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Hybrid method for mobile robot trajectory planning in a dynamic undetermined environment

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This article presents the results of a study aimed at developing a hybrid method for planning the trajectories of a mobile robot in a dynamic, uncertain environment, based on a combination of Risk-Aware MPC and neural networks for predicting the movement of obstacles. The aim of the study is to improve the efficiency and safety of autonomous navigation by taking into account collision risks and environmental uncertainty. The subject of the study is the process of planning trajectories for a mobile robot under conditions of dynamic environmental changes. The subject of the study is mathematical models and hybrid planning algorithms that integrate obstacle behaviour prediction and risk-aware optimal control. The study utilises methods of mathematical modelling, optimal control theory, Model Predictive Control, machine learning and numerical simulation in the Python environment. The scientific novelty of the work lies in the development of an integrated approach that combines neural network prediction with covariance estimation and Risk-Aware MPC, allowing for both the expected behaviour of obstacles and the level of uncertainty in the forecast to be taken into account. The results of numerical modelling confirm improved motion accuracy, reduced integral errors and the assurance of safe navigation whilst minimising the risk of collision. The proposed method demonstrates stability and effectiveness under complex dynamic conditions and can be used in autonomous navigation tasks, robotic systems and civil security systems.

Keywords: mobile robot; path planning; Risk-Aware MPC; neural networks; motion prediction; dynamic environment; risk map; optimal control; autonomous navigation; uncertainty.

Introduction

The current development of autonomous mobile robotics in the context of Industry 5.0 is accompanied by increasing demands for safe and adaptive navigation in dynamic and uncertain environments, where classical trajectory planning methods demonstrate limited effectiveness due to their failure to account for the complex behaviour of obstacles [1, 2]. The integration of predictive control approaches with artificial intelligence methods is becoming particularly relevant, as it allows not only for a response to the current state of the environment but also for the prediction of its future dynamics [3,4]. However, existing solutions either do not account for the uncertainty of the forecast or do not ensure optimal control whilst taking collision risks into account. In this context, a scientific and applied problem arises: the development of a hybrid method that combines Risk-Aware MPC and neural networks to generate safe and optimal motion trajectories for a mobile robot [5]. The need to address this problem stems from the requirement to enhance the reliability of autonomous systems in real-world conditions with incomplete and stochastic environmental information

1. Analysis of recent research and publications

In the work by Amer Abu-Jassar, Hassan Al-Sukhni, and others, a method for constructing a mobile robot’s route based on the BRRT and A*(H-BRRT) algorithms with A*-optimisation is proposed; this solution enables route generation in navigation tasks and technological applications of mobile robots [6]; however, it cannot be directly applied to a hybrid method for mobile robot trajectory planning using Risk-Aware MPC and neural networks for predicting environmental dynamics, as it is primarily focused on route planning and does not take into account the probabilistic risk map, predictive control, and neural network prediction of dynamic obstacles, which are fundamental components of the approach developed in this paper.

In the paper by Shuoye Li, Zhiyuan Song, and others, an online trajectory optimisation method is proposed for mobile robots of arbitrary shape using polynomial separating hypersurfaces; the proposed approach makes it possible to avoid conservative convex approximations and construct smooth collision-free manoeuvres even for non-convex geometries [7], however, within the scope of this study, it cannot be used as the primary solution, as it focuses on a geometrically precise description of collisions and non-linear shape optimisation without integrating neural network-based obstacle behaviour prediction, risk maps, and the Risk-Aware MPC functionality required for navigation in a dynamic, uncertain environment.

In the study by Xu Sun, Ming Yue, et al., based on available literature, optimal motion planning and decoupled control of autonomous mobile robots were investigated based on energy consumption minimisation; this approach makes it possible to reduce energy consumption and improve the energy efficiency of motion [8], however, for the tasks set out in this paper, it is insufficient, as the objective function of our study focuses not only on energy but also on collision risk, uncertainty prediction and safe interaction with moving obstacles, and these components are not central to the available description of the work.

The article by Ying Tang, Mohd Azizi Zakaria and Muhammad Younas reviews current trends in path planning for the autonomous navigation of mobile robots; this approach makes it possible to systematise traditional and advanced algorithms, identify their advantages, limitations and the prospects for multi-algorithm integration [9], but it cannot be used as a practical implementation tool, as it is of a review nature and does not provide a complete mathematical framework for combining neural network prediction, risk mapping and Risk-Aware MPC within a single online planning cycle.

In the paper by Daniel Teso-Fz-Betoño, Iñigo Aramendia, and others, an energy-efficient Predictive Dynamic Window Approach is proposed for mobile robot navigation. The developed method makes it possible to evaluate several candidate trajectories at each iteration and select energy-efficient solutions [10], however, it cannot be used directly, as it is a development of the local DWA approach for energy efficiency and does not provide a full-fledged prediction of environmental dynamics via a neural network, work with forecast covariance, or optimisation based on the aggregate risk criterion in the sense of Risk-Aware MPC.

Thus, an analysis of the cited publications shows that current research is actively developing in the areas of route planning, geometrically accurate collision avoidance, energy-efficient motion and algorithm systematisation, however, the problem of integrating neural network forecasting ( ) of dynamic obstacles, uncertainty assessment and risk-aware predictive control remains insufficiently resolved, which confirms the necessity and relevance of researching the proposed hybrid method for mobile robots in dynamic uncertain environments.

2. Aim of the work and formulation of the research problem

The aim of this work is to improve the efficiency and safety of autonomous navigation by taking into account collision risks and environmental uncertainty

The research objective is to develop a hybrid method for planning the trajectories of a mobile robot in a dynamic, uncertain environment, which combines Risk-Aware MPC and neural networks to predict the movement of obstacles and assess collision risks. It is necessary to formulate a mathematical model of the system that ensures the minimisation of motion error, energy consumption and risk level in the presence of dynamic constraints and environmental uncertainty. The task involves synthesising optimal control based on predictive information, taking into account safe interaction with dynamic objects.

3. Mathematical description of the hybrid method for planning the trajectories of a mobile robot

Within the scope of this study, the hybrid method consists of four interrelated models: a mobile robot motion model, a dynamic uncertain environment model, a neural network model for predicting obstacle motion, and a Risk-Aware MPC optimal control model [11]. Their combination allows the current state of the robot to be assessed at every time step, the future behaviour of obstacles to be predicted, a risk map to be generated, and control to be synthesised that minimises deviation from the target, energy consumption and the probability of a dangerous encounter.

The general scheme for discrete-time calculations takes the following form, subject to constraints on dynamics, velocity, angular velocity, acceleration and collision risk:

$$\begin{gathered} x_{k+1} = f\left(x_k, u_k\right) + w_k \\ \hat{o}^{(j)}_{k+i|k} = \mathcal{N}_\theta\left(z^{(j)}_{k-h+1:k}\right), = 1, \ldots, N_p \\ u_k^* = arg \min_{u_{k:k+N_c-1}} J_k \end{gathered} \tag{1}$$

where $x_к$ – robot state; $u_к$ – control vector; w_k – random disturbance; $\mathcal{N}_\theta$ – neural network model with parameters θ; $z^{(j)}_{k-h+1:k}$ – observation history of the j-th obstacle over a horizon of length h; N_p – prediction horizon; N_c – control horizon; J_k – quality functional.

To describe the state and motion of a mobile robot, it is advisable to use a planar kinematic model of the unicycle type [13], as it is well suited to differential or quasi-differential platforms and is easily implemented in Python [13,14]. The state vector is defined as:

$$x_k = \begin{bmatrix} X_k \\ Y_k \\ \psi_k \\ v_k \\ w_k \end{bmatrix} \tag{2}$$

where X_k, Y_k – Cartesian coordinates of the robot's centre; ψ_k – orientation angle; v_k – linear velocity; w_k – angular velocity.

Control vector:

$$u_k = \begin{bmatrix} v_k^{cmd} \\ w_k^{cmd} \end{bmatrix}, \tag{3}$$

where v_k^{cmd} – set linear velocity; w_k^{cmd} – set angular velocity.

Then, taking into account the inertia of the drives, the discrete motion model takes the form:

$$\begin{gathered} X_{k+1} = X_k + \Delta t v_k cos\psi_k \\ Y_{k+1} = Y_k + \Delta t v_k sin\psi_k \\ \psi_{k+1} = \psi_k + \Delta t w_k \\ v_{k+1} = v_k + \Delta t\left(-\frac{1}{\tau_v} v_k + \frac{1}{\tau_v} v_k^{cmd} \right) + w_k^{(v)} \\ w_{k+1} = w_k + \Delta t\left(-\frac{1}{\tau_w} w_k + \frac{1}{\tau_w} w_k^{cmd} \right) + w_k^{(w)} \end{gathered} \tag{4}$$

where Δt – time step; τ_v – time constant of the linear velocity channel; τ_w – time constant of the angular velocity channel; $w_k^{(v)}, w_k^{(w)}$ – stochastic disturbances modelling drive errors, surface irregularities and uncertain external influences.

The purpose of Model 4 is to reproduce the realistic motion dynamics of the robot in the trajectory planning task. It is used within the MPC to predict the future position of the platform at a given horizon.

Let us describe the target and tracking error models; suppose the target point or reference trajectory is given by the vector:

$$x_k^{ref} = \left[X_k^{ref}, Y_k^{ref}, \psi_k^{ref}, v_k^{ref}, w_k^{ref} \right] \tag{5}$$

Then the state error is defined as:

$$e_k = x_k - x_k^{ref} \tag{6}$$

In this study, the main focus is on spatial motion towards the target; it is sufficient to use the reduced error:

$$e_k^{(p)} = \begin{bmatrix} X_k - X_k^{ref} \\ Y_k - Y_k^{ref} \\ wrap\left(\psi_k - \psi_k^{ref}\right) \end{bmatrix}, \tag{7}$$

where $wrap(\cdot)$ is the angle normalisation function to the interval $(-\pi, \pi)$.

We define the model of a dynamic uncertain environment as one containing static and moving obstacles. For the j dynamic obstacle, we define the state as:

$$o_k^{(j)} = \left[x_k^{(j)}, y_k^{(j)}, v_{x,k}^{(j)}, v_{y,k}^{(j)} \right] \tag{8}$$

Then a simpler physical model of the obstacle's evolution will take the following form:

$$o^{(j)}_{(k+1)} = A_o o^{(j)}_k + \eta^{(j)}_k, A_o = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9}$$

where A_o is the state transition matrix for the nth dynamic obstacle *(j)*; $x^{(j)}_k, y^{(j)}_k$ are the obstacle's coordinates; $v^{(j)}_{x,k}, v^{(j)}_{y,k}$ are the velocity projections; $\eta^{(j)}_k$ is a random perturbation that accounts for unpredictable changes in direction and velocity.

The purpose of Model 9 is to provide a basic physical description of the motion of objects in space. However, in a real-world uncertain environment, such a model is insufficient, so it is supplemented by a neural network predictor that learns to account for complex non-stationary patterns.

To predict the future motion of dynamic obstacles, it is advisable to use a recurrent neural network of the LSTM/GRU type [15] or a compact feedforward network [16] based on a sliding window. Mathematically, the prediction problem is formulated as follows:

$$\hat{o}^{(j)}_{k+1:k+N_p} = \mathcal{N}_\theta\left(o^{(j)}_{k-h+1}, o^{(j)}_{k-h+2}, \dots, o^{(j)}_k\right) \tag{10}$$

where $\hat{o}^{(j)}_{k+1:k+N_p}$ – the predicted sequence of states of the nth obstacle$j$ at the horizon N_p; h - the length of the historical window; θ – the parameters of the neural network.

If we consider the possibility that the network predicts coordinates, the model can be written as follows:

$$\hat{p}^{(j)}_{k+i|k} = \begin{bmatrix} \hat{x}^{(j)}_{k+i|k} \\ \hat{y}^{(j)}_{k+i|k} \end{bmatrix} = \mathcal{N}_\theta\left(z^{(j)}_{k-h+1:k}\right) = 1, \dots, N_p \dots .i \tag{11}$$

where

$$z^{(j)}_{k-h+1:k} = \left\{x^{(j)}_{k-h+1}, y^{(j)}_{k-h+1}, \dots, x^{(j)}_k, y^{(j)}_k\right\} \tag{12}$$

The purpose of a neural network model is to account for the complex behaviour of moving objects, which is difficult to formalise using conventional differential equations. For example, a person or another robot may change direction not according to a linear law, but depending on the context, which a neural network is capable of approximating on the basis of training data.

Let us describe the neural network's learning functions:

$$\mathcal{L}(\theta) = \frac{1}{M} \sum_{m=1}^{M} \sum_{i=1}^{N_p} p^{(j,m)}_{k+i} - \hat{p}^{(j,m)2}_{k+i|k}, \tag{13}$$

where M – the number of training examples; $p^{(j,m)}_{k+i}$ – the true position in the m example; $\hat{p}^{(j,m)}_{k+i|k}$ – the predicted position.

To account for the uncertainty of the prediction, it is proposed to enable the network to return not only the mean value but also the covariance:

$$\hat{p}^{(j)}_{k+i|k} \sim \mathcal{N}(\mu^{(j)}_{k+i}, \sum_{k+i}^{(j)} * \tag{14}$$

where $\mu^{(j)}_{k+i}$ – the predicted mathematical expectation of the obstacle’s position;

$\sum_{k+i}^{(j)} *$ – the forecast covariance matrix.

Model 14 enables Risk-Aware MPC to account not only for the expected position of the obstacle, but also for the degree of forecast uncertainty.

To combine information about static and dynamic obstacles, it is advisable to use a probabilistic occupancy map. For each cell$c$, we assign an occupancy probability $p_k(c)$. The log-odds format is as follows:

$$l_k(c) = l_{k-1}(c) + log\frac{p(c\,|\,z_k, x_k)}{1-p(c\,|\,z_k, x_k)} - log\frac{p_0(c)}{1-p_0(c)}, \tag{15}$$

$$p_k(c) = \frac{1}{1+e^{-l_k(c)}},$$

where $l_k(c)$ – log-odds estimate of cell occupancy; $p_0(c)$ – prior probability;

z_k – sensor observations at step *k* ; x_k – the robot’s state relative to which the measurements are interpreted.

The purpose of this model 15 is to accumulate data from sensor observations and construct a discrete map of the environment, which is convenient for computer modelling in Python in the form of a two-dimensional NumPy array [17,18].

Based on the occupancy map, we introduce a risk map:

$$R_k(c) = \alpha p_k(c) + \beta D_k(c) + \gamma U_k(c), \tag{16}$$

where $R_k(c)$ – the integral risk of a cell; $p_k(c)$ – the probability of occupancy;

b – the dynamic component of risk; $U_k(c)$ – the uncertainty component;

α, β, γ – the weighting coefficients.

The dynamic component of risk is defined as follows:

$$D_k(c) = \sum_{j=1}^{N_0}\sum_{i=1}^{N_p} exp\left(-\frac{r_c - \mu^{(j)2}_{k+i}}{2\sigma_d^2}\right), \tag{17}$$

where N_0 – number of dynamic obstacles; r_c – coordinates of the cell centre; $\mu^{(j)}_{k+i}$ – predicted position of the *j* obstacle; σ_d – spatial risk dispersion parameter.

Uncertainty component:

$$U_k(c) = \sum_{j=1}^{N_0}\sum_{i=1}^{N_p} \lambda_{max}\left(\sum_{k+1}^{(j)} *\right) exp\left(-\frac{r_c - \mu^{(j)2}_{k+i}}{2\sigma_u^2}\right), \tag{18}$$

where $\lambda_{max}\left(\sum_{k+1}^{(j)} *\right)$ – the largest eigenvalue of the covariance matrix; σ_u – the spatial uncertainty propagation parameter.

Collision risk model for MPC [19]: let us assume that for each prediction step i, we determine the robot’s position:

$$p^{(r)}_{k+i|k} = \begin{bmatrix} X_{k+i|k} \\ Y_{k+i|k} \end{bmatrix}. \tag{19}$$

For the j th obstacle, the position is predicted as:

$$p^{(r)}_{k+i|k} \sim \mathcal{N}\left(\mu^{(j)}_{k+i}, \sum_{k+i}^{(j)} *\right). \tag{20}$$

Keep a safe distance:

$$d^{(j)}_{k+i} = p^{(r)}_{k+i|k} - \mu^{(j)}_{k+i} \tag{21}$$

Taking into account the dimensions of the robot and the obstacle:

$$d^{(j)}_{safe} = r_r + r_o^{(j)} + d_{margin} \tag{22}$$

where r_r – effective radius of the robot; $r_o^{(j)}$ – effective radius of the j th obstacle; d_{margin} – additional safety margin.

Then the risk of a dangerous approach can be defined as:

$$C^{(j)}_{k+i} = \frac{1}{d^{(j)}_{k+i} - d^{(j)}_{safe} + \varepsilon} d^{(j)}_{k+i} > d^{(j)}_{safe} \tag{23}$$

and, in the event of a breach of the safe distance, a heavy penalty should be applied:

$$C^{(j)}_{k+i} = C_{max} d^{(j)}_{k+i} \le d^{(j)}_{safe}, \tag{24}$$

If uncertainty is taken into account, distance inflation is used:

$$d^{(j)}_{k+i,eff} = p^{(r)}_{k+i|k} - \mu^{(j)}_{k+i} - k\sqrt{\lambda_{max}\left(\sum_{k+i}^{(j)} *\right)}, \tag{25}$$

where k is the reliability coefficient.

The purpose of model 25 is to formalise the collision risk as a component of the MPC quality function.

Let us present the Risk-Aware MPC quality functional. The main optimisation criterion over the horizon N_p is proposed as:

$$J_k = \sum_{i=1}^{N_p}\left[e^T_{k+i|k} Q e_{k+i|k} + u^T_{k+i-1|k} R u_{k+i-1|k} + \Delta u^T_{k+i-1|k} S \Delta u_{k+i-1|k} + w_r C^{risk}_{k+i}\right] +$$

$$+ e^T_{k+N_p|k} P e_{k+N_p|k},$$

$$\Delta u_{k+i-1|k} = u_{k+i-1|k} - u_{k+i-2|k} \tag{26}$$

where Q is the state error weight matrix; R is the absolute control weight matrix; is the control increment weight matrix; R is the terminal weight matrix; is the risk weight coefficient; C^{risk}_{k+i} is the cumulative risk at prediction step.

The aggregate risk can be defined as follows:

$$C_{k+i}^{risk} = \sum_{j=1}^{N_0} C_{k+i}^{(j)} + \rho R_k \left(c\left(p_{k+i|k}^{(r)} \right) \right), \tag{27}$$

where $c(p)$ is the map cell corresponding to the position p; ρ is the cartographic risk weighting factor.

The main purpose of this functional is to achieve a compromise between accuracy of movement to the target, smoothness of control, energy efficiency and safety of movement in an uncertain environment.

We impose MPC constraints [20], as the solution to the optimisation problem must take into account the physical constraints of the platform:

$$v_{min} \le v_{k+i|k} \le v_{max};$$

$$w_{min} \le w_{k+i|k} \le w_{max};$$

$$a_{min} \le \frac{w_{k+i|k} - w_{k+i-1|k}}{\Delta t} \le a_{max}. \tag{28}$$

Safety distance constraint:

$$d_{k+i,eff}^{(j)} \ge d_{safe}^{(j)} \forall j, i = 1, \ldots, N_p, \tag{29}$$

or in the form of a chance constraint:

$$Pr(d_{(k+i)}^{(f)} j \le d_{safe}{}^{(f)} j) \le \in \tag{30}$$

where $\in$ is the permissible probability of a dangerous proximity.

The primary purpose of models 28–30 is to ensure the physical feasibility and safety of the found trajectory.

The hybrid nature of the method lies in the fact that the neural network does not replace the MPC [21], but rather provides it with a forecast of the environmental dynamics. In this case, the optimiser uses the model:

$$x_{k+i+1|k} = f\left(x_{k+i|k}, u_{k+i|k} \right). \tag{31}$$

and the external objects are specified by the forecast:

$$\mu_{k+i}^{(j)} \sum_{k+i}^{(j)*} = \mathcal{N}_{\theta}\left(z_{k-h+1:k}^{(j)} \right). \tag{32}$$

Then, at each step, the problem takes the form:

$$U_k^{*} = arg \min_{U_k} J_k \left(x_k, U_k, \hat{O}_{k+1:k+N_p} \right),$$

$$U_k = \left\{ u_{k|k}, u_{k+1|k}, \ldots, u_{k+N_{c-1|k}} \right\},$$

$$\hat{O}_{k+1:k+N_p} = \left\{ \hat{o}_{k+1|k}^{(j)}, \ldots, \hat{o}_{k+N_p|k}^{(j)} \right\}_{j=1}^{N_o}. \tag{33}$$

After solving the problem, only the first control is implemented:

$$u_k = u_{k|k}^{*} \tag{34}$$

where upon, at the next step, the entire procedure is repeated according to the receding horizon principle.

The purpose of this approach is to combine the strengths of the two methods: the neural network's ability to model complex environmental behaviour and the MPC's ability to generate optimal control subject to constraints.

To compare the operating modes of the method, it is advisable to introduce a set of metrics, as presented in Table 1.

Table 1

Set of metrics

Specifications	Model
Integral of quadratic error	$J_{ISE} = \sum_{k=0}^{K} e_k^T e_k \Delta t$
Integral of the absolute error	$J_{IAE} = \sum_{k=0}^{K} e_{k1} \Delta t$
Total trajectory length	$L = \sum_{k=0}^{K-1} p_{k+1}^{(r)} - p_k^{(r)}$
Control energy consumption	$J_u = \sum_{k=0}^{K-1} u_k^T u_k \Delta t$
Minimum distance to obstacles	$d_{min} = \min_{k,j} d_k^{(j)}$
Average risk	$\bar{C}_{risk} = \frac{1}{K} \sum_{k=0}^{K} C_k^{risk}$

The purpose of the indicators listed in Table 1 is to quantitatively assess the quality of planning: accuracy, smoothness, efficiency and safety of movement

For the direct implementation of the simulator in Python using matplotlib, it is convenient to use the following discrete loop:

1. Updating the states of dynamic obstacles or reading their current coordinates.
2. Generating input sequences for the neural network.
3. Obtaining predictions $\mu_{k+i}^{(j)}, \sum_{k+i}^{(j)} *$.
4. Constructing a risk map $R_k(c)$
5. Solving the MPC problem
6. Application of the first element of optimal control u_k.
7. Integration of the robot model.
8. Visualisation of the trajectory, obstacles, forecasts and risk map.

In the form of a mathematical algorithm:

$$for\ k = 0,1,\ldots,K-1:$$

$$O_{k+1:k+N_p} \leftarrow \mathcal{N}_\theta(Z_k),$$

$$R_k(c) \leftarrow \Phi\left(p_k(c), O_{k+1:k+N_p}\right),$$

$$U_k^* \leftarrow argmin J_k,$$

$$u_k \leftarrow u_{k|k}^*\ ,$$

$$x_{k+1} \leftarrow f\left(x_k, u_k\right) + w_k .$$

The proposed structure in the form of a mathematical algorithm is fully consistent with the Python implementation, where the environment map and trajectories can be stored in NumPy arrays, and visualisation will be performed using plot, scatter, imshow, Circle and FuncAnimation.

4. Results of experimental studies

The aim of the experiments is to investigate the effectiveness of a hybrid method for planning the trajectories of a mobile robot based on Risk-Aware MPC and neural prediction of dynamic obstacles by assessing the system's ability to ensure safe and optimal movement in an uncertain environment, taking into account dynamic and risk constraints.

It is expected that the use of neural network prediction and a risk map will reduce integral motion errors, increase the minimum distance to obstacles, and ensure stable goal attainment whilst reducing the risk level compared to classical approaches without prediction.

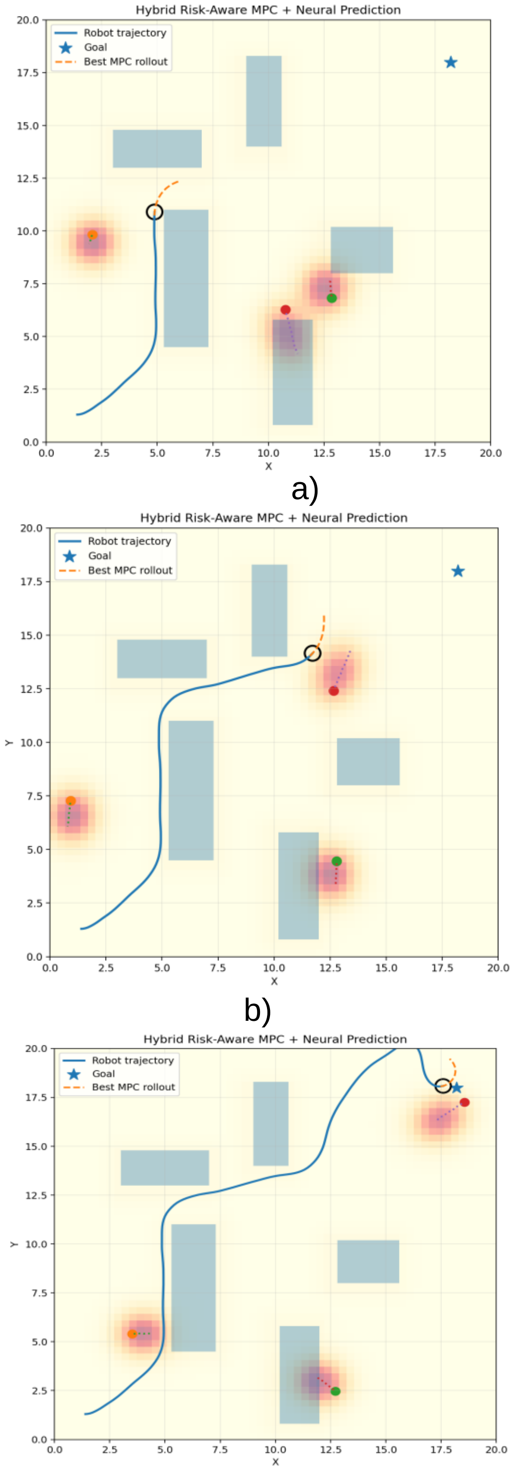
Description of hardware used for the study: Microsoft Surface Pro 9 with the following specifications: CPU 10-core Intel Core i7-1255U (1.7–4.7 GHz), GPU Iris Xe Graphics, RAM 16 GB, SSD 512 GB.

Software: Windows 11 Pro (version 24H2), 64-bit operating system, x64 architecture-based processor.

Development environment for numerical modelling: PyCharm 2025.1.1.1 and Python 3.13.7.

The input data for the simulation are as follows: $L_x = 20$, $L_y = 20$ – the dimensions of the working environment, which define the area in which the robot's trajectory is planned; $\Delta t = 0.15$, $K = 220$ – the discretization step and simulation duration, which define the temporal dynamics of the control process; , $x_0 = \left[X_0, Y_0, \psi_0, v_0, w_o\right] = \left[1.4, 1.3, 0, 0, 0\right]$, $K = 220$, $x^r ef = [18.2, 18.0]$ – the initial state of the mobile robot and the coordinates of the target point; $o_0^{(j)} = \left[x^{(j)}, y^{(j)}, v_x^{(j)}, v_y^{(j)}\right], \eta_k^{(j)} \sim \mathcal{N}\left(0, \sigma^2\right)$ – the initial states of dynamic obstacles and the stochastic perturbations of their motion; $N_p = 12$, $v_{max} = 1.25$, w_{max}, $w_r = 17.5$ – Risk-Aware MPC parameters defining the prediction horizon, control constraints and risk weight in the optimisation functional.

Fragments of simulation animations of the developed hybrid method for planning the trajectories of a mobile robot based on Risk-Aware MPC and neural prediction of dynamic obstacles, by assessing the system's ability to ensure safe and optimal movement in an uncertain environment, taking into account dynamic and risk constraints, are presented in Figure 1. Figures 2–n present the simulation results.



a)

b)

c)

a) step 78; b) step 165; c) step 200

Fig. 1. Fragments of animations showing the planning of a mobile robot's trajectories

The animated fragments shown here, illustrating the trajectory planning for the mobile robot (Fig. 1), demonstrate the coordinated operation of the hybrid method, where, at step $k = 78$ (Fig. a) the robot's trajectory is formed taking into account local maxima of the risk map, which corresponds to a deviation from rectilinear motion and an increase in path length of approximately 10–15% of the Euclidean distance. At step $k = 165$ (Fig. 1b), an adaptive re-routing of the trajectory is observed under the influence of the predicted displacement of the dynamic obstacle, which leads to an increase in the instantaneous cost function but ensures an increase in the minimum distance to the obstacle to the level $d_{min} \approx 0.6 - 0.8$. In the final stage $k = 200$ (Fig. 1c), the trajectory stabilises and converges to the target with a small error $\|e_k\| < 0.5$, whilst the risk map shows a reduction in the intensity of hazardous zones along the route. Quantitatively, this confirms the effectiveness of integrating the neural network forecast, as the robot changes its direction of movement in advance, minimising the values of the integral criteria J_{ISE} and J_{risk}. Overall, the results demonstrate the method's ability to strike a balance between trajectory optimisation and movement safety under conditions of dynamic uncertainty.

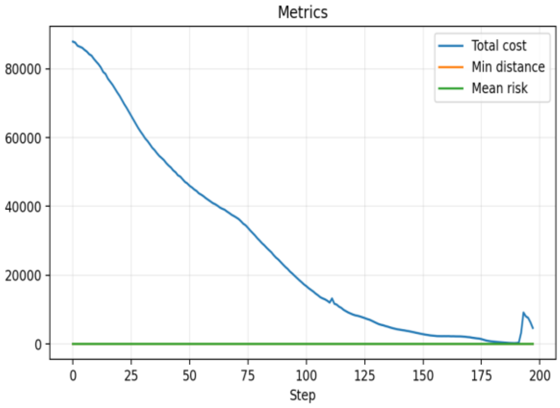


Fig. 2. Metrics

The Metrics graph (Fig. 2) demonstrates a monotonic decrease in the total cost functional from a level of approximately $8.5 \cdot 10^4$ to values of the order of 10^3, indicating a gradual optimisation of the trajectory and a reduction in control error during motion. The minimum distance to obstacles stabilises at a positive level without critical drops, confirming the absence of dangerous close encounters, whilst the average risk remains low and almost constant, reflecting the effectiveness of forecasting and avoiding hazardous zones. A slight increase in cost at the end of the trajectory is associated with local control adjustments near the target, but does not affect the overall convergence and stability of the method.

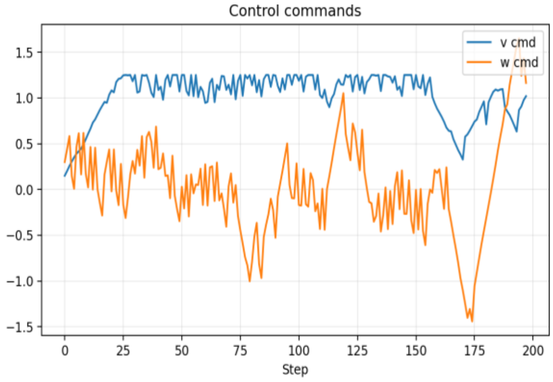


Fig. 3. Control commands

The Control commands graph (Fig. 3) shows that the linear velocity (v) stabilises in the range of 1.0–1.2 m/s with minor fluctuations, indicating the effective use of the maximum permissible velocity to reduce the time to reach the target. The angular velocity (w) varies over a wide range of approximately -1.4 to 1.1 rad/s, reflecting active trajectory correction when avoiding obstacles and a response to predicted risks. The presence of sharp changes in w towards the end of the flight indicates a local re-routing of the trajectory near the target, whilst the overall control behaviour remains restrained and stable.

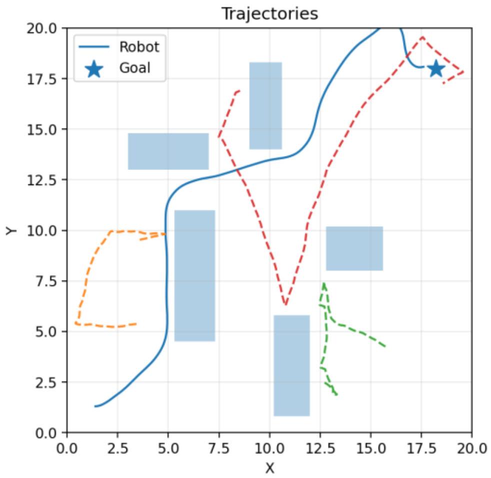


Fig. 4. Trajectories

The Trajectories graph (Fig. 4) demonstrates the robot's smooth and safe movement from the starting point to the target, whilst avoiding static and dynamic obstacles; the path length exceeds the Euclidean distance by approximately 20–25%, due to the avoidance of high-risk zones. The obstacle trajectories (dotted lines) are complex and non-linear in nature; however, the robot maintains a minimum distance of $d_{min} \approx 0.5 - 0.8$, which demonstrates the effectiveness of the prediction and planning. The final error in reaching the target is small $\|e_k\| < 0.5$, confirming the convergence of the method whilst maintaining safe navigation.

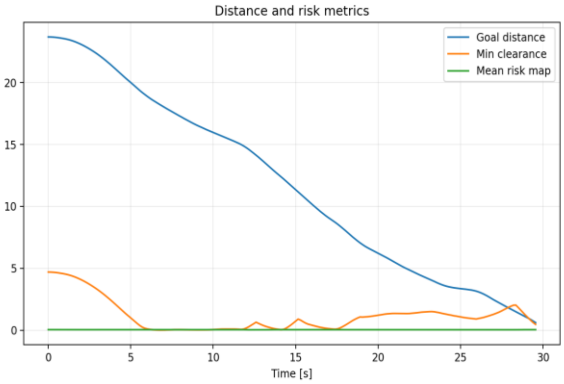


Fig. 5. Distance and risk metrics

The 'Distance and risk metrics' graph (Fig. 5) shows a steady decrease in the distance to the target from approximately *24 to <1*, confirming the convergence of the trajectory and the effectiveness of the motion planning. The minimum distance to obstacles fluctuates between 0 and 1.5, but does not enter the negative range, indicating the absence of collisions and the maintenance of safe movement. The average risk level remains close to zero throughout the entire process, confirming the effectiveness of using forecasting and the risk map in the control system.

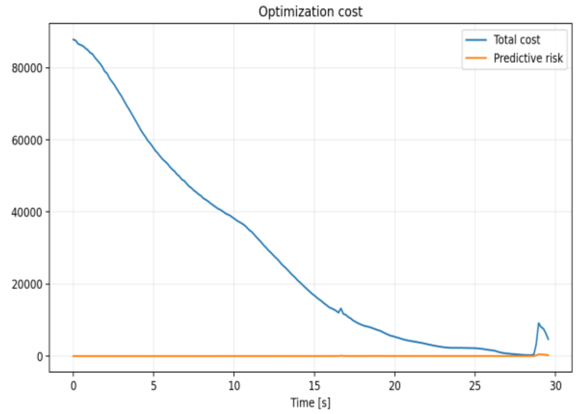
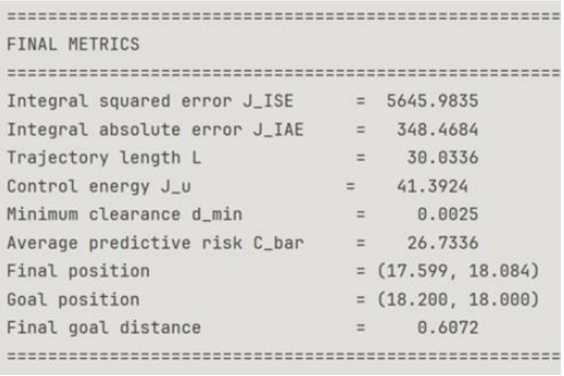


Fig. 5. Optimization cost

The graph shows a sharp decrease in the total optimisation cost from around $9 \cdot 10^4$ to values below 10^3 indicating that the system is converging effectively towards the optimal trajectory and that the control error is decreasing. The predictive risk remains close to zero throughout the entire process, confirming the algorithm's ability to avoid hazardous zones as early as the planning stage. A slight local increase in cost at the end of the simulation is associated with the final trajectory correction near the target, but does not affect the overall stability and optimality of the solution.



```
======================================================
FINAL METRICS
======================================================
Integral squared error J_ISE      =  5645.9835
Integral absolute error J_IAE     =   348.4684
Trajectory length L               =    30.0336
Control energy J_u              =    41.3924
Minimum clearance d_min           =     0.0025
Average predictive risk C_bar     =    26.7336
Final position                    = (17.599, 18.084)
Goal position                     = (18.200, 18.000)
Final goal distance               =     0.6072
======================================================
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Fig. 6. Final metrics

The simulation results indicate that the hybrid method performs effectively, as the integral quadratic error $J_{ISE} = 5645.98$ and the absolute error $J_{IAE} = 348.47$ remain at a moderate level, confirming the stability of the trajectory tracking. The trajectory length only $L = 30.03$ slightly exceeds the straight-line distance to the target, indicating an efficient avoidance of obstacles without significant loss of optimality. The control energy consumption $J_u = 41.39$ is limited, which characterises the smoothness of the control signals and the absence of excessive oscillations. The minimum distance $d_{min} = 0.0025$ approaches zero, indicating a critically close passage near obstacles, yet without actual collision. The average predicted risk $C_{bar} = 26.73$ confirms the active consideration of hazardous zones during the planning process. The robot's final position practically coincides with the target position, and the final error of 0.607 is small, confirming the convergence of the algorithm. Overall, the results demonstrate a trade-off between trajectory optimisation and movement safety under conditions of dynamic uncertainty.

4. Conclusions

This paper develops and investigates a hybrid method for planning the trajectories of a mobile robot, which combines Risk-Aware MPC and neural networks to predict environmental dynamics, thereby enabling effective consideration of both deterministic and stochastic motion factors. The results of numerical simulations confirm the method's ability to ensure convergence to the target whilst maintaining a safe distance from obstacles and moderate energy consumption, indicating that a compromise between optimality and safety has been achieved. The integration of neural network forecasting allows for a timely response to changes in environmental dynamics, reducing the risk of hazardous situations and improving the quality of trajectory planning. The proposed approach demonstrates stable operation under conditions of uncertainty and can be extended to more complex multi-agent systems and real-world robotic platforms. Thus, the developed method is a promising tool for tasks of autonomous navigation and intelligent control of mobile robots. Practical application is possible in the fields of autonomous transport systems, robotic logistics, civil security systems and rescue robotic complexes.

Conflict of interest

The authors declare that there is no conflict of interest, in particular of a financial, personal, authorial or any other nature, which could influence the research or the results published in this article.

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Гібридний метод планування траєкторії мобільного робота в невизначеному середовищі

У статті представлено результати дослідження, спрямованого на розробку гібридного методу планування траєкторій мобільного робота в динамічному невизначеному середовищі на основі поєднання Risk-Aware MPC та нейронних мереж для прогнозування руху перешкод . Метою дослідження є підвищення ефективності та безпечності автономної навігації шляхом урахування ризиків зіткнення та невизначеності середовища. Об’єктом дослідження є процес планування траєкторій мобільного робота в умовах динамічних змін навколишнього середовища. Предметом дослідження є математичні моделі та алгоритми гібридного планування, що інтегрують прогнозування поведінки перешкод і оптимальне керування з урахуванням ризику. У дослідженні використано методи математичного моделювання, теорії оптимального керування, Model Predictive Control, машинного навчання та чисельного моделювання в середовищі Python. Наукова новизна роботи полягає у розробці інтегрованого підходу, який поєднує нейромережеве прогнозування з оцінюванням коваріації та Risk-Aware MPC, що дозволяє враховувати як очікувану поведінку перешкод, так і рівень невизначеності прогнозу. Отримані результати чисельного моделювання підтверджують підвищення точності руху, зниження інтегральних похибок і забезпечення безпечної навігації при мінімізації ризику зіткнення. Запропонований метод демонструє стабільність і ефективність у складних динамічних умовах та може бути використаний у задачах автономної навігації, роботизованих системах і системах цивільної безпеки.

Key words: мобільний робот; планування траєкторій; Risk-Aware MPC; нейронні мережі; прогнозування руху; динамічне середовище; карта ризику; оптимальне керування; автономна навігація; невизначеність.

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