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## **Development of a model for constructing the optimal trajectory of the gripping device of a collaborative robot-manipulator taking into account the influence of the cargo mass and energy consumption**

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In the current conditions of the development of Industry 5.0, collaborative robotic systems play a key role in increasing the efficiency of production processes, ensuring flexibility and safety of interaction with humans. One of the most critical elements of such systems is the gripping device of a collaborative robot-manipulator, which performs precise and energy-dependent actions in a complex dynamic environment. Given the constant change in the mass of the transported cargo and the limited energy resources, there is an urgent need to develop an optimal trajectory of movement that takes into account not only geometric constraints, but also the energy feasibility of the manipulator's movement. The relevance of the study is due to the need to reduce energy consumption when performing tasks of gripping and transporting objects in the presence of spatial obstacles, which is important for increasing the autonomy and productivity of robotic systems.

The object of the study is the process of spatial movement of the gripping device of a collaborative robot-manipulator. The subject of the study is the optimization of the trajectory of movement taking into account dynamic constraints, variable cargo mass and energy consumption. The methods of mathematical modeling, numerical integration, energy analysis and spatial visualization of trajectories were used in the study. The basis for the formalization of the trajectory construction process is a system of motion dynamics equations, on which optimality conditions are imposed, taking into account energy consumption and avoidance of collisions with obstacles. The purpose of the study is to build a mathematical model and implement an algorithm for forming the optimal trajectory of the gripping device in 3D space, which allows minimizing energy consumption when transporting objects, taking into account their mass and existing obstacles. The results of the study include the construction of the optimal trajectory under given spatial constraints, substantiation of its effectiveness based on comparison with variable trajectories, as well as numerical confirmation of the reduction of energy consumption during the movement. The resulting model demonstrates the potential for implementation in collaborative robot control systems in real-time conditions.

Based on the developed mathematical model, further research using the Pontryagin maximum principle in continuous time is recommended to develop analytical solutions and improve the control system for more complex trajectory planning problems under variable loads and complex obstacles.

**Key words:** collaborative robot, gripping device, optimal trajectory, energy consumption, load mass, motion modeling, Pontryagin principle, manipulator, environmental obstacles, adaptive control, numerical modeling, energy efficiency, robotic system.

### **Introduction**

In the current conditions of rapid development of industrial technologies, within the framework of the Industry 5.0 concept, the creation of robotic systems capable of flexible, energy-efficient and safe interaction with humans is becoming increasingly relevant [1-3]. Collaborative robot-manipulators, unlike traditional automated industrial systems, must work in a common space with the operator, adapting to variable loads, complex dynamic conditions and high requirements for safety and energy efficiency [4-7]. This necessitates the development of new mathematical models and control

methods that allow not only to form motion trajectories with given parameters, but also to ensure their optimality according to a number of criteria, in particular, minimizing energy consumption and taking into account the influence of external factors, such as the mass of the load [8, 9]. Such approaches acquire particular importance in the context of cyber-physical production systems, where the robot manipulator must not simply perform predetermined actions, but make dynamic decisions in real time based on sensor data, forecasts, and interaction with other production elements [10-13]. The construction of the optimal trajectory of the gripping device in such conditions turns into a complex task that requires the use of modern mathematical modeling and optimal control equipment. The application of the Pontryagin maximum principle allows us to formulate the optimization problem in continuous time, taking into account the physical constraints of the system, load parameters, and energy characteristics of the drive [14, 15]. The results of the study have significant practical significance for increasing the efficiency of the functioning of collaborative robots in various areas – from precision assembly to medical and service applications. Thus, the development of a model for constructing the optimal trajectory of the gripping device of a collaborative robot manipulator, taking into account the influence of the load mass and energy consumption, is a relevant and necessary area of research that meets the challenges of Industry 5.0 and contributes to the integration of intelligent systems into new generation production processes.

## 1. Related works

In the work of Mart, L., a combinational tactile device for the implementation of object transfer between a human and a robot is proposed, which allows to improve the physical interaction at the moment of gripping and takes into account the load and safety parameters [16]. This solution can be useful in modeling contact at the moment of gripping the load, but it does not cover the dynamics of motion and energy consumption.

In the study of Ciupe, V., a pneumatic actuator is developed for the automation of a manual process, where the load and effort parameters are important [17]. These approaches can partially complement the modeling of energy consumption, but do not take into account the optimality of the trajectory.

In the work of Heuer, C., the concept of automated assembly of heavy elements with the participation of a collaborative crane and robots is presented [18], which demonstrates the need to take into account the mass of the object and its effect on the movement, but does not focus on the formalization of the trajectory in continuous time.

Kermenov, R. developed a method for constructing near-optimal trajectories based on ISO standards for joint tissue transport with a human [19]. This study is directly relevant because it takes into account motion constraints, path optimization, and speed, and can be adapted to gripping operations.

Soudani, M. conducted a comparative analysis between industrial and collaborative robots in terms of accuracy, safety, and energy saving [20], which supports the need for energy-efficient models, although no specific methods for trajectory optimization are proposed.

Brian, W. used Franka Emika as a haptic device to assess the response of load and action in real time [21], which can be integrated as feedback in the trajectory model.

Terras, N.'s study considers the integration of deep learning-based computer vision systems for real-time control [22], which allows for object, shape, and mass

estimation, useful for modeling variable mass loads.

Vu, M.'s paper proposes an online trajectory replanning method for capturing irregularly shaped objects [23], which directly addresses the dynamic trajectory optimization problem, although it does not consider energy consumption.

Zhang, B.'s paper describes the implementation of a shape, mass, and restricted access-aware robotic tomato picking system [24], focusing on energy efficiency, which can be used to analyze such manipulations.

In the publication of Alabbas, Z., neural network models for adaptive control in prostheses are created [25], which, although not a manipulator in the classical sense, allows for adapting movements based on feedback - a principle that can be used in the construction of optimal control.

In the work of Yu, J., a strategy for adaptive control of a space robot with separation of dynamics is proposed, which takes into account the change in the mass of the object and is applied in variable conditions [26], which most closely corresponds to the topic of the study both in terms of dynamics modeling and energy control.

In general, the analysis shows that the topic of optimal control of the trajectory of the gripping device taking into account energy consumption and cargo mass is relevant in the context of the development of collaborative robotics. Existing approaches either focus on tactile interaction or consider the adaptation of shapes and positions, but do not formally combine mass dynamics, energy consumption and trajectory in continuous time. This emphasizes the need for further research in the direction of developing holistic mathematical models that take into account all the mentioned factors, as well as software implementation of real-time planning and adaptation algorithms to ensure effective and safe operation of collaborative manipulators in variable conditions.

**The aim of the study** is to build a mathematical model and implement an algorithm for forming the optimal trajectory of the gripping device in 3D space, which allows minimizing energy costs when transporting objects, taking into account their mass and existing obstacles.

## 2. Development of a mathematical model for constructing the optimal trajectory of the gripping device of a collaborative robot-manipulator using the Pontryagin maximum principle in continuous time

Let's formalize the problems, let the robot manipulator have  $n=3$  links (degrees of freedom), then the state of the system at time  $t$  can be described by the following state vector:

$$\mathbf{x}(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \dot{\theta}_3(t) \end{bmatrix} \in \mathbb{R}^6 \quad (1)$$

where:  $\theta_i(t)$  – rotation angle of the  $i$ -th joint (rad);

$\dot{\theta}_i(t)$  – angular velocity of the  $i$ -th joint (rad/s).

The control actions for each link can be described by the following expression:

$$\mathbf{u}(t) = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \\ \tau_3(t) \end{bmatrix} \in \mathbb{R}^3 \quad (2)$$

where:  $\tau_i(t)$  – moment of force (control signal) for the  $i$ -th drive (N m).

The dynamics of the manipulator, taking into account the mass of the cargo  $m_L$ , will be presented in the form of system dynamics using modifications of the Euler-Lagrange model in the following form [27,28]:

$$\mathbf{M}(\theta)\ddot{\theta} + \mathbf{C}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta, m_L) = \mathbf{u} \quad (3)$$

where:  $\theta_i(t)$  – rotation angle of the  $i$ -th joint (rad);

$\dot{\theta} \in \mathbb{R}^3$  – angular velocity vector, first derivatives of generalized coordinates;

$\mathbf{M}(\theta) \in \mathbb{R}^{3 \times 3}$  – inertia matrix (dimensional 3x3). Depends on the current position of the links  $\theta$  and takes into account the mass, geometric characteristics of the robot and the mass of the load  $m_L$ . This matrix describes how changing accelerations in the links affect the dynamic forces;

$\mathbf{C}(\theta, \dot{\theta}) \in \mathbb{R}^{3 \times 3}$  – the matrix of Coriolis and centrifugal forces takes into account nonlinear forces that arise when links rotate at certain speeds and models the interaction between the moving parts of the manipulator during movement;

$\mathbf{G}(\theta, m_L) \in \mathbb{R}^3$  – vector of gravitational moments, depends on the position of the links and the mass of the load  $m_L$ , this vector determines the forces that must be applied to compensate for the influence of gravity at the current position of the robot and the load;

$\mathbf{u}$  – vector of control actions, moments (forces) applied to each actuator/motor of the manipulator, these controlled vectors ensure the implementation of a given trajectory or behavior.

Expression 3 describes the full dynamic model of the manipulator in the space of generalized coordinates taking into account the mass of the load, gravitational influence, inertia and internal dynamic interactions. It is the basis for building an optimal or adaptive control system, in particular for trajectory planning, energy-efficient control and ensuring safe interaction with a person in the conditions of Industry 5.0.

The representation of the manipulator model in a state representation is necessary for formalizing and simplifying the analysis of system dynamics, which allows for the effective use of modern methods of automatic control, in particular optimal, adaptive or robust. The state representation provides convenience in numerical modeling in MATLAB/Simulink environments and allows you to convert a system of second-order differential equations to a first-order system, which is a standard for control algorithms [29]. This approach simplifies the construction of observers, filters and diagnostic systems, allowing you to more accurately take into account changes in dynamics, for example, when changing the mass of the load or the configuration of the robot. The state form within the framework of these studies has the following form:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), m_L) = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \mathbf{M}^{-1}(\theta)(\mathbf{u} - \mathbf{C}(\theta, \dot{\theta})\dot{\theta} - \mathbf{G}(\theta, m_L)) \end{bmatrix} \quad (4)$$

where:  $\mathbf{M}^{-1}(\theta)(\mathbf{u} - \mathbf{C}(\theta, \dot{\theta})\dot{\theta} - \mathbf{G}(\theta, m_L))$  – the second-order component (angular acceleration), which is obtained from the equation of motion and determines the change in velocity according to the dynamics of the system.

The proposed state form (4) allows us to describe the full dynamics of the manipulator as a system of first-order equations, which is necessary for the construction and implementation of optimal or adaptive control algorithms, as well as for the analysis of stability and energy consumption in the process of movement.

According to the purpose of the study, to minimize the total energy consumption for the movement period  $[0, T]$ , to approach the given final configuration  $x_f$ , to ensure smoothness of movement using the following optimization function:

$$J = \int_0^T \left( \frac{1}{2} \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) + \frac{1}{2} (\mathbf{x}(t) - \mathbf{x}_f)^T \mathbf{Q} (\mathbf{x}(t) - \mathbf{x}_f) \right) dt \quad (5)$$

where:  $J$  – a scalar quantity that characterizes the total "cost" or quality of a trajectory over the time of movement from the initial moment to the final time  $T$ ;

$\mathbf{u}(t)$  – vector of control actions (moments or forces) applied to the manipulator links at time  $t$ , it directly affects the energy consumption of the system;

$\mathbf{R} > 0$  – is a positive definite weight matrix that sets the "price" of control actions. The larger the values in  $\mathbf{R}$ , the large values of  $\mathbf{u}(t)$  are penalized, the system will strive for energy-saving control;

$\mathbf{x}_f$  – vector of the desired (target) state that the system should achieve, this could be, for example, the position and velocity at the endpoint of the trajectory;

$\mathbf{Q} > 0$  – a positive definite weight matrix that determines the importance of the accuracy of reaching the target state, it sets the priority for different components of the state vector - for example, positions may be more important than velocity, or vice versa;

$(\mathbf{x}(t) - \mathbf{x}_f)^T \mathbf{Q} (\mathbf{x}(t) - \mathbf{x}_f)$  – a quadratic expression that estimates the deviation from the target state at time  $t$  taking into account the weight coefficients of the matrix  $\mathbf{Q}$ .

The proposed expression 5 is classical in optimal control problems, in particular in the linear-quadratic (LQR) approach, and allows us to formally find a balance between the energy efficiency of the motion and the accuracy of achieving the desired configuration of the manipulator [30,31].

We introduce the vector of conjugate variables (costate vector), which is used in optimal control to formalize the problem according to the Pontryagin maximum principle, its purpose is to evaluate the sensitivity of the optimization functional  $J$  to changes in the dynamics of the system, it plays the role of "shadow prices" or "cost gradients" with respect to each state of the system:

$$\boldsymbol{\lambda}(t) = [\lambda_1(t) \dots \lambda_6(t)]^T \quad (6)$$

where:  $\lambda_i(t)$  – is responsible for the corresponding coordinate of the state vector  $x_i(t)$  and reflects how much a change in this coordinate at a specific point in time affects the total cost (quality criterion) of the system.

Thus, the lambda vector is a mathematical tool that connects the current state of the system, control actions, and the optimal path in time space necessary to construct a control action that minimizes a given functional.

Then the Hamiltonian will have the following form:

$$\mathcal{H}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}, t) = \frac{1}{2} \mathbf{u}^T \mathbf{R} \mathbf{u} + \frac{1}{2} (\mathbf{x} - \mathbf{x}_f)^T \mathbf{Q} (\mathbf{x} - \mathbf{x}_f) + \boldsymbol{\lambda}^T \mathbf{f}(\mathbf{x}, \mathbf{u}, m_L) \quad (7)$$

where:  $\mathbf{x}$  – the system state vector, which contains generalized coordinates (for example, link angles) and their derivatives (angular velocities), determines the current state of the manipulator;

$\mathbf{u}$  – vector of control actions (moments) that affect the movement of the manipulator, the optimal values of this vector are determined from the conditions for minimization of the Hamiltonian;

$\boldsymbol{\lambda}$  – a vector of conjugate variables (costate vector), which estimates the

sensitivity of the functional to state changes and is used to construct optimal control equations;

$t$  – time, is an independent variable that describes the development of the process over time;

$R$  – the matrix of weights of control actions, symmetric and positive definite, it reflects the importance of minimizing energy costs or control efforts;

$Q$  – the state deviation weight matrix, symmetric and positive semidefinite, determines how important it is to reduce the deviation of the current state from the desired one  $x_f$ ;

$x_f$  – the desired (target) state of the system to which the manipulator should strive;

$f(x, u, m_L)$  – a function that describes the dynamics of the system in the form of a state equation, where  $m_L$  is the mass of the load that affects the motion of the manipulator through inertial characteristics.

Thus, Hamiltonian (7) combines energy costs, accuracy of goal achievement and internal dynamics of the system, serving as the basis for constructing optimal control in the problem of minimizing the quality functional.

Let us describe the necessary optimality conditions:

- state dynamics, is the basis for formulating optimality conditions, since it determines how the state of the system changes under the action of control influences in time. Without it, it is impossible to construct a conjugate system of equations and Hamiltonian, which are necessary for finding the optimal control trajectory according to the Pontryagin maximum principle:

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial \lambda} = f(x, u, m_L) \quad (8)$$

where:  $\dot{x}$  – the derivative of the state vector with respect to time, which describes how the state of the system changes, i.e. the rate of change of positions and velocities of the manipulator links;

$\frac{\partial \mathcal{H}}{\partial \lambda}$  – the partial derivative of the Hamiltonian with respect to conjugate variables, which, under the conditions of the Pontryagin maximum principle, is equal to the state function of the system; it determines the state dynamics based on optimal control;

$f(x, u, m_L)$  – a vector state function that describes the physical model of the system's behavior in terms of its dynamics: position and velocity change according to control actions  $u$ , the current state  $x$ , and the mass of the cargo  $m_L$ ;

$x$  – state vector, which includes coordinates (angular positions) and their derivatives (angular velocities) of the manipulator links;

$u$  – vector of control actions, i.e. moments of forces applied to the manipulator actuators;

$m_L$  – the mass of the load, which is taken into account in the model as a parameter that affects the dynamics of the system and changes its inertial properties.

Expression (8) formalizes how the system responds to control influences taking into account the mass of the cargo when constructing the optimal trajectory.

- conjugate dynamics is necessary to determine the change in the vector of conjugate variables in time, which correspond to the sensitivity of the quality criterion to changes in the state of the system, it is a component of the optimality conditions according to the Pontryagin maximum principle and allows you to find such control

actions that minimize the given functional. Through conjugate dynamics, the connection between the optimization criterion and the dynamics of the system is implemented, ensuring the achievement of the optimal result within the given constraints.

$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial x} = -\mathbf{Q}(x - x_f) - \left(\frac{\partial f}{\partial x}\right)^T \lambda \quad (9)$$

where:  $x(t)$  – system state vector at the current time;

$x_f$  – the desired (target) state of the system that needs to be achieved;

$\mathbf{Q}$  – the matrix of weights of the functional, which determines the importance of the accuracy of achieving the target state, deviations  $x - x_f$ ;

$f(x, u, m_L)$  – state dynamics of the system, which depends on the current state, control actions and load mass;

$\frac{\partial f}{\partial x}$  – the matrix of partial derivatives of the dynamics of the system over states (jacobian), which shows how a change in state affects the change in the rate of the state;

$\left(\frac{\partial f}{\partial x}\right)^T \lambda$  – the gradient part, which describes the inverse effect of state changes on the vector of conjugate variables.

Expression (9) defines how the vector of conjugate variables changes over time based on the current state of the system, deviations from the target state, and the structure of the system dynamics.

- optimal control, necessary to find such a control effect that minimizes the given quality functional, taking into account physical constraints and system dynamics. It determines the best trajectory of the gripping device in continuous time, ensuring a balance between energy consumption and accuracy of achieving the target position, in the context of the Pontryagin maximum principle, optimal control is a key element for constructing extremum conditions:

$$\frac{\partial \mathcal{H}}{\partial u} = \mathbf{R}u + \left(\frac{\partial f}{\partial u}\right)^T \lambda = 0 \Rightarrow u^*(t) = -\mathbf{R}^{-1} \left(\frac{\partial f}{\partial u}\right)^T \lambda(t) \quad (10)$$

where:  $u(t)$  – vector of control actions (moments or forces) applied to the manipulator links at a given time  $t$ , this is the control that needs to be found to minimize the quality functionality;

$u^*(t)$  – optimal control satisfying the condition of minimum of the Hamiltonian according to Pontryagin's maximum principle;

$\mathbf{R}$  – weight matrix of dimension  $n \times n$ , which determines the weight of energy costs in the quality functional; a symmetric, positive definite matrix that penalizes large control actions;

$\mathbf{R}^{-1}$  – inverse matrix to  $\mathbf{R}$ , is used to calculate the analytical expression of optimal control;

$f(x, u, m_L)$  – the vector state function describing the dynamics of the system taking into account the influence of the load mass  $m_L$ , depends on the state  $x(t)$ , the control  $u(t)$  and the load mass;

$\left(\frac{\partial f}{\partial u}\right)^T$  – transposed matrix of the partial derivative of the dynamics function under control, used in the expression for calculating the influence of control on the Hamiltonian;

$\lambda(t)$  – vector of conjugate variables (costate variables), which is a solution to conjugate dynamics and reflects the sensitivity of the functional to changes in the state of the system over time.

The vector of conjugate variables (10), which is the solution of conjugate dynamics and reflects the sensitivity of the functional to changes in the state of the system over time.

We know the constraints on control and states, they are necessary to ensure the physical realism and safety of the functioning of the collaborative robot-manipulator in real-world conditions, which allow preventing the exceeding of permissible moments and angular velocities, which can lead to damage to the drive mechanisms or a decrease in accuracy, as well as constraints important for avoiding collisions with people or objects.

- constraints on moments:

$$|\tau_i(t)| \leq \tau_{i,max} \quad (11)$$

where:  $\tau_i(t)$  – control torque applied to the  $i$ - th actuator at time  $t$ ;

$\tau_{i,max}$  – the maximum permissible torque value that can be realized without overloading or damaging the drive.

- angular velocity restrictions:

$$|\dot{\theta}_i(t)| \leq \dot{\theta}_{i,max} \quad (12)$$

where:  $\dot{\theta}_i(t)$  – instantaneous angular velocity of the  $i$ - th link at time  $t$ ;

$\dot{\theta}_{i,max}$  – a limit value that ensures safe, stable and controlled operation of the drive without overheating, vibrations or loss of positioning accuracy.

- collision avoidance, forms constraints on control and states for constructing a safe trajectory of movement of the collaborative robot's gripping device is to ensure the avoidance of collisions with obstacles or elements of the environment. To implement this requirement, it is advisable to include special tools in the mathematical model of optimal control that simulate "undesirable" areas of space using the potential field method, in which obstacles are considered as sources of repulsive potential, which creates a force field that repels the robot's trajectory from dangerous zones. In this case, the cost function  $J$  (5) is supplemented by the repulsive potential  $V_{obs}(x)$ , which increases as the obstacle is approached, which forces the optimal motion plan to bypass these areas:

$$J = \int_0^T \left( \frac{1}{2} \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) + \frac{1}{2} (\mathbf{x}(t) - \mathbf{x}_f)^T \mathbf{Q} (\mathbf{x}(t) - \mathbf{x}_f) + V_{obs}(x) \right) dt \quad (13)$$

where:  $V_{obs}(x)$  – repulsive potential, which depends on the distance to the obstacle, with  $V_{obs}(x) \rightarrow \infty$  as  $x$  approaches the obstacle, forcing the optimal trajectory to bypass the forbidden regions.

Taking into account the energy consumption for performing movements and the mass of the transported load, affect both the accuracy and efficiency of controlling the trajectory of the gripping device, especially in the context of adaptive control in variable production conditions. The mass of the load directly affects the dynamics of the entire system: with its increase, inertia increases, the load on the motors changes and additional moments of force are created in the joints of the manipulator. Failure to take this parameter into account can lead to failure to reach the set point, overloading of the drives or instability in operation. Energy consumption is a criterion of optimality, it is important not only to perform the task accurately and safely, but also with minimal energy consumption.

The mass of the load  $m_L$  is taken into account in  $\mathbf{M}(\theta)$  and  $\mathbf{G}(\theta, m_L)$ , which affects the dynamics, and energy consumption is modeled as:

$$E = \int_0^T \sum_{i=1}^3 (\tau_i(t) \cdot \dot{\theta}_i(t)) dt \Rightarrow \text{include in } J \text{ through } \mathbf{u}^T \mathbf{R} \mathbf{u} \quad (14)$$

where:  $E$  – total energy consumption of the manipulator for the entire time of

movement from the initial moment  $t = 0$  to the final moment  $t = T$ , measured in J;

$\tau_i(t)$  – the control moment (or force producing rotation) at the  $i$ -th joint at time  $t$ . It is a component of the control vector  $u(t)$ , measured in N m;

$\dot{\theta}_i(t)$  – angular velocity of the  $i$ -th joint at time  $t$ , the rate of change of angle  $\theta_i$  over time. Measured in rad/s;

$\tau_i(t) \cdot \dot{\theta}_i(t)$  – instantaneous mechanical power consumed in the  $i$ -th drive, its integration gives the total work (energy);

$u^T R u$  – quadratic form of control actions in the quality functional  $J$ , where  $R$  – is the weight matrix that determines the importance of energy consumption in the optimization problem.

Thus, the energy consumption  $E$  reflects the physical content of the work performed by the manipulator, and through the mathematical form  $u^T R u$  are included in the optimization criterion, which allows reducing energy consumption when constructing motion trajectories.

To clearly define the initial and target states of the system, which is critical for the formulation of the optimal trajectory control problem of the collaborative robot gripper. The initial state  $x(0)$  reflects the actual position, velocity, and configuration of the manipulator at the start of the movement, while the final state  $x_f$  defines the desired position and dynamic parameters that need to be achieved at time  $T$ :

$$x(0) = x_0, \quad x(T) \approx x_f \quad (15)$$

where:  $x(0) = x_0$  – the initial state vector of the system, which includes the initial values of the joint angles  $\theta_i(t)$  and their angular velocities  $\dot{\theta}_i(t)$ ;

$x(T) \approx x_f$  – vector of the desired final state that the system should reach at a point in time  $T$ .

Due to these conditions, the algorithm for constructing the optimal trajectory is able not only to plan the movement according to the constraints, but also to guarantee the achievement of the target configuration taking into account energy consumption, cargo mass and safe interaction with a person. The obtained mathematical description using the Pontryagin maximum principle allows finding optimal control actions for constructing an energy-efficient and dynamically consistent trajectory of the gripping device of a 3-link collaborative robot. Taking into account the cargo mass, system dynamics and constraints ensures physically realistic behavior of the manipulator. Such models are the basis for constructing optimal control solvers and can be implemented numerically through the shooting method [32], indirect numerical optimal control methods (Indirect methods) or training neural networks for approximating the control policy [33].

### 3. Modeling the optimal trajectory of the gripping device of a collaborative robot-manipulator and analyzing the results obtained

The purpose of the experimental study is a comprehensive assessment of the effectiveness of methods for planning and optimizing the trajectories of the gripping device of a collaborative robot-manipulator in a three-dimensional workspace, taking into account dynamic, energy, and spatial constraints.

Within the framework of the experimental study, the following tasks were set:

- to investigate the formation of the optimal trajectory of the gripping device in the absence of spatial restrictions in the working area;
- to implement a trajectory planning algorithm taking into account the presence

of obstacles and ensuring safe bypassing of prohibited areas;

- to conduct a comparative analysis of the results of the basic and improved approaches in terms of the shape of the trajectories, stability of movement and the amount of energy consumption;

- to assess the impact of obstacles in the working environment on the energy efficiency and smoothness of the gripping device's movement;

- to formulate conclusions regarding the effectiveness of the application of trajectory optimization algorithms taking into account the environment configuration.

Description of hardware for conducting the study: Microsoft Surface Pro 9 with the following parameters: CPU Deca-core Intel Core i7-1255U (1.7 – 4.7 GHz), GPU Iris Xe Graphics, RAM 16Gb, SSD 512.

Software: Windows 11 Pro (version 24H2) OS type 64-bit operating system, processor based on x64 architecture.

Development environment for the program for numerous modeling PyCharm 2025.1.1.1 and programming language Python 3.13.7

For the first stage of experimental modeling, key input parameters were determined that describe the conditions of the movement of the gripping device of the collaborative robot-manipulator. The total duration of the simulation is 10 seconds, which was divided into 100 uniform time intervals, which allows us to take into account the change in the state of the system in a continuous mode. The mass of the load attached to the gripping device is set at 2 kg, which affects the dynamics of movement and energy consumption. To simulate energy consumption, an energy consumption coefficient is introduced, the value of which is 0.1, and it is taken into account when calculating the optimal trajectory. The initial position of the effector is given as a point with coordinates (0, 0, 0), and the final target is the point (5, 5, 5), which simulates the need to move the load in three-dimensional space. The working environment is provided with obstacles that simulate the constraints in the real environment: the first of them has a center at the point (2, 2, 2) with a radius of 1 unit, and the second one at the point (4, 3, 3) with a radius of 0.8 units. These obstacles play an important role in the formation of trajectories, forcing the system to look for bypass paths taking into account optimal control. The obtained results of modeling the optimal trajectories of the gripper of the collaborative robot-manipulator are presented in Figure 1, and the calculation results are presented in Figure 2.

The presented 3D graph (Fig. 1) shows the results of the first stage of experimental modeling of the trajectories of the gripping device of the collaborative robot-manipulator. It can be seen that the optimal trajectory (marked by a red line) is straight, which indicates the minimization of energy consumption without taking into account the influence of existing obstacles in the workspace. The existing obstacles in the form of gray translucent spheres did not affect the formation of the optimal route, which indicates the absence of an implemented model for their bypass in the calculation of energy-efficient movement. Alternative trajectories, represented by dashed lines of different colors, have significant fluctuation around a straight line, which demonstrates potential deviations in movement due to the action of random factors or irregular control. It is visually noticeable that some of the alternative trajectories intersect with prohibited zones, which confirms the need to include restrictions on obstacles in the optimization process. Despite this, the optimal route demonstrates the shortest path between the initial and final points, but ignores possible collision risks. The analysis conducted allows us to conclude that the implemented approach is basic and requires improvement - in particular, the inclusion of motion planning algorithms

taking into account the configuration of the environment and objects that impede direct movement. The results of the first experiment are important for understanding the basic characteristics of the system's motion and serve as a starting point for further research with the implementation of advanced optimization methods and obstacle avoidance.

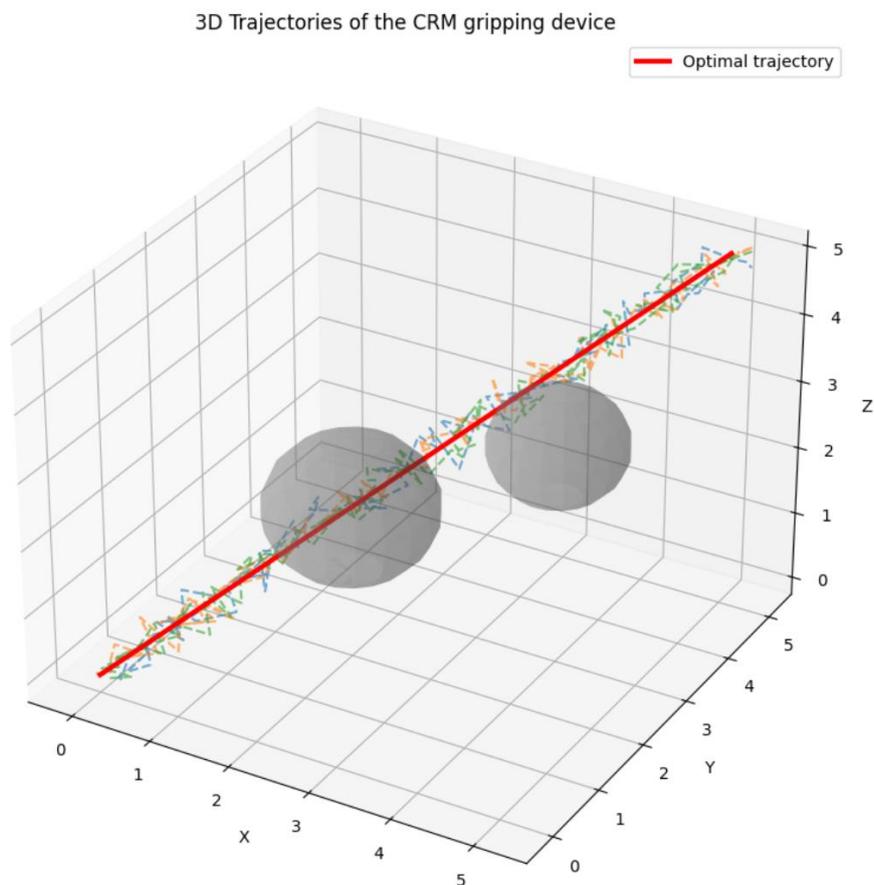


Fig. 1. 3D graph of the results of the first stage of experimental modeling of the trajectories of the gripping device of the collaborative robot manipulator

Optimal trajectory taking into account the weight of the cargo  $m_L = 2.0$   
Estimated energy consumption on this trajectory: 1.5

Fig. 2. Results of energy loss calculation of the first experiment

For the second stage of experimental simulation, key input parameters were determined that describe the conditions of the movement of the collaborative robot-manipulator's gripper. The total simulation time is 10 seconds, divided into 100 uniform steps, which provides sufficient discretization for analyzing the dynamics of the system. The mass of the load moved by the collaborative robot is 2.0 kg, which affects the inertial properties of the system. The energy consumption coefficient is set at 0.1, which allows us to take into account energy consumption during trajectory optimization. The initial position of the gripper is at the point with coordinates (0.0, 0.0, 0.0), and the final target position is at the point (5.0, 5.0, 5.0), which determines the desired trajectory of movement. In the working area, the presence of two spherical obstacles with centers at points (2.5, 2.5, 2.5) and (3.5, 3.5, 3.5), respectively, with radii of 1.2

and 1.0 units, which create constraints for constructing a safe route, is simulated. The simulation results of the second experiment are shown in Figure 3, and the results of calculating energy losses of the second experiment are shown in Figure 4.

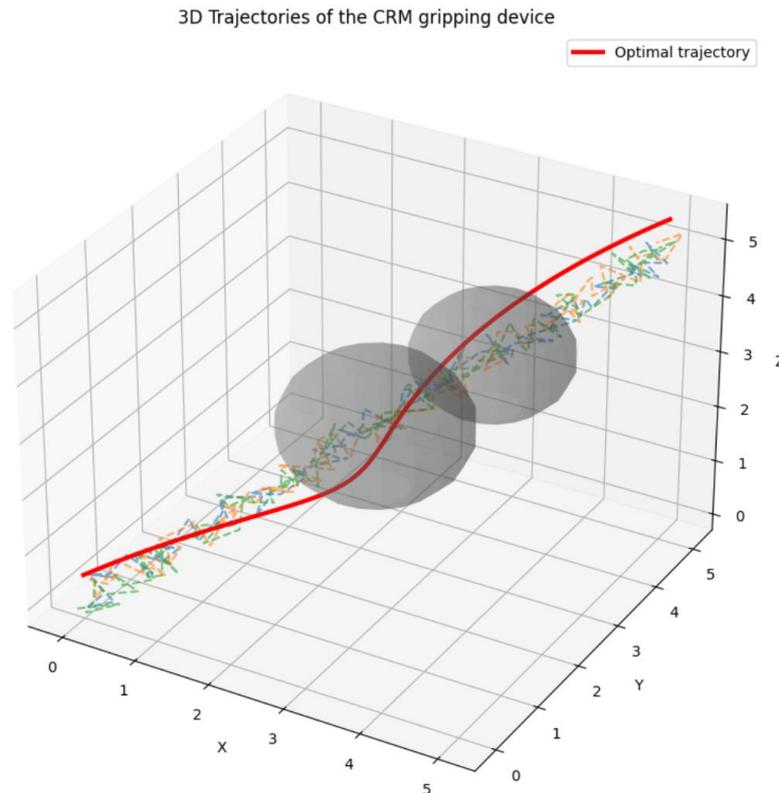


Fig. 3. 3D graph of the results of the first stage of experimental modeling of the trajectories of the gripping device of the collaborative robot manipulator

Optimal trajectory taking into account the weight of the cargo  $m_L = 2.0$   
Estimated energy consumption on this trajectory: 1.8141

Fig. 4. 3D graph of the results of the first stage of experimental modeling of the trajectories of the gripping device of the collaborative robot manipulator

The 3D graph shows the trajectories of the collaborative robot manipulator's gripping device in the presence of obstacles, taking into account a more advanced route optimization algorithm (Fig. 3). The optimal trajectory (red line) now takes on a curved shape, demonstrating the bypass of the zones marked with gray spheres — these are probably prohibited or dangerous areas for movement. Unlike the previous experiment, where the movement was carried out in a straight line without taking into account spatial restrictions, here a clear formation of a safe route with minimizing the risk of collision is observed, which indicates the inclusion of the obstacle-taking functionality in the trajectory planning. Alternative trajectories, represented by dotted lines, still demonstrate variability in the system's movement due to internal oscillations or noise, but are located much more densely around the optimal line, which indicates improved stability of the control system. Visually, the smoothness of the optimal route is observed, which indicates a decrease in peak loads on the actuators and an increase in energy efficiency. The results of this stage of the experiment indicate the effective

implementation of the obstacle avoidance algorithm without losing the target route efficiency, as well as the potential for further adaptation in real conditions of the collaborative robot. The obtained data confirm the correctness of the chosen approach to building trajectories taking into account the environment and open up prospects for scaling the model to more complex spatial configurations.

#### 4. Conclusions

The article presents a mathematical model for constructing the optimal trajectory of the gripper of a collaborative robot-manipulator taking into account the influence of the load mass and energy consumption. The model takes into account the physical characteristics of the system, including the payload mass and the energy consumption coefficient, which allows for realistic modeling of the dynamics of movement in the presence of spatial obstacles. Based on numerical analysis, the effectiveness of the developed trajectory was confirmed, which ensures safe movement in a complex environment and minimizes energy costs. Visualization of the obtained trajectories indicates the ability of the system to adapt the movement to spatial constraints and achieve a given goal while observing dynamic constraints. The results obtained can be the basis for further optimization of trajectories in conditions of variable load mass and in real time. It is recommended to further expand the research by applying the Pontryagin maximum principle in continuous time to analytically obtain optimal control actions in complex dynamic scenarios.

#### Conflict of interest

The authors declare that they have no conflict of interest, in particular financial, personal, authorial or any other nature, which could affect the research, as well as the results published in this article.

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## **Розробка моделі побудови оптимальної траєкторії руху захватного пристрою колаборативного робота-маніпулятора з урахуванням впливу маси вантажу та витрат енергії**

У сучасних умовах розвитку індустрії 5.0 колаборативні роботизовані системи відіграють ключову роль у підвищенні ефективності виробничих процесів, забезпеченні гнучкості та безпеки взаємодії з людиною. Одним із найбільш критичних елементів таких систем є захватний пристрій колаборативного робота-маніпулятора, який виконує точні й енергозалежні дії у складному динамічному середовищі. Зважаючи на постійну зміну маси вантажу, що транспортується, та обмеженість енергетичних ресурсів, виникає нагальна потреба в розробці оптимальної траєкторії руху, яка враховує не лише геометричні обмеження, а й енергетичну доцільність руху маніпулятора. Актуальність дослідження зумовлена потребою в зменшенні енергоспоживання під час виконання завдань захвату й транспортування об'єктів у присутності просторових перешкод, що є важливим для підвищення автономності та продуктивності роботизованих систем.

Об'єктом дослідження є процес просторового переміщення захватного пристрою колаборативного робота-маніпулятора. Предметом дослідження виступає оптимізація траєкторії руху із врахуванням динамічних обмежень, змінної маси вантажу та енергоспоживання. У межах дослідження було використано методи математичного моделювання, чисельного інтегрування, енергетичного аналізу та просторової візуалізації траєкторій. Основу формалізації процесу побудови траєкторії становить система рівнянь динаміки руху, на яку накладено умови оптимальності, враховуючи витрати енергії та уникнення зіткнень із перешкодами.

Метою дослідження є побудова математичної моделі та реалізація алгоритму формування оптимальної траєкторії руху захватного пристрою у 3D-просторі, яка дозволяє мінімізувати енергетичні витрати при транспортуванні об'єктів з урахуванням їхньої маси та наявних перешкод. Результати дослідження включають побудову оптимальної траєкторії в умовах заданих просторових обмежень, обґрунтування її ефективності на основі порівняння з варіативними траєкторіями, а також чисельне підтвердження зниження енергетичних витрат під час реалізації руху. Отримана модель демонструє потенціал до впровадження в системи управління колаборативними роботами в умовах реального часу.

На основі розробленої математичної моделі рекомендовано проведення подальших досліджень з використанням принципу максимуму Понтрягіна в неперервному часі з метою розробки аналітичних рішень та вдосконалення системи керування для більш складних задач траєкторного планування в умовах змінних навантажень та складних перешкод.

**Ключові слова:** колаборативний робот, захоплювальний пристрій, оптимальна траєкторія, енергоспоживання, маса вантажу, моделювання руху, принцип Понтрягіна, маніпулятор, перешкоди навколишнього середовища, адаптивне керування, числове моделювання, енергоефективність, роботизована система.

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