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MATHEMATICAL MODEL AND SOFTWARE DEVELOPMENT FOR MODELING ENGINES WITH PERIODICAL WORKFLOW USING AN ARTIFICIAL INTELLIGENCE SERVICE

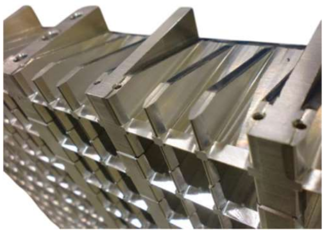
*This paper examines the problem of constructing a mathematical model of a reed valve suitable for use in zero-dimensional thermodynamic and one-dimensional gas-dynamic models of a pulsejet engine's working cycle. **The subject of the study** is the reed valve of a pulsejet engine. It is shown that simple single-mass classical reed valve models lack sufficient accuracy and therefore require verification and adjustment. However, despite the large number of publications on the reed-valve topic, the number of studies containing experimental reed lift curves sufficient for direct verification in a frequency range close to pulsejet engine operating conditions remains limited. **The objective of this study** is to develop a single-mass mathematical model of a reed valve using artificial intelligence, motion based on a mechanical analogy of the "mass-spring-damper" type, but supplemented by a system of correction coefficients. Objective: to develop a single-mass mathematical model of a reed valve, an algorithm, and a program for numerical calculation of the reed motion, to adjust the model parameters based on experimental data, to test the model performance for different designs and in different operating modes, and to evaluate its reliability and applicability. **Research methods.** To solve the equations of reed motion, a numerical integration algorithm was used, considering the displacement limits and energy losses during seating and in the region of maximum lift. Published pressure and reed-lift diagrams for steel and non-metallic reeds were used as an experimental benchmark. **Results.** The model parameters were tuned using the experimental lift curves at operating frequencies of 79.5 Hz, 90.5 Hz, and 152 Hz. It was found that the proposed model provides good agreement between the calculated and experimental values of reed lift in this range of operating frequencies, with a relative error of 7.4-11.3%. It is shown that the main differences between the regimes are concentrated only in the region of large reed displacements. **Conclusions.** The developed single-mass reed valve model with correction coefficients occupies an intermediate position between the simplest unadjusted engineering models and resource-intensive spatial models, making it suitable for engineering analysis and the preliminary design of pulsejet engines.*

Keywords: *UAV, pulse jet engine, reed valve, single-mass model, petal lift, numerical calculation, simulation, verification*

Introduction

The classic reed valve is a characteristic element of the intake system of the engines with periodic workflow (pulsejets and 2-stroke internal combustion engines) due to its simple design, low mass of moving parts, and ability to automatically match flow rate to the current pressure drop. Historically, the reed valve design (Fig. 1) originates from the valve grid unit of the classic Argus pulse jet engine [1], and its subsequent engineering development in calculations and applied models for two-stroke internal combustion engines is largely due to the initial work of Queen's University Belfast [2].

The relevance of reed valve modeling is currently determined not so much by theoretical interest as by the practical challenges of perspective high speed UAV pulse jet engine design. For example, the already developed and operational online model of a pulsejet engine, Pulsejet-Sim [3], uses a simple reed valve model [4]. While this proved quite useful for initial engineering assessments, for certain ranges of engine geometric dimensions, the simplicity of the petal model became one of the causes of discrepancies between the calculated and actual system properties.



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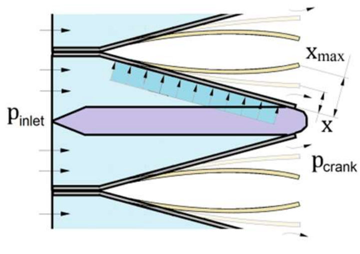

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Fig. 1. Classic valve grid with reed valves (a) and its design scheme (b)

Under these conditions, the need for a more reliable, yet computationally compact and accessible, petal valve model becomes important. Therefore, this paper proposes not to complicate the problem to the level of spatial modeling, but to attempt to preserve the simple mechanical analogy of the petal, making it suitable for practical engineering applications for maintaining the compatibility of the valve model with zero-dimensional (0D) thermodynamic problems and simple one-dimensional (1D) gas-dynamic engine models.

1. Literature review and problem statement

Numerous publications exist on reed valves of various types, including those similar to the valve design under study and operating in the 70–150 Hz operating frequency range. One of the earliest system sources detailing a valve design of this type is the work of Dr. G. Diedrich, dedicated to the Argus power plant of the V-1 cruise missile [1]. This work is of particular value as a historical document, but also as a primary source for the engineering approach to creating a reed valve grid operating as part of pulse jet engine. A book by Queen's University of Belfast Professor G.P. Blair [2] not only examines the reed valve as an important element of gas exchange in a two-stroke engine but also directly points to its origins in the Argus pulse jet engine valve system. An important step in the development of this approach was the improved model of reed valve behavior for a two-stroke engine [5], aimed at a more accurate description of petal motion under real-world operating conditions. This attempted to move from simple estimates to a more sophisticated description of petal motion. However, even at this stage, it became clear that the accuracy of classical engineering models' agreement with experiment remains an open question, especially when considering not only the integral characteristics but also the petal' lift curves as main parameter.

Nevertheless, in subsequent developments, petal valve single-mass models became the most widely used (Fig. 2). They describe petal motion using a mechanical analogy such as "mass-spring-damper" [6]. This approach proved particularly convenient for two-stroke engines, as it allowed the valve model to be incorporated into the overall calculation scheme of the operating process without resorting to 3-dimensional modeling.

In this group of study [7], gas flow through a two-stroke engine's reed valve are researched and theoretical and experimental data, providing a link between valve dynamics and the gas-dynamic characteristics of the intake system, are compared. However, a typical limitation also appears here: even with experimental data, primary attention is often focused on flow rate or integral characteristics [4], while direct verification using petal lift curves is less comprehensive and/or not given sufficient attention.

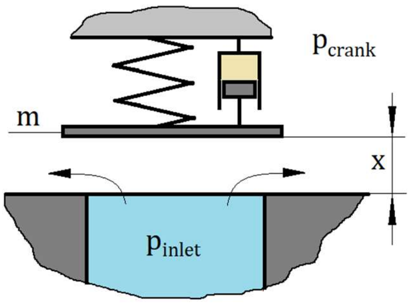

Fig. 2. A Simplified single-mass reed valve calculation model

As a result, the use of classical single-mass models creates a dual situation. On the one hand, classical models can be viewed as a necessary and methodologically important foundation, as they enable the construction of compact models easily compatible with the description of the working process of a pulse jet engine. These models are simple, fast, and convenient for engineering calculations. However, the issue of their reliable verification, especially using direct experimental petal lift curves [8], remains only partially resolved. When switching to new operating modes, a different process frequency, a different petal size, or a different valve grid geometry, it remains unclear in advance how much they maintain their accuracy. This is especially important for the present problem, where the reed valve of a pulse jet engine is studied over a wide frequency range.

For these reasons, the simplest single-mass model of the "mass-spring-damper" type does not always accurately describe the actual petal motion, especially with large deviations, variable pressure drops, and complex interactions with the air flow. This has led to the emergence of studies that retain the general nature of the model but introduce additional nonlinear elements, correction factors, and refined force laws.

A typical example of this approach is nonlinear models of reed valves [9]. Unlike simple single-mass models, this approach attempts to account for the more complex nature of reed valve motion and improve the model's fit to the real process.

Thus, nonlinear models can be viewed as a next step in the development of classical engineering models. Such studies are of particular interest because they occupy an intermediate position between classical single-mass models and resource-intensive three-dimensional modeling approaches. They demonstrate that even without switching to 3-D modeling, enough improvement in the reliability of the description is possible [10].

However, even with nonlinear models, the same principled question remains: how fully and by what criteria are they verified experimentally. If model improvement is assessed primarily by integral characteristics or indirect parameters, the question of the correctness of the description of the reed valve lift itself remains open.

On the other hand, an analysis of the work performed on the above-mentioned models leads to an important conclusion: the key issue lies not only in choosing a linear or nonlinear form of equations, but also in the availability of a reliable experimental baseline that allows for tuning and validating the model using petal lift curves. Without such a baseline, even a complex model will have limited applicability when transferred to other operating modes and other valve sizes or designs.

The difficulties with further development of single-mass models may have led to the emergence of three-dimensional (3D) modeling of reed valves, using FEA (Finite Element Analysis) numerical simulations as another promising area of research. The transition to these types of models was largely driven by the fact that a simple single-mass design does not allow for accounting for distributed petal deformation, changes in its deflection, local flow structure, and the influence of flow on the opening and closing dynamics.

In one of the early studies in this area [11], reed valve modeling was included in the CFD calculation of a two-stroke engine. This work is important as an example of the transition from an engineering mechanical analogy to a more detailed description of the interaction between the petal and the flow. A similar approach was subsequently developed in the form of a 2D model incorporating the full engine geometry [12].

Further development of this approach entailed a significant increase in the complexity of the problem, as the petal valve had to be considered in conjunction with the spatial gas flow as CFD-approach (Computational Fluid Dynamics), which together constituted a new class of models – FSI (Fluid-Structure Interaction). In some studies [13], the problem is really posed as a coupled one: gas flow and petal lift are calculated jointly (Fig. 3), allowing for the distributed nature of deformation and more complex forms of petal-flow interaction to be taken into account.

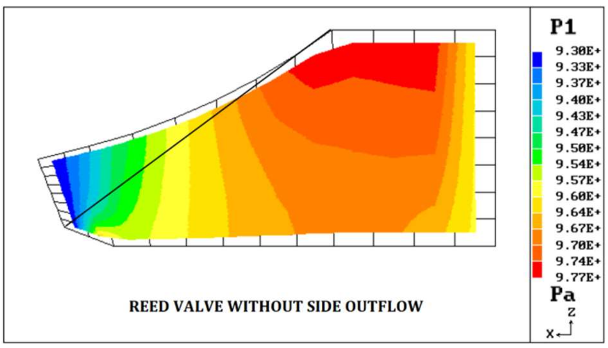

Fig. 3. An example of instantaneous pressure distribution along a reed valve, obtained using CFD modeling [7], demonstrates the need to adjust the single-mass model constructed for ideal conditions

A similar approach is also presented in materials from engine manufacturers [14], where FSI modeling of a reed valve is accompanied by experimental validation and is aimed at obtaining a description practically applicable for a two-stroke engines. Such studies are of great importance for the present study for several reasons. For example, to obtain a more accurate description of the process, it is indeed necessary to move from classical single-mass models to spatial FSI models. On the other hand, such studies demonstrate which effects are not captured by a simple single-mass model: distributed petal deformation, spatial bend, local flow structure, the influence of channel shape, and unsteady flow interaction.

However, it is precisely here that the main limitation of spatial models in the context of this study becomes apparent. For engineering problems within thermodynamic (0D) and one-dimensional (1D) gas-dynamic models of engine operation, such reed valve models prove excessively complex and resource-intensive. A spatial description of the reed valve itself requires a spatial description of the flow through the valve grid and adjacent duct [15], which significantly complicates the solution. As a result, FSI models should not be considered a direct replacement for concentrated mass engineering models, but rather as a more detailed, but significantly more complex, alternative.

A separate and fairly extensive group of publications consists of studies on reciprocating compressor reed valves, as they examine in detail the impact processes of petal-seat interaction, the influence of the lift limiter, the spatial forms of petal motion, the flow in the gap between the petal and seat, and the coupled interaction of the deformable valve with the flow.

It is in this area that FSI approaches, as well as experimental verification of petal motion, are comparatively widely developed. Thus, studies [16, 17] focus on the combined calculation of flow and petal deformation, as well as the comparison of numerical results with experiment. A number of publications additionally analyze spatial motion effects, including the torsional component of petal lift and impact processes during interaction with bounding elements [18]. More detailed formulations consider full-fledged three-dimensional unsteady flow through a compressor valve [19], which allows for the most complete and accurate description of the flow dynamics and petal deformation.

However, the direct use of such models for the classic reed valve of a pulse jet engine cannot be considered correct. This is due not only to the difference in the application area, but also to the fundamentally different formulation of the problem. In piston compressors, the reed valve operates in a different geometry, under different boundary conditions, with a different petal orientation relative to the flow, and a different flow structure. The free space between the petal and the seat, the height of the stop, the contact with the retaining and guiding elements, and the specific kinematics of the valve all play a significant role. Therefore, even with the external similarity of the petal, the mechanics and gas dynamics of a

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compressor reed valve are not identical to the mechanics of an angle reed valve in a pulse jet or two-stroke engine.

Thus, a review of the literature leads to the following conclusion. On the one hand, classic single-mass reed valve models remain useful and even necessary for engineering calculations. On the other hand, the available literature lacks data to assess their accuracy and reliability in a range close to the operating conditions of a pulse jet engine. Detailed FSI models partially fill this gap, but prove excessively complex for applied calculations. This contradiction defines the goal and objectives of this study.

2. The objective and tasks of the study

The objective of this study is to develop and verify a single-mass model of angular reed valve motion. This model is suitable for use in engineering thermodynamic (0D) and one-dimensional gas dynamic (1D) models of the pulsejet operating process and ensures consistency with experimental petal lift curves.

To achieve this objective, the following tasks are solved in this study:

1) Develop a single-mass mathematical model of angular reed valve motion based on a mechanical analogy with the introduction of correction coefficients;

2) Implement an algorithm for numerically calculating reed valve motion suitable for subsequent incorporation into an existing pulse jet engine model and its further development toward a 1D description of the flow path;

3) Adjust the model parameters using experimental data to ensure maximum agreement between the calculated and experimental petal lift curves.

4) test the performance of the same model structure for different designs in several operating modes and evaluate which parameters are universal and which require mode-specific adjustments;

5) analyze the physical meaning of the remaining discrepancies between calculations and experiments and provide recommendations on the possible application of the developed model.

3. The study materials and methods

The key feature of this work was the extensive use of artificial intelligence at all stages – from source analysis, problem formulation, model type selection, algorithm creation, computational program development, and calculation execution to retrieving experimental data, digitizing it, and converting it to the required graphical format, comparing experimental and calculated data, and performing parametric calculations to select correction coefficients.

As a result, all work in this study was conducted exclusively in ChatGPT, the professional version Plus. Moreover, in addition to the scientific component, a secondary objective of this work was to demonstrate the potential and advantages of using AI in scientific research compared to the traditional "manual" search for sources, writing custom algorithms and programs, or using existing standard programs.

3.1. Single-mass mathematical model of reed valve

In this paper, the calculation algorithm was derived solely from a discussion of the problem in ChatGPT, and its main principles are presented below. So, the reed valve motion is described by a classical single-mass model with a single generalized coordinate x(t), corresponding to the lift of the petal free end (Fig. 1). A single-mass "mass-spring-damper" system (Fig. 2) is used as the basic dynamic scheme, for which the equation of motion is written in the well-known standard form of a single-mass oscillatory system [20]:

$$m_{eff}\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_p(t), \qquad (1)$$

where x(t) is lift of the free end of the petal, m; $\dot{x}(t)$ is lift speed, m/s; $\ddot{x}(t)$ is acceleration, m/s^2; m_{eff} is effective mass of the petal, kg; viscous damping coefficient, N•s/m; k is effective stiffness, N/m; $F_p(t)$ isequivalent pressure force, N.

In a single-mass system, instead of directly specifying the coefficient c, a more convenient parameter can be introduced: the relative damping coefficient ζ. Indeed, for a linear single-mass system, the critical damping [21] is equal to

$$c_{cr} = 2\sqrt{km_{eff}}\,. \qquad (2)$$

If the relative damping coefficient is specified as the ratio

$$\zeta = \frac{c}{c_{cr}}. \qquad (3)$$

then the coefficient c is immediately obtained from formula (2):

$$c = 2\,\zeta\sqrt{km_{eff}}. \qquad (4)$$

On the other hand, to determine the acting forces, the petal should be considered as an equivalent cantilever beam with a rectangular cross-section. For such beam, the moment of inertia of the cross-section is determined as [22]:

$$I = \frac{b_{reed}t_{reed}^3}{12}, \qquad (5)$$

where b_{reed} is the petal width, m; t_{reed} is petal thickness, m.

The effective working length of the petal is determined by the expression:

$$L_{eff} = L - L_{support}, \qquad (6)$$

where L is the total petal length, m; $L_{support}$ is the petal support length, m.

For a cantilever beam with a concentrated force at the free end, the linear stiffness is determined using the formula from the classical Euler-Bernoulli bending theory through the relationship between the force and deflection of the cantilever beam at the free end [23]:

$$k_0 = \frac{3EI}{L_{eff}^3}, \quad (7)$$

where E is the modulus of elasticity of the petal material, Pa.

Since a real valve petal does not operate as an ideal cantilever beam, a correction stiffness coefficient$C_k$is introduced into the model. This coefficient is an empirical model coefficient that takes into account the differences between a real petal and an idealized beam and is determined during tuning using experimental data. Based on this, the effective stiffness of the beam is taken to be

$$k = C_k k_0. \quad (8)$$

The physical mass of the petal is determined as

$$m_{phys} = \rho_s b_{reed} t_{reed} L, \quad (9)$$

where ρ_s is the density of the petal material, kg/m^3.

For a cantilever beam in the first mode of vibration, the effective mass at the free end of the cantilever beam is approximately taken to be equal to the standard approximation used in single-mass approach [24]:

$$m_{eff,0} = 0.236\ m_{phys}. \quad (10)$$

Since the mass distribution and vibration mode of a real petal differ from the idealized cantilever scheme, a mass correction factor C_m is introduced – an empirical coefficient determined during model adjusting, after which the final value is:

$$m_{eff} = C_m m_{eff,0}. \quad (11)$$

The petal motion is excited by a pressure difference:

$$\Delta p(t) = p_{up}(t) - p_{down}(t), \quad (12)$$

where $p_{up}(t)$ is the pressure on the opening side of the petal, Pa; $p_{down}(t)$ is the pressure on the opposite side of the petal, Pa.

In this study, two options for defining the pressure drop were considered:

$$\Delta p = p_{atm} - p_{crank}; \Delta p = p_{inlet} - p_{crank}. \quad (13)$$

where p_{atm} is the ambient pressure at the inlet, p_{inlet} is the pressure in the channel upstream of the valve, and p_{crank} is the pressure in the chamber downstream of the valve.

Obviously, when the valve opens, the pressure drop is determined by the first formula. However, as air inflows, the pressure in the channel upstream of the reed valve drops, and the drop at the petal itself should be calculated using the second formula.

To convert the pressure drop to an equivalent force, the effective area of the petal load is introduced:

$$A_{eff} = b_{reed} L_{eff}. \quad (14)$$

The basic pressure force is written as the usual relationship between force and pressure through area:

$$F_{p,0}(t) = A_{eff}\Delta p(t). \quad (15)$$

For a real valve, the pressure does not act on the petal with equal effectiveness during the opening and closing stages. This is clearly due to the pressure redistribution along the petal's length (Fig. 3), changes in the gap geometry, and differences in the flow during opening and reseating. Therefore, to calculate the pressure force, two independent coefficients are introduced into the model:

$$F_p(t) = \begin{cases} C_{p,open} A_{eff} \Delta p(t), \phi(x), & \Delta p > 0 \\ C_{p,close} A_{eff} \Delta p(t), & \Delta p \le 0. \end{cases} \quad (16)$$

where $C_{p.open}$ is the effective force coefficient during opening; $C_{p.closed}$ is the effective force coefficient during closing; $\varphi(x)$ is the correction function.

These coefficients allow the actual distributed aerodynamic force to be taken into account in the equivalent force of the single-mass model. They have the following physical meaning: if the coefficient is equal to 1, it is assumed that the entire pressure drop acts on the effective area of the petal, and if the coefficient is less than 1, the actual force effect is decreased.

The correction function $\varphi(x)$ takes into account the decrease in the effective force effect on the petal from pressure forces as the petal lift increases in the open mode. In accordance with the nature of the pressure change along the petal (Fig. 3), the following function form was adopted:

$$\phi(x) = 1 - \beta\eta^n, \quad (17)$$

where β is the force attenuation coefficient; n is the exponent defining the nonlinearity of the force distribution along the petal length; x_{ref} is the characteristic lift scale, m; η is the normalized petal lift (η = 0-1,0):

$$\eta = \begin{cases} 0, & x \le 0 \\ x/x_{ref}, & 0 < x < 1. \\ 1, & x \ge 1 \end{cases} \quad (18)$$

Since the lift cannot be negative, the constraint $x \ge 0$ is adopted. When the petal reaches the seat, the law of restitution of the petal's velocity upon rebound from the seat is used:

$$\dot{x}^+ = -e_{seat}\dot{x}^-, \quad (19)$$

where: e_{seat} is the coefficient of restitution upon landing on the seat; $\dot{x}^-$ is the impact velocity; $\dot{x}^+$ is the rebound velocity after the impact, which is generally less than the impact velocity.

Data on the interaction of materials can be used as a physical guide to determine the approximate value of the coefficient of restitution e_{seat}. For example, for the impact interaction of metal plates with steel bodies, the coefficient of restitution is approximately 0.4 [25].

However, this value can only be taken as a first approximation, since the direct transfer of these data to an angled petal valve requires further clarification.

Additionally, and in a similar manner, an upper lift limit $x \le x_{stop}$ is introduced, and when it is reached

$$\dot{x}^{+} = -e_{stop}\dot{x}^{-}, \tag{20}$$

where x_{stop} is the maximum petal lift, m; e_{stop} is the effective loss coefficient in the region of maximum deflection. This parameter in the model is not necessarily interpreted as a consequence of impact with a real mechanical stop. For an angled reed valve, it can describe in equivalent form the set of losses and nonlinear effects in the region of large deflections.

Thus, the model has two groups of parameters. These are parameters based on classical beam mechanics and vibration theory I, k_0, m_{phys}, $m_{eff,0}$, c, and parameters introduced in this paper as correction coefficients C_k, C_m, $C_{p,open}$, $C_{p,close}$, β, n, x_{ref}, x_{stop}, e_{seat}, e_{stop}. This separation makes the model both physically understandable and suitable for adjusting using experimental petal lift curves.

3.2. Numerical integration algorithm and model software implementation

The numerical solution of the petal motion equation can be performed by time integration with initial conditions. The current values of the pressure drop across the petal, as well as the current position, velocity, and acceleration of the single-mass system, were used as input data at each step.

The initial conditions assume that at the initial time $t = 0$ the petal's lift x_i and velocity $\dot{x}_i$ are zero. Then, at the next calculation step, the current pressure drop Δp_i is first determined, after which the equivalent pressure force $F_{p,i}$ is calculated using the formulas given in Subsection 3.1. The petal acceleration is then determined from the equation of motion (Newton's second law):

$$\ddot{x}_i = \frac{F_{p,i} - c\dot{x}_i - kx_i}{m_{eff}}. \tag{21}$$

After this, the velocity and petal lift at the next step are calculated using an explicit second-order Runge-Kutta finite-difference scheme [3] with a specified time step. When calculating the motion, the above-mentioned physical limitations of the model on lift and velocity, and also recovery, are taken into account. This means that after each integration step, an additional check is performed to ensure that the petal lift limitations are met, with the velocity recalculated upon impact with the seat or upon reaching the maximum lift.

To improve the stability and comparability of the results, the integration step was chosen to be significantly shorter than the characteristic time of the oscillation cycle and the time of change in the pressure drop. This allowed for the correct reproduction of both the petal opening phase and its reverse motion upon closing.

The software implementation of the algorithm is written in the Python environment as a separate calculation module, allowing for the use of various pressure drop specifications. This calculation organization allowed for the use of a single integration algorithm and diagram representation for different problem formulations (Fig. 4).

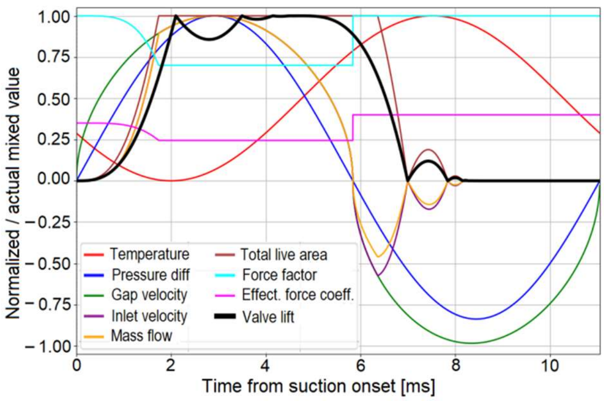


Fig. 4. Instantaneous changes in relative parameters (pressure, temperature, speed, air flow, opening cross-section, correction factors, petal lift) over time, simulated in ChatGPT using a pre-configured reed valve model operating within the valve grid of a pulse jet engine [26]

To compare the calculated and experimental dependencies, all pressure and lift curves were first converted by ChatGPT to a single phase reference frame. The origin was the instant of occurrence of the opening pressure differential, corresponding to the onset of lobe opening. The experimental and calculated dependencies were then interpolated onto a single time grid, ensuring a correct comparison of the curve shapes, maximum lift, and phase positions of characteristic points.

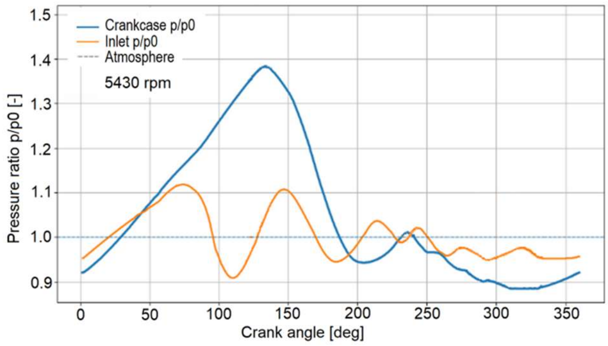
Thus, the implemented numerical integration algorithm provides an explicit time-domain solution to the reed valve motion equation, takes into account physical limitations along the petal's travel, allows for adjustment (correction) in accordance with experimental data, and can be directly used as a computational block within a more general pulse jet engine model.

3.3. Experimental baseline and parameter adjusting procedure

To verify the model, at a minimum, reed valve dimensions, pressure drop, and petal lift curves are required. However, when searching for experimental data for verification, it became clear that there were very few studies suitable for this task. As a result, the experimental baseline in this study was the pressure drop and petal lift curves provided in [2, 5, 9], as the most suitable for directly comparing the calculated and experimental lift curves in a form sufficient for verifying the single-mass model.

The analysis utilized two operating modes for a steel valve petal, corresponding to rotational speeds of 5430 rpm (90.5 Hz) and 9150 rpm (152 Hz), and one mode at 4470 rpm (79.5 Hz) for a petal made of a non-metallic material such as Glassfiber. With help of ChatGPT, the experimental curves were pre-digitized, then graphically cleaned, local artifacts removed, and brought to a periodically consistent form over the interval of one complete cycle.

Particular attention was paid to matching the edge sections of the curves, as discontinuities at the junction of the beginning and end of a cycle would lead to phase distortions during subsequent comparison of the calculated and experimental curves (Fig. 5).



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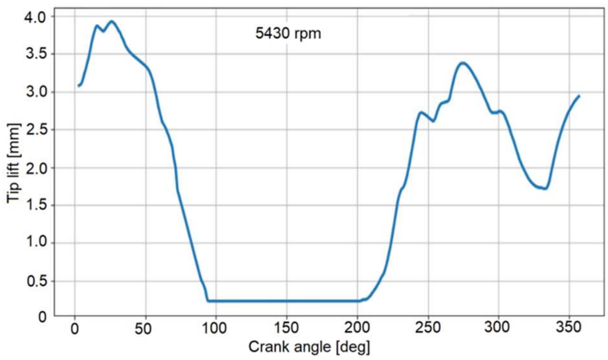

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Fig. 5. An example of experimental pressure drop (a) and valve petal lift (b) diagrams [5] for 90,5 Hz regime (digitized by ChatGPT)

To ensure accurate comparison of the calculated and experimental curves, all curves were referenced to a single phase reference system. The origin was the moment of occurrence of the opening pressure drop, corresponding to the onset of petal opening. After phase referencing, the pressure and lift values were interpolated onto a single calculated time grid. This ensured accurate comparison not only of the maximum lift but also of the curve shape as a whole, including the initial opening phase, the section of maximum deflection, and the nature of the petal's return to the seat.

The model parameters were adjusted to ensure maximum agreement between the calculated and experimental lift curves. In this case, the values of the correction coefficients C_k, C_m, $C_{p.open}$, $C_{p.close}$, β, n, x_{ref}, e_{seat} and e_{stop} were selected.

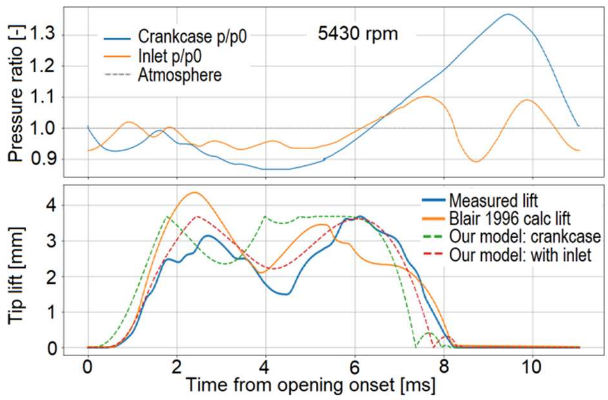
The primary focus was not only on matching the maximum lift but also on coordinating other characteristics: the moment of opening, the rate of lift increase, the position and magnitude of the maximum, the shape of the descending portion of the curve, and the nature of the petal's seating.

Thus, the model was tuned not just at a single point or a single integral parameter, but across the entire x(t) curve within the cycle under consideration.

4. Results

4.1. Model verification using experimental data for a steel petal

In the first stage, the single-mass model was verified using experimental data [5] for a steel petal. The calculated curves were compared with the experimental petal lift curves after preliminary phase locking and time scale matching (Fig. 6).



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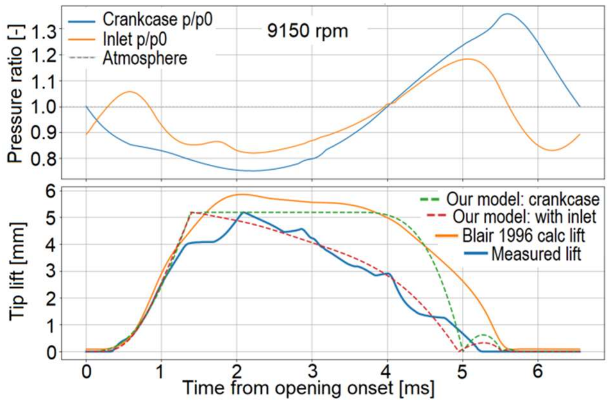

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Fig. 6. Verification of the proposed model, performed in ChatGPT using experimental data [5] for a steel petal and operating modes of 90.5 Hz (a) and 152.5 Hz (b)

For the 90.5 Hz mode, the proposed model ensured almost complete agreement between the calculated and experimental curves. Agreement was achieved not only in the maximum lift but also in the overall shape of the dependence, including the initial opening moment, the rise section, the position of the maximum, and the subsequent closing. This demonstrates that the single-mass circuit with the introduced correction coefficients is capable of adequately describing the dynamics of a steel petal in this mode.

Good agreement between calculations and experiment was also achieved for the 152.5 Hz mode. Furthermore, it was determined that the basic structure of the model remains functional even at a higher process frequency. At the same time, to achieve the most complete agreement, it was necessary to modify the correction coefficients in the region of large petal deviations. This demonstrated that the main difference between the two modes lies not in the basic inertial-elastic model, but in the description of petal behavior in the range of maximum petal lift value.

Comparison with the calculated curves presented in [5] also shows that the single-mass model with correction coefficients proposed in this paper provides closer agreement with experiment for both modes (Fig. 6).

4.2. Model verification using experimental data for a non-metallic valve petal

In the second stage, the model was verified using data from [9] for a glassfiber petal at a process frequency of f = 79.5 Hz (Fig.7).

As a result of adjusting the model for the reed valve, good agreement was achieved between the calculated and experimental curves for key parameters: the opening and closing phase, the order of maximum petal lift, the general shape of several lifts during the cycle, and the moment of final seating of the petal on the seat.

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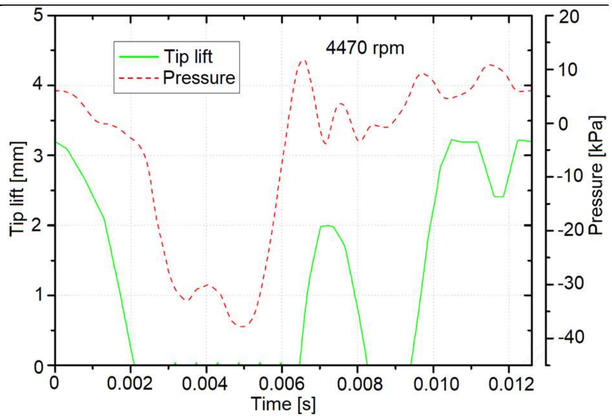
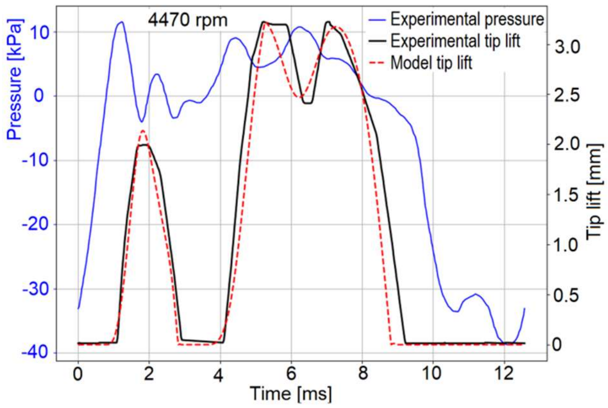
Thus, the single-mass model was successfully applied not only to a steel petal but also to a petal made of a different material and with a different geometry. This result is very important, as it demonstrates that the model's performance is not limited to a single specific case. A single-mass design with correction coefficients maintains the ability to match experimental data even for a glassfiber petal, that is, with significant changes in the elastic and mass characteristics of the system.



a



b

Fig. 7. Digitized in ChatGPT pressure-lift diagram [9] for a non-metallic reed valve (a) and comparison of the calculated lift diagram with the experimental one (b) at a given experimental pressure drop

4.3. Model adjusting results and model accuracy assessment

One of the main results of this study is the creation of a table of model settings for different materials and operating modes (Table 1). This table allows us to move from the partial matching of individual curves to a meaningful comparison of model parameters and an analysis of physic process feature.

The table shows the initial geometric and mechanical parameters of the valve petals, operating modes, and sets of correction coefficients that ensure model consistency with experimental data for three cases: a steel petal at 90.5 Hz and 152.5 Hz, and a non-metallic glassfiber petal at 79.5 Hz.

The model error assessment yields the following values for the mean model error (Mean Absolute Percentage Error MAPE) and the systematic bias per cycle (Table 2).

Thus, the model shows an absolute error of 7.4-11.3% with a systematic bias of -6.4 to 5.2% over the considered range of operating modes, reed valve dimensions, and materials.

5. Discussion

The results obtained demonstrate that the proposed single-mass reed valve model is generally capable of reproducing experimental petal lift curves for both the steel reed valve [5] and the glassfiber RV2 reed valve [9]. This means that this simple model, combined with correction coefficients, can be used as a practical basis for engineering modeling of the reed valve.

A comparison of the two modes for the steel reed valve revealed that most of the coefficients remain stable with frequency changes, with only the loss coefficient remaining the most sensitive parameter in the range of large reed valve deflections. This transforms the resulting single-mass model from a specific fitting model into a tool for comparative analysis of reed valve dynamics.

A comparison of the third set of parameters revealed another pattern: when switching from a steel reed valve to a fiberglass reed valve, the stiffness, mass, damping, and force coefficients change significantly. In particular, the fiberglass petal required lower values for the coefficients Ck and Cm, significantly greater relative damping ζ, a weaker effective opening force, and a stronger effective closing force.

Table 1

Adjusting the model coefficients for steel and glassfiber petals

Parameter / coefficientname	Symbol	Equation No.	Steel reed[5]		Glassfiber reed [9]
Initial data					
Reed width, m	b_{reed}	—	0.0227		0.0220
Reed length, m	L	—	0.0320		0.0290
Reed thickness, m	t_{reed}	—	0.00020		0.00046
Effective working length, m	L_{eff}	—	0.0280		0.0290
Young’s modulus, Pa	E	—	2.07×10^{11}		2.15×10^{10}
Material density, kg/m^3	ρ_s	—	7800		1850
Correction factors and coefficients					
Stiffness correction factor	C_k	(8)	0.75		0.63
Mass correction factor	C_m	(11)	1.50		1.15
Relative damping ratio	ζ	(3), (4)	0.00		0.20
Opening force coefficient	$C_{p,open}$	(16)	0.35		0.22
Closing force coefficient	$C_{p,close}$	(16)	0.40		0.75
Lift-force weakening coefficient	β	(17)	0.30		0.00
Exponent of weakening law	n	(17)	1.5		2.0
Seat restitution coefficient	e_{seat}	(19)	0.40		0.10
Operating mode and final adjustment					
Rotationalspeed, rpm	n_{rpm}	—	5430	9150	4770
Frequency, Hz	f	—	90.5	152.5	79,5
Upper-limit loss coefficient	e_{stop}	(20)	0.35	0.05	0.10

Table 2

Model error assessment

Reed	f, Hz	MAPE, %	Syst. bias, %
Steel	90,5	11,3	5,2
Steel	152,5	8,5	0,2
Glassfiber	79,5	7,4	-6,4

Meanwhile, for the RV2 petal, there was virtually no need for additional weakening of the opening force as lift increased, since the coefficient β was accepted equaled zero.

However, the main result of the study is not so much the agreement between the calculated and experimental diagrams, but rather the creation of the table (Table 1) of model settings for different materials and operating modes.

The Table 1 allows us to see which coefficients remain relatively stable and which change significantly from one case to another. For the steel petal, most of the coefficients remain similar at two frequencies, and the most sensitive parameter is the upper-limit loss coefficient in the region of large deviations. For the non-metallic petal, there is not only the loss coefficient change, but also the coefficients of stiffness, mass, damping, and force action. Consequently, the model cannot be considered a scheme with a universal set of coefficients; however, its structure and calibration of the coefficients allow it to be adapted to a specific material and operating mode.

The obtained result, including an error of 7.4-11.3% with a systematic bias from -6.4 to 5.2% (Table 2), does not reveal any unique properties of the model. However, it does demonstrate the model's overall stability and the possibility of controlling its properties using correction coefficients. Consequently, the achieved result can be considered an extension of the classical engineering approach based on targeted tuning using experimental data.

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From a practical standpoint, this determines the model's applicability. It remains significantly simpler than 3-dimensional FSI models, does not require calculations of distributed petal deformation or spatial air flows, and is therefore suitable for inclusion in zero-dimensional and one-dimensional models of pulsating motors [26]. This is its main advantage: with a relatively simple structure, it allows agreement with experiment for comparable dimensions and within a similar operating range.

However, the model contains limitations. It does not describe the distributed nature of petal lift and does not account for higher vibration modes. Therefore, its application should be limited to the range of modes and designs for which verification or similarly meaningful coefficient tuning has been performed. Thus, the proposed single-mass model should be considered as an intermediate, however practically useful option between the simplest un-adjusted engineering schemes and extremely resource-intensive FSI models.

The method for implementing the work using artificial intelligence at all stages should be separately assessed. The obtained result demonstrates that proper problem formulation, continuous monitoring, and guidance of AI actions in model development and verification at all stages ensures not only high speed and quality of research, but also a high degree of elaboration of the processes within the adopted concept. This demonstrates the exceptionally high, but still underestimated, potential of using AI in scientific research.

Conclusions

1. Analysis of the completed research revealed that both classic single-mass engineering models and more complex three-dimensional models exist for reed valves. Simple models cannot reliably describe reed motion without verification and specialized adjusting, while three-dimensional models fit poorly into engineering thermodynamic and one-dimensional gas-dynamic models of the pulse jet engine operating process.
2. A single-mass mathematical model of reed valve motion was developed with help of AI ChatGPT, based on mechanical analogy of the "mass-spring-damper" type, but supplemented with a system of correction coefficients that account for the deviation of actual reed valve behavior from the idealized cantilever configuration.
3. A computational program in the Python environment was completely written by AI ChatGPT, implementing an algorithm for the numerical integration of the valve petal motion equation, suitable for use in the existing thermodynamic model of a pulse jet engine and its further development towards a one-dimensional gas dynamic description of the operating process.
4. Using experimental data, the parameters of the developed model were adjusted to the petal lift curves, and the values of the correction coefficients were determined. Testing the model for a steel petal at 90.5 and 152.5 Hz, as well as a non-metallic glassfiber petal at 79.5 Hz, showed that the proposed single-mass approach provides sufficiently accurate agreement between the calculated and experimental dependencies. The model error turned out 7.4-11.3% with a systematic bias from -6.4 to 5.2%, suggesting that the model represents a reasonable development of the classical engineering approach based on targeted tuning using experimental data.
5. It was established that the main differences between the studied modes are concentrated in the range of large petal deviations, with the parameter responsible for energy losses in the maximum petal lift zone being the most sensitive to changes. This suggests that the remaining discrepancies are mainly associated with nonlinear boundary effects. As a result, when properly configured, the proposed single-mass model with correction coefficients occupies an intermediate position between the simplest un-adjusted engineering models and resource-intensive FSI models and can be recommended for use in engineering analysis and preliminary design of pulse jet engines.
6. The obtained result demonstrates that a proper problem statement, continuous monitoring, and guidance of AI efforts in model development and verification at all stages ensures not only high speed of work and quality of research, but also a high degree of elaboration of processes within the adopted concept.
7. Future research is planned to address issues of model verification and adjusting based on experimental flow rate characteristics of a reed valve, with subsequent implementation of the model in pulse jet engine computational models, as well as to quantitatively evaluate the advantages of its use in place of un-adjusted classic single-mass reed valve models.

Conflict of Interest

The author declares that he has no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

Financing

This study was conducted without financial support.

Data Availability

The manuscript has no associated data.

Use of Artificial Intelligence

The author has used artificial intelligence technologies within acceptable limits to provide his own verified data, as described in the research methodology section.

The author has read and agreed to the published version of this manuscript.

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РОЗРОБКА МАТЕМАТИЧНОЇ МОДЕЛІ ТА ПРОГРАМНОГО ЗАБЕЗПЕЧЕННЯ ДЛЯ МОДЕЛЮВАННЯ ДВИГУНІВ З ПЕРІОДИЧНИМ РОБОЧИМ ПРОЦЕСОМ З ВИКОРИСТАННЯМ СЕРВІСУ ШТУЧНОГО ІНТЕЛЕКТУ

О. Е. Хрулєв

У роботі розглянуто завдання побудови математичної моделі пелюсткового клапана, придатної для використання в термодинамічних нуль-мірних і одновимірних газодинамічних моделях робочого процесу пульсуючого повітряно-реактивного двигуна. **Предмет дослідження**: пелюстковий клапан пульсуючого двигуна. Показано, що прості одномасові класичні моделі пелюстки мають недостатню точність, унаслідок чого вимагають верифікації та налаштування. Однак, незважаючи на велику кількість публікацій з пелюсткових клапанів, кількість робіт, що містять експериментальні криві підйому пелюстки, достатніх для прямої верифікації в діапазоні частот, близьких до умов роботи пульсуючого двигуна, залишається обмеженим. **Метою роботи** є створення, за допомогою штучного інтелекту, одномасової математичної моделі руху пелюстки, основаної на механічній аналогії типу "маса-пружина-демпфер", але доповненою системою коригуючих коефіцієнтів. **Завдання**: розробити одномасову математичну модель пелюсткового клапана, алгоритм і програму чисельного розрахунку руху пелюстки, виконати налаштування параметрів моделі за експериментальними даними, перевірити працездатність моделі для різних конструкцій та на різних режимах роботи, а також оцінити її достовірність та застосовність. **Методи дослідження**. Для вирішення рівнянь руху пелюстки використано алгоритм чисельного інтегрування з урахуванням обмежень щодо підйому та втрат енергії при посадці на сідло та в області максимального підйому. В якості експериментальної бази використані опубліковані діаграми тиску та підйому для сталевої та неметалічної пелюсток. **Результати**. Виконано налаштування параметрів моделі за експериментальними кривими підйому для режимів 79,5 Гц, 90,5 Гц і 152 Гц. Установлено, що запропонована модель забезпечує гарне узгодження розрахункових і експериментальних значень підйому пелюстки в даному діапазоні режимів роботи з похибкою 7,4-11,3%. Показано, що основні відмінності між режимами зосереджено лише в сфері великих відхилень пелюстки. **Висновки**. Розроблена одномасова модель пелюсткового клапана з коригуючими коефіцієнтами займає проміжне положення між найпростішими неналаштованими інженерними моделями та ресурсомісткими просторовими моделями, що дозволяє рекомендувати її для застосування в задачах інженерного аналізу та попереднього проєктування пульсуючих двигунів.

Ключові слова: пульсуючий повітряно-реактивний двигун, пелюстковий клапан, одномасова модель, підйом пелюстки, чисельний розрахунок, моделювання, верифікація.

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