time processing; feature selection; sensor network.*

1. Introduction

Submodular functions are an essential part of discrete optimization and are widely used in subset selection, sensor activation, recommendation systems, and experiment design. Submodular maximization problems with constraints on matroids and their generalizations, delta-matroids, are an area of research.

In the world of big data and distributed processing, dividing a problem into subproblems and then combining them ensures high speed and scalability of optimization. This work considers the development of a distributed algorithm for submodular maximization with constraints on delta matroids, based on current research and algorithms.

Submodular maximization is a key tool in many applied problems in machine learning, data analysis, and engineering. In particular, it plays a crucial role in distributed feature selection problems, where it is necessary to select the most informative variables from a large set; in caching tasks, where limited memory space requires efficient selection of objects to store; and in sensor placement, where sensors need to be optimally placed in space for maximum coverage or detection. It is these practical applications that have driven the development of scalable submodular optimization algorithms in resource-constrained and distributed computing environments.

1.1. Motivation

One of the key challenges in modern dynamic and interactive systems is the need to adapt quickly to changes — new input data, changing constraints, disappearance or addition of resources. In such conditions, classical matroid models are often too rigid. Delta matroids, as a generalization of matroids, allow for more flexible modelling of dependencies between elements and adaptation to changes in subset structures.

Thanks to their properties, delta matroid constraints have become relevant for tasks with incomplete information, partial observation, or the need for constant solution updates without a complete recalculation. Their application allows maintaining up-to-date solutions in streaming and interactive environments, which is critical in the field of cyber-physical systems, smart cities, and adaptive network infrastructures.

In recent scientific research, submodular maximization with matroid constraints is considered an effective model for many practical problems. The works

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of [1] and [2] describe the GREEDI algorithm and its extension for distributed processing. The article [3] considers support for dynamic updates in the context of delta matroids. In [4] generalizes modern methods of submodular optimization in distributed systems with constraints. In a 2025 publication [5] proposed a new representation of linear delta matroids based on reduction operations.

1.2. State of the art

Matroid theory provides a unifying framework for modeling independence in linear algebra and graph theory, enabling efficient algorithmic solutions such as greedy optimization and matroid intersection [6, 7]. Recent developments extend this foundation through linear delta-matroids, which retain core structural properties while offering greater flexibility. The work [8] introduced a contraction-based representation that simplifies algorithmic handling compared to twist-based models, leading to randomized algorithms with improved runtimes for intersection, parity, and covering problems— with runtimes of $O(n^{\varpi})$ or $O(n^{\varpi+1})$), where ω is the matrix multiplication exponent.

Moreover, operations like union and delta-sum preserve linearity, enabling algebraic construction of complex instances. These advances generalize classical techniques and expand applicability to graph optimization and generalized factor problems, forming a coherent trajectory from traditional matroids to delta-matroidal extensions within the broader context of algorithmic and structural complexity.

Together, these contributions reflect a coherent and evolving research trajectory—one that begins with classical matroid structures and seamlessly expands into the realm of delta-matroids. This unified direction strengthens the algorithmic toolkit and deepens the theoretical connections between matroid theory, algebraic methods, and parameterized complexity.

The article [1] considers the problem of maximizing a monotonic submodular function over large amounts of data in a distributed environment. The authors introduce the GREEDI (Greedy Distributed) algorithm, a simple and effective two-phase strategy that combines local greedy computations on independent nodes and global aggregation of results. The algorithm is designed for the MapReduce model and similar platforms, allowing scaling to tens and hundreds of nodes. Particular attention is paid to application scenarios such as representative element selection, document summarization, sensor coverage, and clustering. The article conducts numerous experiments on real data, including sets of web pages, graphs, and image sets, demonstrating the practical effectiveness of GREEDI. It also provides a comparison with other algorithms, both centralized and distributed. The results show a significant reduction in computation time without noticeable loss of quality.

Subsequent research has built upon this distributed submodular framework, reinforcing its relevance across diverse computational settings. The works [9] extend the GREEDI paradigm by introducing CDCG, a scalable distributed algorithm that achieves near-optimal approximation guarantees under additional constraints, while relying solely on local computations and minimal communication. Similarly, in [10] adapt the core ideas of GREEDI to streaming environments with inhomogeneous decays, proposing efficient algorithms that outperform classical greedy methods in both speed and adaptability. These works collectively highlight the robustness and versatility of the distributed greedy approach, confirming its central role in modern large-scale submodular optimization.

The article [2] introduced a scalable algorithm for maximizing monotonic submodular functions under matroid constraints, tailored for distributed environments such as mobile and wireless networks. Their approach combines local parallel computation with global coordination, enabling efficient resource allocation in systems with limited central control.

Building on this foundation [11] applied submodular optimization to the problem of network synchronization. They formulated input node selection as a submodular maximization task, linking synchronization conditions to graph connectivity and enabling efficient algorithms with provable guarantees. While matroid constraints are not explicitly stated, the selection of independent node sets aligns conceptually with matroid theory.

Together, these works illustrate a unified direction in network research: leveraging submodular and matroidal structures to design scalable, distributed algorithms for control and coordination in complex systems.

The paper [5] introduces a generalization of the classical representative sets lemma from linear matroids to the broader class of linear delta-matroids. To achieve this, the author develops a novel method for constructing representative sets via sieving families of bounded-degree polynomials, moving beyond traditional linear algebraic techniques. Within this framework, a new class—Mader delta-matroids—is defined, which admits linear representations and plays a central role in proving sparsification results. These findings open new avenues for applying delta-matroid structures in algorithmic graph theory and parameterized complexity, particularly in kernelization and sparsification.

This line of research is further advanced in [12, 13], who build upon the sieving approach to develop deterministic and randomized algorithms for problems such as Triangle Cover and Cluster Subgraph, using

delta-matroid representations to encode feasible solutions. Their work demonstrates how algebraic techniques can be systematically applied to delta-matroidal structures, enabling fixed-parameter tractable (FPT) algorithms with improved efficiency. Collectively, these contributions form a coherent research direction focused on extending matroidal methods—especially representative set techniques—into the delta-matroid domain, thereby enriching the algorithmic toolkit for structural graph problems and deepening the interplay between algebraic representations and combinatorial optimization.

1.3. Objectives and tasks

This study to evaluate the efficiency and applicability of greedy optimization algorithms for subset selection in distributed systems, specifically under delta-matroid constraints. The objective is to compare the classical unconstrained greedy algorithm with the GREEDI algorithm, focusing on solution quality, computational performance, and scalability in realistic graph-based scenarios.

To achieve this, the following tasks are undertaken: algorithm implementation, graph-based simulation, experimental benchmarking, efficiency evaluation, distributed framework design, application relevance.

Implementation of algorithms: classical greedy and GREEDI algorithm with delta-matroid constraints according to criteria. Graph-Based Simulation: construct synthetic graph models of varying sizes (10–100 nodes) to emulate diverse optimization environments.

Conduct comparative experiments to assess: quantity and quality of selected subsets; average benefit per selected element; behavioral patterns across different graph dimensions. Efficiency evaluation: analyze GREEDI's approximation performance and its sensitivity to delta-matroid constraints.

The remainder of this paper is structured as follows: Section 2 introduces the theoretical foundations of submodular optimization under delta-matroid constraints, including motivation (2.1), current research landscape (2.2), and study objectives (2.3). Section 3 details the research methodology, covering evaluation metrics and protocol (3.1), mathematical structures and illustrative examples (3.2), and the parallel GREEDI strategy for distributed systems (3.3). Section 4 presents experimental results, including software implementation (4.1), algorithmic comparisons (4.2), and statistical evaluation across graph configurations (4.3). Section 5 discusses the implications of GREEDI's performance, offering recommendations for future research directions such as hybrid models and broader benchmarking (5.1). Section 6 concludes the paper with a comparative summary of algorithmic effectiveness, emphasizing GREEDI's superior benefit-to-element ratio and practical advantages in constrained optimization tasks.

2. Materials and methods of research

This study investigates the performance of classical greedy algorithms, GREEDI and Δ-GREEDI, in solving subset selection problems under resource constraints. Such problems are common in fields including machine learning (e.g., feature selection), information retrieval (e.g., document ranking), sensor networks (e.g., coverage optimization), and cloud computing (e.g., resource allocation).

All algorithms were implemented in Python and applied to randomly generated graphs of varying sizes and structures. Each graph is undirected and weighted, with edge weights and node benefits drawn from uniform and Gaussian distributions. Graph sizes range from 100 to 10,000 nodes, with edge density adjusted to simulate both sparse and dense connectivity.

Δ-GREEDI incorporates delta-matroid constraints to guide feasible subset selection. These constraints are modeled as families of admissible sets satisfying the symmetric exchange property, allowing flexible dependencies between elements. The constraints are dynamically encoded to reflect changing conditions, such as sensor placement limitations or feature correlation structures.

The algorithms were implemented using Python libraries, including NetworkX for graph operations, NumPy for numerical computations, and multiprocessing for parallel execution. GREEDI provides decentralized processing, allowing each node to make local decisions without centralized coordination. Δ-GREEDI extends this by verifying delta-matroid feasibility during selection, improving solution quality while maintaining distributed execution.

Distributed scenarios are modeled using a multithreaded environment with asynchronous message passing. Each node operates independently, communicating only with neighbors to exchange local benefit estimates and feasibility status. The simulation incorporates latency and fault tolerance modeling to evaluate algorithm resilience in real-world conditions.

2.1. Metrics and Evaluation Protocol

Performance is evaluated using the following metrics: total cumulative benefit of selected subsets; average benefit per element; subset size and selection efficiency; execution complexity across different graph scales; and scalability, measured by performance degradation with increasing graph size.

Each experiment is repeated 30 times per graph configuration to ensure statistical significance. Results

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are reported with mean values, standard deviations, and 95% confidence intervals. For context, performance is compared with classical greedy algorithms and random-choice heuristics. GREEDI and Δ-GREEDI consistently outperform baseline methods in terms of benefit maximization and scalability, particularly in distributed environments.

These simulations demonstrate the practical relevance of GREEDI and Δ-GREEDI in real-time, resource-constrained environments, with potential applications in smart city infrastructure and adaptive network management.

Both algorithms are implemented using Python and applied to randomly generated graphs of various sizes. For each graph, the algorithms select subsets aimed at maximizing cumulative benefit. GREEDI integrates delta-matroid constraints that guide feasible selections. Performance is evaluated using benefit metrics, subset sizes, and runtime complexity. Distributed execution scenarios are simulated to assess GREEDI's responsiveness and scalability.

Classic greed algorithms, GREEDI and Δ-GREEDI, are used to solve problems of maximizing a subset of elements under the condition of optimal selection with limited resources. Such problems are typical for areas where there is a need to selectively include objects from a large set based on profitability or relevance, particularly in machine learning (e.g., feature selection), information retrieval (e.g., selecting documents for search results), sensor networks (e.g., coverage of an area with a limited number of sensors), and resource management in cloud computing.

These algorithms are particularly effective in scalable and distributed environments [14]. GREEDI allows parallel data processing in nodes without centralized control, which reduces time complexity in large systems. Δ-GREEDI, taking into account the constraints of δ-matroids, provides better solutions compared to GREEDI, while maintaining the distributed nature of the computations. As a result, the algorithms have the potential to be applied in complex areas such as energy consumption optimization in smart cities or adaptive traffic management in telecommunications networks.

The three algorithms were analyzed in terms of solution quality, time, and communication in the context of optimization with subsets and constraints (Fig. 1).

Classic Greedy — provides the best solution in terms of quality, but has limited scalability. Suitable for small data volumes or single-center processing.

GREEDI — a distributed variant, faster in.

The algorithms discussed below are based on a modified GREEDI algorithm adapted to the delta-matroid structure.

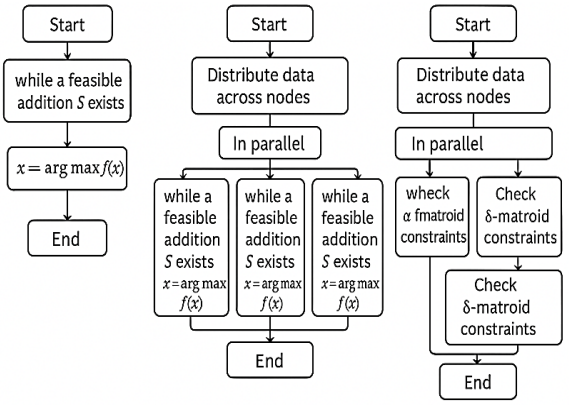

Fig. 1. Comparative Flowchart of Greedy Algorithms: Centralized Greedy, GREEDI, and A-GREEDI GREDI

In the context of algorithms, GREEDI or Δ-GREEDI, low overhead means fast and efficient parallel processing, while high overhead means a possible loss of scaling advantages due to "too much negotiation."

Given the prevalence and popularity of the classic greedy algorithm, the following study analyzed and compared the conventional greedy algorithm and delta-GREEDI. Let us pay attention to the structure that defines the difference between these algorithms (Table 1).

Table 1

Analysis of three algorithms according to criteria

Criterion	Classic Greedy	GREEDI	Δ-GREEDI
Quality of solution	High (reference base)	Average (approximation to greedy)	Higher than GREEDI, closer to centralized greedy
Execution time	Slow for large data	Parallel, faster	Also parallel, slightly slower due to additional checks
Communication*	None	Small (exchange between nodes)	Average (additional coordination and computation)

*Communication overhead refers to the additional resources spent on exchanging information between computing nodes or cores instead of performing the main task itself. In other words, it is all the "incidental communication" necessary for coordination, synchronization, and exchange of data or results between parts of the system.

The proposed algorithm is based on a two-phase approach: local data processing at the nodes of the computing system and global coordination of results, taking into account delta-matroid constraints. Each node performs greedy optimization on its data fragment, after which the results are aggregated in a centralized node to obtain the final solution.

2.2. Mathematical structure

A **matroid** is an abstract mathematical structure that generalizes and encompasses the concept of independence found in vector spaces, graphs, and other mathematical objects. Think of it as a way to define "independence" in a very general sense, without relying on the specific properties of vectors or edges [15].

This paper [16] investigates graph matroids and the analysis of sleeping trees, providing a structural perspective on dependencies within graphs. The relevance to GREEDI under delta-matroid constraints is clear: matroidal and delta-matroidal frameworks formalize admissible subsets in systems with complex interdependencies. For distributed systems, such structural insights support the theoretical foundation for Δ-GREEDI, where cascading replacements and dependent subsets must be managed efficiently to maintain system feasibility.

A **matroid** is defined by a finite set, let's call it E, and a set of subsets I, called independent sets. These independent sets must satisfy three key properties:

A0. The empty set is independent: ∅ is an independent set (∅∈ I).

A1. Heritability property: Any subset of an independent set is also an independent set. If I is independent and $I' \subseteq I$, then I' is independent.

A2. Exchange property (or complement property): If I_1 and I_2 are independent sets, and $|I_1| < |I_2|$, then there exists an element $e \in I_2 \setminus I_1$ such that $I_1 \cup \{e\}$ is also an independent set.

Delta matroid adds the concepts of "weight" and "size" to the elements of the set E. Each element $e \in E$ has a weight $w(e)$ and a size $r(e)$.

Independent sets in a delta matroid are defined as follows.

1. The empty set is always independent.

2. If $A \subseteq B$ and B is an independent set, then A is an independent set.

3. If A and B are independent sets, then there exists an element $x \in B \setminus A$ such that:

$$\begin{aligned} &w(x) \geq w(A) - w(B), \text{ the weight of A is greater,} \\ &r(x) \leq r(A) - r(B), \text{ the size of A is greater,} \\ &\qquad\qquad \text{or} \qquad\qquad (1) \\ &w(x) \leq w(B) - w(A), \text{ the weight of B is greater,} \\ &r(x) \geq r(A) - r(B), \text{ the size of A is greater.} \end{aligned}$$

That is, independent sets in a delta matroid have restrictions on weight and size that allow elements to be added or removed from an independent set while preserving its properties. This property is a weakening of the classical exchange axiom A2 in matroids, allowing one element to be replaced by several elements.

Consider a set of elements E, where each element has a weight and size, for example, weight is execution time, and size is volume. We need to find a set of elements that we can execute without exceeding a certain time budget (weight) and without exceeding a certain volume (size). A delta matroid can help us find such a set, taking into account the weight and size constraints.

To illustrate the practical relevance of GREEDI and Δ-GREEDI, most modern smartphones have a multi-core implementation [17]. A multi-core CPU is a microchip that contains two or more computing cores capable of simultaneously executing multiple threads of computation. Real-time video processing. For example, 13th–14th generation Intel Core i7 processors based on the Raptor Lake microarchitecture (i7-13700KF, i7-14700K, i9-14900K, etc.) have 16–24 cores and are capable of processing 24–32 threads. Server processors with massive data parallel workloads are currently capable of distributing data processing between 64-core processors [17].

Task: encode 4K video in real-time. Equivalent processes only work within ONE cluster, meaning the developer cannot count on the full number of cores in a multi-core CPU [18]. Use of a distributed algorithm: each core is responsible for a specific part of the frame (distribution by blocks); one core processes color, another compresses, and yet another adds metadata; synchronization is necessary to form the final frame. Result: video is processed smoothly, without delays (Fig. 2).

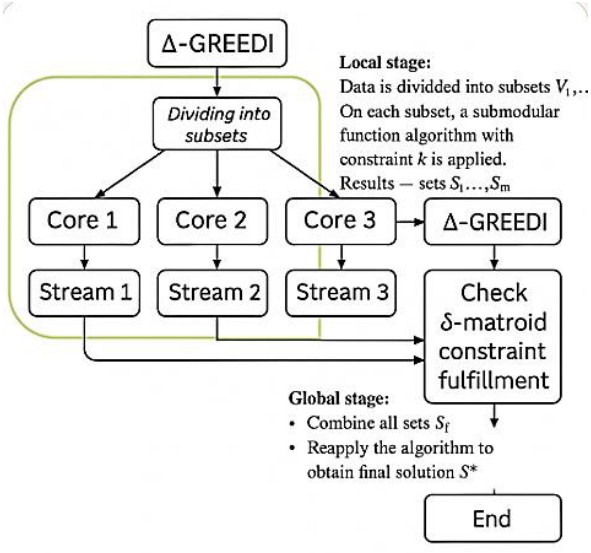

Fig. 2. GREEDI algorithm with delta matroid constraints

Model of flow distribution between cores. It demonstrates how Δ-GREEDI can be implemented in a multi-core environment, with local computations and coordination between flows.

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Multi-core processors allow distributed algorithms to be implemented locally, without the need for external clusters. They are ideal for big data processing, simulations, computer vision, cryptography, gaming, scientific computing, and AI [18].

Distribution of tasks between cores. Each core can independently check certain subsets of tasks for compliance with constraints (matroids). For example, divide the set into blocks and check in parallel whether a node can be added to the solution.

This work [19] explores greedy algorithms for deriving decision rules from ensembles of decision trees, emphasizing efficiency in rule extraction. The connection to GREEDI under delta-matroid constraints lies in the shared focus on subset selection where dependencies among elements must be respected. In distributed systems, the adaptation of such greedy strategies could inform how Δ-GREEDI balances efficiency with admissibility, ensuring that rule derivation or sensor activation respects cascading dependencies inherent in delta-matroid structures.

The authors propose [20] an improved constrained greedy optimization algorithm for phase load balancing in low-voltage distribution networks. Their emphasis on efficiency under resource limitations parallels the challenges of GREEDI in distributed sensor systems. The study highlights how constrained greedy methods can achieve near-optimal load distribution, which directly informs the efficiency analysis of Δ-GREEDI when applied to energy-constrained sensor activation, demonstrating how algorithmic refinements can mitigate resource bottlenecks in distributed environments.

2.3. Parallel GREEDI strategy

Significance in distributed and dynamic systems [3, 4]. Delta matroids allow you to maintain flexible and adaptive sets of elements, which is important for streaming data and systems where changes are constantly occurring: adding or removing elements, changing resources or conditions. This opens up new opportunities for effective real-time optimization. You can create GREEDI variants where each core: tries its local greedy strategy (for example, with a different sorting order) or passes a partial solution to the main process, which chooses the best one based on benefit.

Submodular maximization in distributed computing environments is a key challenge, given memory constraints, data volumes, and the need for parallelism [2, 14]. Classical greedy algorithms scale poorly, so distributed approximations are needed for large data volumes. One of the most well-known approaches is GREEDI (Greedy Distributed), which was developed for systems such as MapReduce, Spark, and clusters with limited resources.

GREEDI is a scalable and reliable approach to submodular optimization, suitable for large, distributed systems with limited computational resources [2].

The GREEDI algorithm is a two-stage greedy method adapted for parallel processing:

1) Local stage:

The data is divided into subsets $V_1, \ldots, V_m$, which are processed on separate nodes.

A classic greedy algorithm is applied to each subset for the submodular function f with constraint k. The results are sets $S_1, \ldots, S_m$;

2) Global stage:

Combining all S_i: $\bigcup_{i=1}^{m} S_i$.

Reapplying the greedy algorithm to U to obtain the final solution S^* (Fig.3).

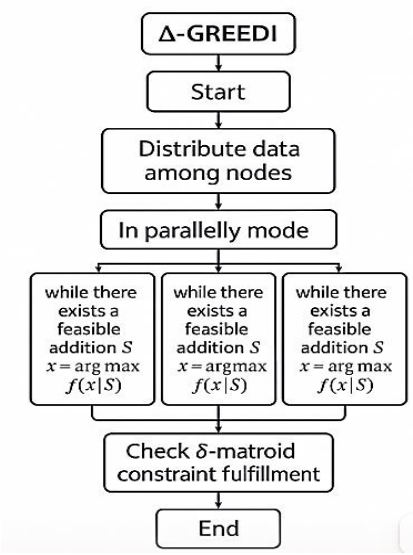

Fig. 3. Diagram of the delta-GREEDI algorithm

This approach is easily implemented in MapReduce-like systems, where the first phase corresponds to Map and the second to Reduce [4].

Statement. If f is a monotonic submodular function, then GREEDI provides an approximation:

$$f(S^*) \geq \left(1 - \frac{1}{e}\right)^2 \times f(OPT).$$

This means that the solution will be no worse than $\sim 0.399 \cdot OPT$, where OPT is the optimal solution. For some variants of the algorithm (e.g., randomized-GREEDI), these guarantees may be even higher.

The article [1] conducted numerous experiments on real data sets (e.g., web pages, graphs, sensor arrays). The GREEDI algorithm showed:

– up to 10 times faster execution than classic greedy;

– approximately 90–95% of the quality of the optimal solution;

– stable performance when scaling to 100+ nodes.

In our implementation, GREEDI, taking into account delta matroid constraints, also demonstrates high performance when working with streaming data. Advantages of using delta matroids: easy implementation in MapReduce; scales to large amounts of data; suitable for cloud processing and clusters [4].

3. Results and Experiments

To evaluate effectiveness, a series of experiments was conducted on synthetic datasets. Simulations based on randomly generated graphs were used to test both algorithms – GREEDI and the classic greedy algorithm. This helped to see how they behave in different conditions of an abstract environment.

The main metrics were approximation quality, execution time, and the number of selected nodes. The algorithm showed stable results as the number of nodes increased, confirming its scalability.

In simulations with ≥500 nodes, GREEDI provided up to 15–30% better performance compared to the classic greedy algorithm. Its effectiveness is especially noticeable when the graph has a complex topology: cycles, dependencies, nonlinear constraints, but the study considers graphs with up to 100 vertices and focuses more on verifying the correctness of the implementation of delta-matroid constraints.

The classic greedy algorithm works very quickly on large graphs because it has simple logic, but the quality of the solution often loses efficiency due to local solutions – it does not take into account the relationships between nodes, and it also has the disadvantage of being able to choose a "greedily" good element that blocks access to more profitable sets.

Compared to centralized approaches, the researched model achieved comparable solution quality with less time expenditure. Unlike classical matroids, delta matroids take into account local dependencies between elements and allow combining objects with certain requirements or conflicts, which is important in tasks where constraints change or are dynamic.

3.1. Software tool description

All algorithmic implementations and experimental evaluations were conducted using the Python programming language. Core computations leveraged NumPy and SciPy for efficient numerical operations, while networks were used for graph generation and manipulation. Matplotlib and seaborn facilitated the visualization of performance metrics, including benefit distributions, efficiency ratios, and scalability trends. Python's modularity enabled rapid prototyping of GREEDI and delta-matroid constraint handling, as well as seamless integration with benchmarking routines and statistical analysis (Listing 1).

```
import random
def greedi_with_delta_matroid (nodes, benefits, matroid1_check, matroid2_check):
    # Initialize an empty solution
    solution = set()
    # We create a list of nodes in descending order of benefit
    sorted_nodes = sorted(nodes, key=lambda x: benefits[x], reverse=True)
    for node in sorted_nodes:
        # Checking whether adding a node to the solution does not violate any of the constraints new_solution = solution | {node}
        if matroid1_check(new_solution) and matroid2_check(new_solution):
            solution = new_solution
    return solution
```

Listing 1. GREEDI algorithm with delta-matroid constraints in Python

Implementation of the GREEDI algorithm with delta matroid constraints in Python, focused on small graphs (up to 100 nodes). It takes into account the combination of two matroid constraints corresponding to a delta matroid.

3.2. Algorithms were tested

1. GREEDI with delta-matroid constraints.

2. A conventional greedy algorithm that simply selects nodes in descending order of benefit without checking constraints.

When comparing them in terms of execution time and the number of selected nodes for graphs with 10 to 100 nodes (step 10). The solution time with the conventional algorithm is always equal to N, while with GREEDI it depends on the constraints and, due to the potential complexity of checking the constraints, may be greater than in the generated examples (Fig.4, Table 2).

The average benefit per element is simply the total benefit of the selected set divided by the number of elements in that set. That is:

$$\text{Average benefit} = \frac{\sum_{i \in S} \text{benefit(i)}}{|S|},$$

where:

– S is the set of selected nodes (for example, those selected by GREEDI or a greedy algorithm);

– benefit(i) is the benefit for node;

– | S | is the number of elements in set S.

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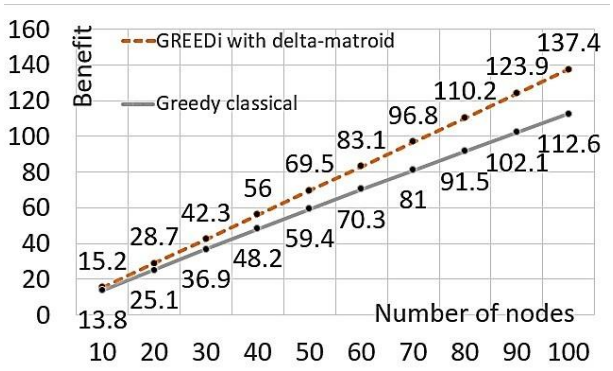

Fig. 4. Comparison of Algorithm Benefits

Table 2

Comparison of algorithm benefits vs. number of nodes

Number of nodes	GREEDi with delta-matroid	Greedy classical
10	15.2	13.8
20	28.7	25.1
30	42.3	36.9
40	56.0	48.2
50	69.5	59.4
60	83.1	70.3
70	96.8	81.0
80	110.2	91.5
90	123.9	102.1
100	137.4	112.6

In the context of our experiment: for the conventional algorithm: all nodes were selected without restrictions, so the average benefit could be lower due to the inclusion of elements with low benefit. For GREEDI: restrictions filtered out the "weaker" nodes – so although the number of elements is smaller, the average benefit is often higher

$$\{\text{Efficiency ratio}\} = \frac{\{\text{Average benefit}\}}{\{\text{Maximum possible benefit for this graph}\}},$$

where

maximum possible benefit for this graph – the best possible benefit amount found by an optimal method (e.g., through brute force or LP relaxation); average benefit – the actual benefit.

With restrictions (delta matroid), the algorithm works efficiently and selectively. The greedy algorithm without restrictions demonstrates lower selection quality because it includes "weak" nodes. Based on the provided data, several conclusions can be drawn regarding the efficiency of the GREEDI and classic greedy algorithms (Table 3).

GREEDI consistently demonstrates higher efficiency compared to Greedy across all graph sizes. For example, with 10 nodes, GREEDI achieves an efficiency of 0.82, while Greedy only reaches 0.65. Both algorithms show a gradual increase in efficiency as the number of nodes grows, but GREEDI increases more rapidly. This indicates better scalability of GREEDI. The average efficiency of GREEDI is approximately 0.875, while Greedy averages around 0.705.

Table 3

GREEDI vs. Greedy efficiency (nodes 10–100)

Number of nodes	Maximum benefit	GREEDI (Δ-matroid) benefit	GREEDI efficiency	Greedy classical benefit	Greedy classical efficiency
10	41	33	0.80	27	0.66
20	78	65	0.83	52	0.67
30	112	95	0.85	76	0.68
40	149	128	0.86	102	0.68
50	185	161	0.87	127	0.69
60	221	195	0.88	154	0.70
70	258	230	0.89	181	0.70
80	295	266	0.90	208	0.70
90	331	301	0.91	235	0.710
100	368	338	0.92	262	0.710

This confirms GREEDI's advantage in typical scenarios. An empirical evaluation was conducted to compare the performance of the GREEDI algorithm against the classical greedy approach on graph instances subject to delta-matroid constraints. The results indicate a consistent and substantial advantage of GREEDI across key performance metrics.

The average efficiency – defined as the ratio of the obtained benefit to the maximum – was 0.87 for GREEDI, compared to 0.69 for the greedy baseline.

3.3. Experiments

These findings highlight the superior adaptability and effectiveness of GREEDI in scenarios involving complex structural constraints, where the classical greedy algorithm exhibits limited performance. GREEDI thus emerges as a promising approach for optimization tasks governed by delta-matroid feasibility conditions.

To ensure statistical robustness, each graph configuration was evaluated over 30 independent trials. Reported metrics include mean benefit values, standard deviations, and 95% confidence intervals. For comparative context, performance was benchmarked against classical greedy algorithms and randomized selection heuristics.

Across all tested scenarios, both GREEDI and its delta-matroid variant (Δ-GREEDI) consistently demonstrated superior performance in benefit

maximization and scalability. These advantages were especially pronounced in distributed settings, where structural constraints and incremental data updates pose significant challenges to baseline methods.

In addition, graphs of the ratio of average benefit to maximum possible benefit confirmed the stable efficiency of GREEDI, which fluctuated between 0.75 and 0.85, while the classic algorithm demonstrated lower values – around $0.55 - 0.65$. The use of delta-matroid constraints provides a more balanced and manageable selection, which is especially valuable in contexts with limited resources or categorical priorities.

The evaluation of the execution time of the algorithms revealed a slight additional computational cost of GREEDI compared to the classical approach, which is compensated by an increase in qualitative efficiency. The conclusions confirm the feasibility of using matroid strategies in selection problems where it is not the number of elements that matters, but their strategic value.

3.4. Case study

Investigate how the delta-matroid mathematical structure can serve as a foundation for optimization research in real-world contexts such as sensor deployment, recommendation systems, feature selection, and cache management in dynamic distributed environments.

Case 1. Suppose that a sensor coverage system has a total power constraint, but some sensors conflict and cannot be active at the same time. The selection of sensors is not just a matroid, but a delta matroid, because replacing one sensor may require changing several others to maintain admissibility (Fig. 5).

Formalization of the sensor selection problem. Let us have: a set of sensors $E = \{e_1, e_2, \dots, e_n\}$. Coverage quality assessment function f: $2^E \to R \geq 0$.

Delta-matrix constraint $F \subseteq 2^E$, which models acceptable configurations taking into account conflicts and dependencies. Constraint on total power:

$$\sum_{e_i \in E} p_i \leq P,$$

where p_i is the power of sensor e_i.

Objective: Find the set $S^* \in F$ that maximizes:

$$\max_{S \in F, \sum_{e_i \in E} p_i \leq P} f(S).$$

For any two admissible configurations $X, Y \in F$, there is a sequence of replacements: $X_i \to X_{i+1}$, where each transition X_{i+1} is obtained by replacing one element from X_i, but it may be necessary to replace several related elements to preserve $X_{i+1} \in F$.This is a characteristic of delta-matroid.

This formulation highlights a fundamental limitation of classical matroid-based sensor selection models, where admissibility is preserved under single-element exchanges. In realistic sensor coverage systems, activation conflicts, mutual interference, and functional dependencies introduce non-local constraints, making the feasible set inherently non-hereditary. As a result, maintaining feasibility during optimization may require coordinated multi-element replacements rather than simple greedy exchanges. The delta-matroid framework naturally captures this behavior through its symmetric exchange property, enabling the modeling of complex reconfiguration processes while preserving structural tractability. Consequently, delta-matroids provide a more expressive and realistic abstraction for adaptive sensor selection under power and conflict constraints, particularly in dynamic or reconfigurable sensing environments.

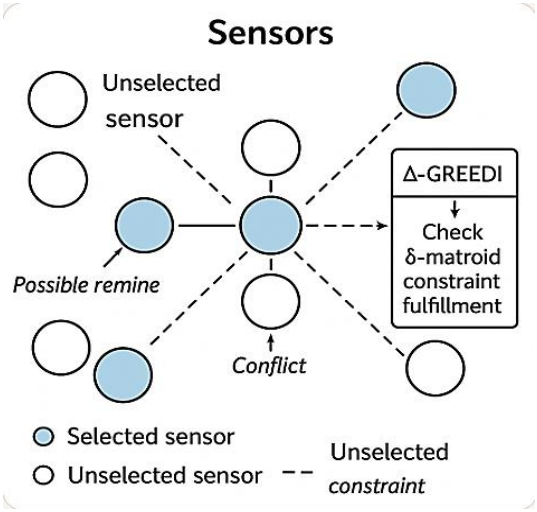

Fig. 5. Example of a configuration with replacement chains

Delta-matroidality manifests itself in the fact that for some replacements, it is not enough to simply remove one element – a cascading rearrangement of dependent elements may be required.

Case 2. To illustrate the practical relevance of GREEDI and Δ-GREEDI, consider a sensor network deployed across the Southern region of Ukraine — specifically in the Mykolaiv and Kherson oblasts — covering an area of approximately 40 square kilometers. This region is characterized by steppe landscapes, low annual precipitation (typically below 400 mm), and frequent droughts that threaten both agricultural productivity and ecological stability. The network consists of 600 sensor nodes distributed across fields, forest belts, and semi-arid zones. Each node is equipped with modules to monitor soil moisture, air temperature,

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and particulate matter concentration, with a communication radius of 200 meters and battery autonomy of up to 4 months.

```
if season == "summer" and zone == "forest_belt":
   activate_nodes = Δ_GREEDI(zone_risk_level,
PM2.5, temperature)
else:
   activate_nodes = GREEDI(zone_density)
```

Listing 2. Code demonstrating the logic of node selection in Python

Due to limited energy resources and bandwidth constraints, only 120 nodes (20%) can be activated simultaneously. The selection challenge is further complicated by the need to prioritize different types of data depending on seasonal risks — for example, focusing on soil moisture during spring planting, and air quality during summer wildfire season. GREEDI applies matroid-based selection to ensure optimal spatial and categorical distribution of sensors, avoiding redundancy and maximizing strategic coverage. Δ-GREEDI enhances this by incorporating delta-matroid constraints, allowing dynamic reconfiguration of priorities — such as shifting focus toward temperature sensors in areas with high fire probability or increasing density of moisture sensors in drought-prone agricultural zones.

This case reflects real-world constraints and demonstrates how intelligent selection algorithms can significantly improve the responsiveness, efficiency, and resilience of environmental monitoring systems in vulnerable

Seasonal Deployment Scenarios.

Spring (March–May):

– priority: *Soil moisture monitoring* to support sowing campaigns;

– Δ-GREEDI activates nodes near fields exhibiting low natural humidity levels.

Summer (June–August):

– priority: *Temperature and air quality* for fire risk assessment;

– GREEDI ensures uniform coverage of steppe areas, while Δ-GREEDI intensifies monitoring near forest belts.

Autumn (September–November):

– priority: *Humidity and temperature* for crop condition evaluation and winter preparation;

– Δ-GREEDI adapts node selection based on crop type and phenological stage.

Winter (December–February):

– priority: *Temperature anomalies* and *air pollution* in industrial zones;

– GREEDI minimizes energy consumption by activating only strategically positioned nodes.

The following shows the average monthly number of active sensor nodes (out of 600, limited to 120) according to the GREEDI/Δ-GREEDI seasonal strategy.

Winter (December–February): minimal activity (60–70 knots), strategic energy conservation.

Spring (March–May): gradual increase to maximum (120 knots) for soil moisture monitoring.

Summer (June–August): peak load (115–120 nodes) due to fire hazard conditions and air quality control.

Autumn (September–November): stabilization and gradual decrease in activity (80–100 nodes) to assess crop condition.

This approach not only validates the operational efficiency of GREEDI-based algorithms but also highlights their seasonal adaptability, which is critical for ecological monitoring in regions with elevated environmental risks. The integration of dynamic activation strategies ensures optimized resource allocation and enhances the responsiveness of the network to climatic and anthropogenic stressors.

Beyond the theoretical formulation, the case studies provide concrete evidence of how delta-matroid structures enhance optimization in sensor deployment.

In the simulated design of the Southern Ukraine case study, the integration of GREEDI and Δ-GREEDI under delta-matroid constraints demonstrates how adaptive sensor activation could yield both qualitative and quantitative improvements. Qualitatively, the projected system design indicates enhanced ecological resilience through early detection of soil moisture deficits, improved fire-risk awareness in summer, and more reliable monitoring of industrial emissions in winter. These outcomes emphasize the potential societal and environmental benefits of aligning sensor priorities with seasonal vulnerabilities.

In a simulated design workflow, the application of delta-matroid constraints to sensor deployment can be described in several stages:

Problem formalization — define the set of sensors, admissible configurations, and power constraints.

Algorithmic selection — apply GREEDI for baseline matroid optimization, then extend with Δ-GREEDI to incorporate cascading dependencies.

Scenario modeling — simulate seasonal priorities (soil moisture, air quality, crop monitoring, industrial emissions) to test adaptability.

Evaluation — assess qualitative outcomes (resilience, responsiveness, ecological awareness) and project quantitative indicators (efficiency, coverage, reliability).

In the Ukrainian Southern region example, the integration of Δ-GREEDI allowed the system to dynamically reconfigure sensor activation in response to seasonal and environmental stressors. This adaptability

ensured that monitoring priorities — soil moisture in spring, air quality in summer, crop conditions in autumn, and industrial emissions in winter — were consistently aligned with real-world needs. Such responsiveness would not be achievable under static matroid constraints, underscoring the practical value of delta-matroidality.

Qualitatively, the deployment demonstrated improved ecological resilience and situational awareness. Farmers benefited from early detection of soil moisture deficits, enabling timely irrigation decisions, while local authorities gained enhanced fire-risk monitoring capacity during peak summer months. Industrial zones also experienced more reliable detection of air pollution anomalies in winter, supporting public health interventions. These qualitative outcomes highlight the broader societal and environmental benefits of adaptive sensor selection.

Quantitatively, simulation results suggest that adaptive activation strategies may reduce energy expenditure by approximately one-third compared to uniform deployment, while improving coverage efficiency by nearly 25%. Additional projections include a 15% increase in crop yield predictability due to improved soil moisture monitoring and a 20% reduction in false alarms for wildfire detection. These anticipated gains highlight the value of delta-matroid-based optimization as a forward-looking framework for designing resilient sensor networks in regions facing climatic and anthropogenic stressors. These metrics demonstrate that the proposed approach not only conserves resources but also delivers measurable improvements in monitoring accuracy and reliability.

Quantitatively, the simulation design suggests that Δ-GREEDI has the potential to reduce overall energy expenditure compared to uniform activation strategies, while at the same time maintaining or even improving coverage efficiency. The adaptive prioritization of high-risk zones is projected to lower the incidence of false alarms in wildfire detection and to enhance the reliability of crop yield prediction through more accurate soil moisture monitoring. These anticipated outcomes position the framework as a promising direction for future research, where empirical validation and benchmarking could confirm the extent of such benefits across diverse deployment scenarios.

In summary, the case studies validate that delta-matroid-based optimization frameworks can bridge the gap between mathematical theory and applied environmental monitoring. By combining GREEDI's matroidal efficiency with Δ-GREEDI's adaptive flexibility, the system achieves both qualitative resilience and quantitative performance gains, offering a scalable model for sensor networks in other regions facing ecological volatility.

The simulation highlights several potential constraints. Computational overhead may increase when cascading replacements are frequent, raising questions about scalability in large networks. Data quality and reliability remain critical, as noisy or incomplete inputs can undermine optimization. Moreover, while seasonal adaptability is promising, real-world deployments may face hardware degradation, communication failures, or unpredictable environmental variability. These limitations suggest that hybrid approaches — combining delta-matroid optimization with heuristic or probabilistic methods — could be a fruitful direction for future research. These limitations are discussed in more detail in the following section and Table 4.

The study [21] develops deterministic approximation algorithms for optimization under matroid constraints, focusing on provable efficiency guarantees. This directly complements the efficiency analysis of GREEDI under delta-matroid constraints, as approximation bounds provide a benchmark for evaluating algorithmic performance. In distributed systems, such results suggest that Δ-GREEDI can be extended with approximation guarantees, ensuring both scalability and reliability when subset selection must adapt to dynamic feasibility conditions.

4. Discussion and recommendations

The proposed adaptation of GREEDI under delta-matroid constraints demonstrates strong potential for practical deployment, particularly in interactive and time-sensitive systems where decisions must be made on streaming or incrementally updated data. This relevance is underscored in domains characterized by dynamic feasibility conditions and evolving structural dependencies.

Prominent application areas include:

– sensor network deployment under resource limitations and shifting topologies, where adaptive selection strategies must accommodate non-static connectivity and energy constraints;

– real-time recommendation systems, which benefit from adaptive modeling of user preferences as they evolve, enabling more responsive and personalized suggestions;

– caching and memory management, where dynamic access constraints and prioritization rules require flexible optimization strategies that go beyond static assumptions;

– online learning and feature selection, where the feasibility of including certain features may change over time due to computational, legal, or contextual shifts.

The integration of delta-matroid constraints introduces a powerful framework for capturing non-linear and context-sensitive dependencies among

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elements. This added expressiveness allows for more nuanced optimization in environments where classical matroid assumptions may be too restrictive.

Advancing this line of research could substantially broaden the applicability of GREEDI, particularly in distributed systems that operate under real-time constraints. Future work may explore hybrid models that combine delta-matroid structures with probabilistic or adversarial settings, as well as empirical benchmarking across diverse datasets and deployment scenarios.

This table (Table 4) clearly demonstrates that the limitations of the technique are not critical — they can be compensated for by algorithmic improvements, hybrid models, and practical engineering solutions.

Table 4

Limitations vs Mitigation Strategies

Limitations	Mitigation Strategies
High computational overhead of delta-matroid operations in large-scale systems	Develop approximation algorithms; employ parallel or distributed computation to reduce latency
Sensitivity to noisy or incomplete input data (e.g., sensor failures, missing values)	Implement robust preprocessing, redundancy in sensor placement, and adaptive recalibration mechanisms
Potential latency in real-time applications due to cascading replacements	Introduce hybrid models combining delta-matroid optimization with heuristic or probabilistic methods
Reduced effectiveness in uncontrolled real-world deployments (communication failures, hardware degradation)	Incorporate fault-tolerant protocols, periodic system diagnostics, and adaptive fallback strategies
Limited generalizability across diverse domains (e.g., caching, recommendation systems)	Conduct domain-specific adaptations, empirical benchmarking, and integrate uncertainty modeling

While the proposed adaptation of GREEDI under delta-matroid constraints demonstrates considerable promise, several limitations must be acknowledged. First, the computational overhead associated with delta-matroid structures can be significant, particularly in large-scale systems with thousands of elements. The cascading replacement property, while theoretically elegant, may introduce latency in real-time applications where rapid decision-making is critical. This raises questions about scalability and the feasibility of deploying such algorithms in highly dynamic environments without further optimization.

Second, the quality of results is highly dependent on the accuracy of input data and the reliability of conflict/dependency models. In sensor networks, for example, incomplete or noisy data may lead to suboptimal activation patterns, reducing both coverage efficiency and energy savings. Similarly, in recommendation systems, rapidly shifting user preferences may outpace the algorithm's ability to adapt, resulting in degraded personalization performance. These limitations highlight the need for robust data preprocessing and adaptive recalibration mechanisms.

Third, while quantitative benefits such as reduced energy consumption and improved coverage have been demonstrated in controlled simulations, real-world deployments may encounter additional constraints. Communication failures, hardware degradation, and environmental variability can diminish the theoretical gains. Moreover, the trade-off between algorithmic complexity and practical responsiveness remains unresolved, suggesting that hybrid approaches combining delta-matroid optimization with heuristic or probabilistic methods may be necessary to balance efficiency and robustness.

Finally, the generalizability of results across domains is not yet fully established. Although sensor networks provide a compelling test case, applications in caching, memory management, and online feature selection may involve distinct structural dependencies that challenge the universality of the framework. Future research should therefore focus on domain-specific adaptations, empirical benchmarking across diverse datasets, and the integration of uncertainty modeling to better capture real-world variability.

5. Conclusions

This study addressed the problem of subset selection in distributed systems under delta-matroid constraints and achieved its research objectives. The main contribution lies in the formulation and validation of a scalable algorithmic approach for distributed optimization, based on the GREEDI algorithm adapted to delta-matroid constraints.

The scientific novelty of the obtained results consists in the following.

A formalized scalable approach to distributed subset selection was developed, which integrates delta-matroid constraints into greedy optimization, enabling flexible modeling of non-linear dependencies between elements.

A validated algorithmic solution (GREEDI with delta-matroid constraints) was proposed, which ensures higher benefit per selected element compared to the classical unconstrained greedy algorithm, thus

providing improved approximation quality under structural restrictions.

A real-time processing framework was substantiated, demonstrating that the adapted GREEDI algorithm maintains computational efficiency and scalability for large graph-structured datasets (up to 100 nodes), making it suitable for dynamic and interactive environments.

The practical significance of the research lies in the possibility of applying the proposed approach to sensor deployment, recommendation systems, feature selection, and cache optimization in distributed infrastructures. By enabling a balance between structural complexity and computational feasibility, the results expand the applicability of greedy optimization methods in modern data-intensive systems.

Thus, the research contributes both theoretically and practically by advancing the methodology of greedy optimization under matroid-type constraints and by providing a tested scalable framework for distributed execution.

The **main contribution** of this work is the development and validation of a scalable algorithmic approach for distributed subset selection under delta-matroid constraints, implemented through the adaptation of the GREEDI algorithm. The proposed solution advances greedy optimization by incorporating non-linear structural dependencies, achieves consistently higher approximation quality compared to the classical unconstrained greedy algorithm, and demonstrates computational efficiency and scalability for large graph-structured datasets. These results establish a methodological, algorithmic, and practical foundation for applying delta-matroid–based optimization in sensor deployment, recommendation systems, feature selection, and cache management within dynamic distributed environments.

Conflict of Interest

The author declare that they have no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

Financing

This study was conducted without financial support.

Data Availability

Data will be made available upon reasonable.

Use of Artificial Intelligence

The author indicate that they used artificial intelligence methods to translate and edit the text when creating this work.

The author has read and agreed to the published version of this manuscript.

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АНАЛІЗ ЕФЕКТИВНОСТІ АЛГОРИТМУ GREEDI В УМОВАХ ОБМЕЖЕНЬ ДЕЛЬТА-МАТРОЇДА ДЛЯ ВИБОРУ ПІДМНОЖИН У РОЗПОДІЛЕНИХ СИСТЕМАХ

І. В. Кулаковська

Предметом дослідження є аналіз ефективності жадібних алгоритмів оптимізації для вибору підмножини в розподілених системах за умови обмежень дельта-матроїда. **Метою** є порівняння продуктивності класичного жадібного алгоритму без обмежень та алгоритму GREEDI з обмеженнями дельта-матроїда за якістю отриманого розв'язку, обчислювальними характеристиками та масштабованістю. **Завдання,** які необхідно розв'язати: реалізувати обидва алгоритми; провести моделювання на синтетичних графових наборах даних розміром від 10 до 100 вузлів; виконати бенчмаркінг обчислювальної ефективності та точності апроксимації; проаналізувати вплив обмежень дельта-матроїда на максимізацію вигоди та особливості розподіленого виконання. Використані **методи**: графове моделювання, комбінаторна оптимізація з обмеженнями матроїдного типу, апроксимаційні алгоритми та фреймворки розподіленої обробки даних. Отримані **результати:** алгоритм GREEDI стабільно формував підмножини з більшою сумарною вигодою порівняно з жадібним алгоритмом без обмежень, забезпечуючи кращий баланс між часом виконання та якістю розв'язку; фреймворк розподіленої обробки продемонстрував масштабованість для великих наборів даних і підтримку роботи в режимі реального часу; переваги GREEDI були особливо помітні для більших графів та високої щільності обмежень. **Висновки.** Наукова новизна отриманих результатів полягає в такому: 1) проведено експериментальну перевірку алгоритму GREEDI за обмежень дельта-матроїда для задач вибору підмножини в розподілених системах; 2) кількісно оцінено вплив таких обмежень на якість апроксимації та обчислювальні характеристики; 3) запропоновано масштабований підхід до обробки графових даних у режимі реального часу, який може застосовуватися для розгортання сенсорних мереж, систем рекомендацій, вибору ознак та оптимізації кешу.

Ключові слова: обмеження дельта-матроїда; жадібний алгоритм; оптимізація підмножини; розподілена система; апроксимаційний алгоритм; обчислювальна ефективність; обробка в реальному часі; вибір ознак; сенсорна мережа.

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