UDC 629.735.33.025.015.4 **doi: 10.32620/aktt.2026.3.02**

Serhii SAVINOV

National Aerospace University
"Kharkiv Aviation Institute", Kharkiv, Ukraine

DETERMINATION OF THE MOMENT OF INERTIA OF THE BEAM WEB STIFFNESS ELEMENT

*The **subject matter** of the article is the shear buckling behavior of a thin-walled beam web in an aircraft structure reinforced by longitudinal stiffeners under transverse shear loading. Particular attention is paid to the influence of the stiffener's rigidity on the critical shear stresses and the web's load-carrying capacity. The **aim** of the article is to develop an analytical approach for determining the required second moment of area of longitudinal stiffeners based on the conditions of global and local shear buckling of the beam web, and to provide a theoretical justification for widely used empirical design relations. The **tasks** to be solved are: to formulate a mechanical model of a stiffened beam web represented as an orthotropic plate with equivalent stiffness characteristics; to derive analytical expressions for critical shear stresses under global buckling conditions; to solve the inverse stability problem for determining the required stiffener second moment of area; to analyze local shear buckling of elementary web panels bounded by stiffeners; to establish a condition for simultaneous occurrence of global and local buckling; and to compare the obtained analytical results with known empirical formulas used in aerospace engineering practice. The **methods** used in the study are based on the classical theory of stability of thin-walled structures, in particular, the theory of shear buckling of orthotropic plates developed in fundamental NACA works, as well as analytical methods of structural mechanics and stability analysis. The following **results** were obtained: an analytical model describing the shear stability of a stiffened beam web was developed; closed-form expressions for determining the required stiffener second moment of area under global shear buckling conditions were derived; a relationship ensuring simultaneous satisfaction of global and local buckling criteria was established; it was shown that the obtained solutions provide a clear physical interpretation of the influence of geometric and material parameters on structural stability; and it was demonstrated that the derived analytical expressions reproduce the characteristic power-law dependence between stiffener rigidity and applied shear load observed in engineering practice. **Conclusions**. The developed analytical approach enables direct evaluation of the required stiffness of longitudinal stiffeners at the preliminary design stage of aircraft structures, accounting for both global and local shear buckling modes. The obtained results can be effectively used for rational structural design and optimization of thin-walled beam elements. The scientific novelty of the results obtained is as follows: an analytical solution to the inverse problem of determining the stiffener second moment of area based on global shear buckling conditions was proposed; a unified criterion for simultaneous consideration of global and local shear buckling was formulated; and a theoretical justification of empirical design dependencies widely used in aerospace engineering was provided on the basis of classical stability theory.*

Keywords: *shear loss of stability; orthotropic panel; thin-walled beam; stiffeners; moment of inertia of a stiffener; cylindrical stiffness.*

1. Introduction

1.1. Motivation

Beam elements with thin webs reinforced by longitudinal stiffeners constitute the fundamental load-carrying components of aerospace structures, including spars, ribs, stringers, and frame members. For such structures, the governing failure mode is often shear buckling of the web, which may occur well before the material reaches its ultimate stress limit [1].

Design practice [2] demonstrates that the stiffness of stiffeners has a significant influence on the critical shear force (or stress) of the beam web and, consequently, on the overall structural mass and efficiency. Insufficient stiffener stiffness leads to premature buckling, whereas excessive stiffness results in unjustified weight penalties. Therefore, the rational selection of the stiffener moment of inertia represents a key problem in optimal structural design.

In established engineering methodologies, stiffener design is frequently performed using empirical [4] or semi-empirical relations [5], including the widely applied formula given in [3], which is commonly used in preliminary and detailed design calculations. However, the physical background of such expressions is not always transparent, and their applicability is generally limited to the assumptions under which they were derived.

In this context, an analytical derivation of design formulas for the stiffener moment of inertia based directly on the shear buckling condition of a pseudo-orthotropic panel representing the beam web with stiffeners is

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

of clear practical and theoretical relevance.

1.2. State of the art

Shear buckling of stiffened thin panels has been extensively studied since the classical NACA investigations, where the beam web was modeled as an orthotropic plate with different cylindrical (bending) stiffnesses in the principal directions. In works such as [2], analytical expressions for the critical shear flow were obtained in terms of the stiffness components D_1, D_2 and the geometric parameters of the panel.

Further development of engineering approaches led to simplified formulas for determining the required stiffener stiffness. A well-known example is the relation presented in [3], which directly links the stiffener moment of inertia to beam geometry, applied load, and material elastic modulus. While such expressions are convenient for practical use, their theoretical transparency is limited, and they do not always allow a clear assessment of the influence of individual structural parameters.

In most sources, the stiffener moment of inertia is treated as a prescribed or iteratively selected parameter. In contrast, the inverse problem determining the required stiffener moment of inertia from the condition of a specified critical shear force has not been sufficiently addressed. This complicates the comparison between analytical models and empirical design formulas.

In addition, several non-aerospace design approaches, widely used in civil engineering [6], shipbuilding [7], and general mechanical engineering [8], provide alternative formulations. However, these methods typically yield overly conservative estimates and lead to excessive structural weight when applied to aerospace structures.

1.3. Objectives and tasks

The objective of the present work is to derive an analytical expression for the required stiffener moment of inertia of a beam based on the shear buckling condition of an orthotropic panel, and to compare the resulting formula with established relations used in aerospace engineering practice.

To achieve this objective, the following methodology is employed:

- an analytical framework based on the classical stability theory of orthotropic plates;
- a beam web model incorporating the contribution of stiffeners to the cylindrical (bending) stiffness of the equivalent panel;
- algebraic transformation of the buckling criterion to solve the inverse problem with respect to the stiffener moment of inertia.

The study addresses the following specific tasks:

- formalization of the influence of stiffeners on the stiffness components of the orthotropic panel;
- derivation of an analytical relationship for the stiffener moment of inertia as a function of the critical shear force;
- analysis of the obtained expression and comparison with classical empirical formulas.

2. Materials and methods of research

2.1. Global Shear Buckling of the Beam Web

When a transverse shear force acts on a thin-walled beam, the stress state of the web is governed predominantly by shear stresses. For beams with stiffened webs, buckling typically occurs in shear and may be interpreted as global shear buckling of an orthotropic panel reinforced by longitudinal stiffeners.

In classical studies [2], the beam web is modeled as an orthotropic plate whose bending (cylindrical) stiffness differs in mutually perpendicular directions. This orthotropy arises from the presence of stiffeners, which significantly increase the panel bending stiffness in the direction perpendicular to their orientation. In general, both vertical and horizontal stiffeners may be considered; however, a typical beam configuration contains only vertical stiffeners.

The critical shear flow for an orthotropic panel may be written in the form

$$N_{xy\,cr} = c_a \cdot \frac{\sqrt[4]{D_1 \cdot {D_2}^3}}{\left(\frac{b}{2}\right)^2},$$

where c_a is a coefficient accounting for panel aspect ratio and boundary conditions [2];

b is the characteristic dimension of the panel in the buckling wave direction;

D_1 and D_2 are the cylindrical (bending) stiffness components of the orthotropic panel.

The stiffness component D_1 is determined solely by the web properties. For an isotropic material,

$$D_1 = \frac{E_w \cdot t^3}{12 \cdot \left(1 - {\vartheta_w}^2\right)},$$

where E_w is the Young's modulus of the web material;

t is the web thickness;

ϑ_w is the Poisson's ratio of the web material.

The stiffness component D_2 accounts for both the web stiffness and the contribution of stiffeners and may be expressed as

$$D_2 = D_1 + \frac{E_s \cdot I_s}{d},$$

where E_s is the Young's modulus of the stiffener material;

I_s is the stiffener second moment of area about the axis parallel to the web;

d is the stiffener spacing.

This expression clearly illustrates the physical meaning of orthotropy: in the absence of stiffeners ($I_s = 0$), the panel is isotropic; as the stiffener stiffness increases, the bending stiffness in the corresponding direction increases accordingly.

The critical shear stress associated with global buckling follows from the critical shear flow:

$$\tau_{cr}^{global} = \frac{N_{xy\,cr} \cdot b}{b \cdot t} = \frac{N_{xy\,cr}}{t} = c_a \cdot \frac{\sqrt[4]{D_1 \cdot {D_2}^3}}{t \cdot \left(\frac{b}{2}\right)^2}.$$

The obtained expression directly relates the geometric parameters of the web, the stiffener stiffness, and the boundary conditions to the critical shear stresses associated with global buckling. In this formulation, the problem of determining the required stiffener stiffness is reduced to an inverse problem of stability theory, in which the stiffener moment of inertia is determined from the condition of achieving a prescribed level of critical shear stress.

This approach is particularly convenient for further analytical investigation, since it enables a direct comparison between relationships derived from the classical theory of orthotropic plates and well-known engineering or empirical formulas used in practical stiffener sizing. In contrast to purely empirical design relations, the present formulation preserves a clear link to the underlying mechanics of orthotropic plate stability.

The remaining orthotropic stiffness components do not influence the final expression for shear buckling. For an isotropic web, the following identity holds:

$$D_3 = D_{12} + 2 \cdot D_{66} = D_{12} + D_1 - D_{12} = D_1,$$

where the relation between D_{12} and D_{66} is governed by the Poisson's ratio of the web material. Consequently, the shear buckling problem reduces to analyzing the influence of stiffener stiffness on the orthotropic bending stiffness component D_2. This substantially simplifies the analytical treatment and clearly isolates the mechanical role of the stiffeners.

Substituting the expressions for D_1 and D_2 into the formula for the critical shear flow yields a relationship that directly connects the critical load with the stiffener moment of inertia. Subsequent algebraic manipulation makes it possible to transform the direct problem (determination of the critical shear force) into the inverse problem, in which the stiffener moment of inertia is determined from the requirement that a prescribed critical shear force be sustained.

As a result of these transformations, the following analytical expression for the stiffener moment of inertia is obtained:

$$I_s = \frac{\left(\sqrt[3]{\frac{\left(\frac{V \cdot b}{4 \cdot c_a}\right)^4}{\left(\frac{E_w \cdot t^3}{12 \cdot (1 - {\vartheta_w}^2)}\right)}} - \left(\frac{E_w \cdot t^3}{12 \cdot (1 - {\vartheta_w}^2)}\right)\right) \cdot d}{E_s}.$$

This expression explicitly demonstrates the dependence of the required stiffener stiffness on the web geometry, material elastic properties, and applied shear load. The formula possesses a clear physical interpretation. An increase in shear force V or panel width b leads to a power-law increase in the required stiffener moment of inertia, reflecting the nonlinear sensitivity of shear buckling to load intensity and panel dimensions. Conversely, an increase in web thickness t significantly enhances the intrinsic bending stiffness D_1 (through the cubic dependence on thickness), thereby reducing the stiffness requirements imposed on the stiffeners.

For comparison, the following empirical formula is widely used in design practice [2]:

$$I_s \approx 2.29 \cdot \frac{d}{t} \cdot \left(\frac{V \cdot b}{33 \cdot E}\right)^{\frac{4}{3}}.$$

This empirical relation exhibits a similar power-law dependence between the stiffener moment of inertia and the applied shear load, which indicates consistency in the general scaling behavior. However, it is derived from the generalization of experimental data rather than from first principles.

A comparison between this empirical expression and the analytical formula derived in the present work makes it possible to:

– identify the theoretical basis underlying empirical coefficients,

– determine the range of applicability of empirical design rules, and

– assess the degree of consistency between empirical approaches and classical orthotropic plate stability theory.

Figure 1 presents the dependence of the required stiffener moment of inertia on web thickness, comparing the existing empirical relation (I_{eg}) with the analytical expression derived in the present study (I_{ag}), while keeping the remaining parameters constant. The following parameters are used (as in [14]): $V = 4448$ N, $d = 0.508$ m, $h = 0.762$ m. Both the web and the stiffener are assumed

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

to be made of an aluminum alloy.

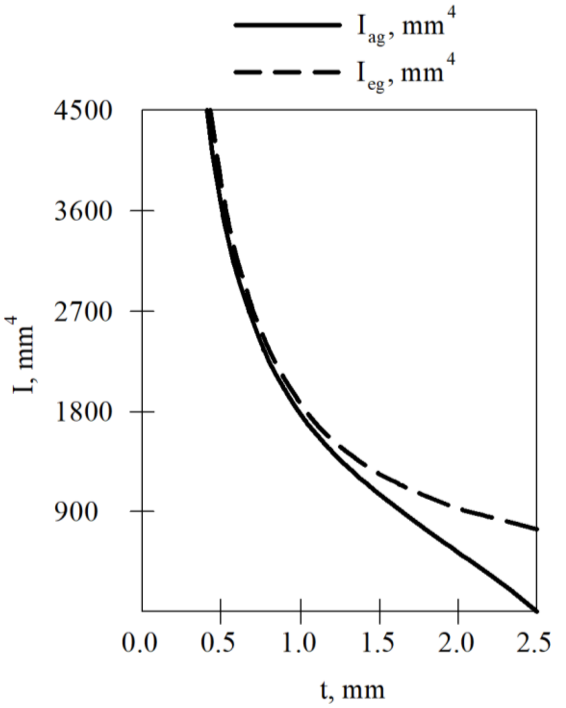

Fig. 1. Comparison of the derived and existing relationships for stiffener moment of inertia

2.2. Local Buckling of the Beam Web

In addition to global shear buckling of the beam web considered as an orthotropic panel, stiffened thin-walled structures may also experience local buckling of individual web segments bounded by stiffeners and beam flanges. This type of instability is characterized by the formation of localized buckling modes that do not extend over the entire panel but are confined to a single web bay (cell). In contrast to global buckling, which involves interaction between the web and stiffeners over a large region, local buckling develops within geometrically limited areas and is primarily governed by the properties of the web itself.

Local buckling may be modeled as shear buckling of an isotropic plate simply supported along its contour, with a characteristic dimension defined as [16]

$$b = \min(h, d),$$

where h – is the web height,

d – is the stiffener spacing.

This definition reflects the fact that the buckling half-wave is constrained either by the web height or by the distance between adjacent stiffeners, depending on which dimension is smaller. Consequently, the geometry of the web bay plays a decisive role in determining the local stability limit.

Within the framework of linear elasticity theory, the critical shear stress associated with local buckling may be written as:

$$\tau_{cr}^{local} = \eta_s \cdot \frac{\pi^2}{12 \cdot \left(1 - \vartheta_w{}^2\right)} \cdot k_s \cdot E_w \cdot \left(\frac{t}{b}\right)^2,$$

where k_s is the shear buckling coefficient, which depends on the aspect ratio of the web cell and the boundary conditions along its edges [9];

η_s is a reduction factor accounting for inelastic buckling beyond the proportional limit in the case of local instability (is given in [10] for hinged mounting, in [11] for fixed condition, in [12] for elastic supports and in [13] for various load cases);

E_w, t, ϑ_w are respectively the Young's modulus, thickness, and Poisson's ratio of the web material.

This expression shows that local buckling is governed primarily by the web geometry and material properties. It does not directly depend on stiffener stiffness, except through the influence of stiffeners on the effective cell dimension b_l. In other words, stiffeners define the geometric boundary of the buckling region but do not directly contribute to the bending stiffness of the locally buckling plate.

From the standpoint of rational structural design, it is undesirable for one buckling mode to occur significantly earlier than the other. If local buckling occurs at stresses substantially lower than the global buckling stress, the stiffeners are underutilized, and their stiffness is not effectively exploited. Conversely, if global buckling occurs first, the web thickness may be unnecessarily large, leading to excessive mass without improving structural efficiency.

For this reason, engineering practice frequently adopts the simultaneity condition for global and local buckling [1]:

$$\tau_{cr}^{local} = \tau_{cr}^{global}.$$

The critical stress associated with global shear buckling, derived in the previous section, is:

$$\tau_{cr}^{global} = c_a \cdot \frac{\sqrt[4]{D_1 \cdot D_2{}^3}}{t \cdot \left(\frac{b}{2}\right)^2} = 4 \cdot c_a \cdot \frac{\sqrt[4]{D_1 \cdot D_2{}^3}}{t \cdot b_g^2},$$

where b_g is the characteristic half-wave dimension associated with global buckling.

By equating the global and local critical stresses, one obtains an equation that relates the cylindrical stiffness components of the equivalent orthotropic panel to the geometric parameters of the local web cell. The simultaneity condition therefore reduces to an inverse problem of determining the stiffener moment of inertia required to ensure the simultaneous occurrence of both buckling modes.

After performing the necessary algebraic transformations, the following analytical expression for the stiffener moment of inertia is derived:

$$I_s = d \cdot \frac{\frac{E_w \cdot t^3}{12 \cdot \left(1 - \vartheta_w{}^2\right)} \cdot \left(\sqrt[3]{\frac{\eta_s{}^4 \cdot \pi^8 \cdot b_g^8}{4^4 \cdot c_a{}^4 \cdot \eta_g{}^4} \cdot \frac{k_s{}^4}{b_l^8}} - 1\right)}{E_s},$$

which defines the minimum required stiffener stiffness ensuring simultaneous global and local shear buckling.

Here, η_g is the reduction factor accounting for inelastic effects in global buckling beyond the proportional limit [15]. The presence of both η_g and η_s in the expression allows the model to incorporate post-proportional material behavior consistently for both instability modes.

The obtained relation provides a direct analytical tool for evaluating the influence of:

- beam geometry (web height, stiffener spacing, global half-wave dimension);
- web and stiffener material properties;
- boundary conditions;
- and inelastic reduction factors,

on the rational selection of the stiffener moment of inertia.

Structurally, the derived expression is consistent with established engineering design formulas used in practice. In particular, empirical relations that relate stiffener stiffness to cell dimensions and web thickness [2] may be written in the form

$$I_s = \frac{0.0217 \cdot d \cdot t^3}{\left(\frac{d}{h \cdot \sqrt{K_s}}\right)}.$$

Although empirical in origin, such expressions exhibit a structural similarity to the analytically derived formula, especially in the cubic dependence on web thickness and the proportionality to stiffener spacing. This confirms the mechanical consistency between empirical practice and classical stability theory.

Figure 2 illustrates the dependence of the required stiffener moment of inertia on web thickness, comparing the existing empirical relation (I_e) with the analytical expression derived in the present study (I_a), while keeping the remaining parameters constant (see Figure 1).

3. Results and Discussion

The derived analytical expression for the stringer moment of inertia I_s is obtained directly from the classical formula [2] for the critical shear flow of an orthotropic panel. Unlike empirical or semi-empirical design relations, the present derivation explicitly accounts for the coupling between the bending stiffness of the skin (web) and the discrete contribution of longitudinal stringers through the cylindrical stiffness component D_2. In this way, the reinforcing effect of stringers is introduced in a physically consistent manner within the framework of classical plate stability theory.

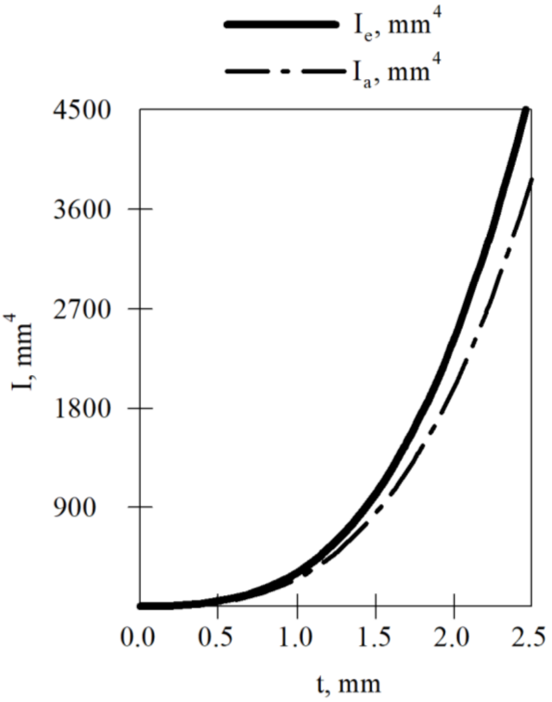

Fig. 2. Comparison of the derived and existing relationships for stiffener moment of inertia

A key feature of the first proposed formula is that the contribution of the stringers enters the buckling condition in a nonlinear manner, specifically through a cubic term. This reflects the physical fact that resistance to shear buckling is governed not only by the local stiffness of the plate but also by the global bending rigidity provided by the reinforcement system. As a consequence, even a moderate increase in stringer stiffness may result in a disproportionately large increase in the critical shear load. This nonlinear sensitivity is an inherent property of orthotropic panel stability and cannot be captured adequately by purely linear design approximations.

By isolating the stringer moment of inertia from the buckling condition, a closed-form analytical expression for I_s has been obtained. This enables direct evaluation of the required stringer stiffness for a prescribed shear load without resorting to iterative procedures. Importantly, the resulting formula preserves the original structure of the classical NACA solution for shear buckling and therefore remains fully consistent with the assumptions of classical plate theory.

It should be emphasized that the obtained value of the moment of inertia represents the minimum required stiffness necessary to prevent global buckling of the web

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

under the given transverse shear force and geometric parameters. Any lower stiffness would lead to instability prior to reaching the specified load level.

A comparison with the widely used engineering expression [3] shows that both relationships exhibit the same functional dependence on the applied shear load V and panel width b, namely a power-law behavior with exponent 4/3. This agreement confirms that the empirical formula implicitly reflects the same fundamental buckling mechanisms as the analytical model. However, the present derivation provides a clear theoretical justification for the value of this exponent and explains the origin of the numerical coefficients appearing in empirical design rules.

In contrast to the existing formula, which assumes isotropic behavior and condenses several physical effects into a single empirical constant, the derived expression explicitly incorporates the web modulus of elasticity E_w, the rib modulus of elasticity E_s, the web Poisson's ratio ϑ_w and the geometric parameter of stiffener spacing d. This makes the formulation particularly suitable for modern aerospace structures, where skins and stringers may be manufactured from different materials (including hybrid or composite configurations) or optimized with respect to spacing and stiffness distribution.

Furthermore, the analytical form clearly demonstrates the limiting behavior of the system. In the limit of vanishing stringer stiffness, the critical shear load asymptotically approaches the value corresponding to an unstiffened plate. Conversely, for very large values of I_s, the structural response becomes dominated by the stringers. Such asymptotic tendencies are concealed within simplified empirical relations but are fundamentally important for understanding load-transfer mechanisms and for developing rational optimization strategies.

Overall, the proposed formula may be regarded as a theoretically substantiated generalization of existing design rules. It enables transparent sensitivity studies with respect to material and geometric parameters and can serve as a reliable basis for preliminary sizing of stiffened panels subjected to shear loading particularly in applications where classical empirical coefficients are used outside their original calibration range.

A distinctive feature of the second formula is that it provides the minimum stiffener moment of inertia required to ensure equality of the critical stresses for local and global buckling. If the first formula demonstrates that global buckling does not occur at the specified load level, it logically follows that local buckling will also not occur under the same conditions. Thus, the second formulation provides a unified stability criterion ensuring simultaneous safety with respect to both instability modes.

Figure 3 presents, as an example, the dependence of the required stiffener moment of inertia on web thickness, comparing existing engineering expressions and the formulas derived in the present study (see Figure 1).

Additionally, the data on the Floor Beam from [17], [18], [19], [20] and [21] was processed and plotted on Figure 3. The data points show that, in practice, the beam stiffener have a higher moment of inertia than required. However, this value provides a safety margin.

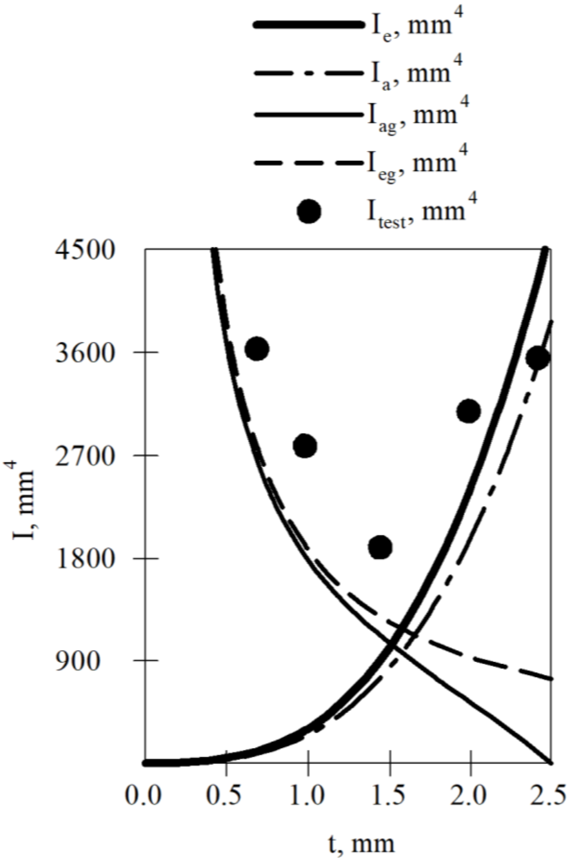

Fig. 3. Comparison of derived and existing relationships

As shown in Figure 3, the discrepancy between the existing and derived relations increases with increasing web thickness. This behavior likely results from the fact that the stiffness of the web itself was partially neglected or simplified in the development of the existing empirical relations. In contrast, the present formulation retains the explicit contribution of web bending stiffness, leading to increasing divergence at higher thickness values.

The plasticity reduction factors η_s and η_g may be determined from material stress-strain diagrams based on the Ramberg-Osgood equation. However, the use of such coefficients for materials with atypical deformation diagrams is not recommended without careful validation. It should be emphasized that all equations derived in this work are strictly exact within the framework of linear elastic behavior.

The coefficient c_a is purely geometric in nature and may be determined graphically, numerically (for example, using the finite element method), or from the condition of minimum total potential energy of the deforming system.

It should also be noted that the limiting behavior of the obtained expressions is physically consistent. For example:

- at zero stiffener spacing, zero web thickness, or zero web modulus of elasticity, the simultaneity condition yields a required stiffener moment of inertia equal to zero. In such cases, instability occurs in both the web and the stiffener at arbitrarily small load levels.

- at zero stiffener modulus of elasticity, the simultaneity condition requires an infinite value of the moment of inertia to compensate for the loss of stiffness. This conclusion also applies to the condition of preventing buckling altogether.

These limiting cases confirm the internal consistency and physical correctness of the derived analytical relations.

4. Conclusions

This study presents an analytical investigation of the influence of stiffeners on the shear buckling behavior of the web of a thin-walled beam and solves the inverse problem of determining the required stiffener moment of inertia to ensure a prescribed load-carrying capacity.

Based on the classical theory of shear buckling of orthotropic panels, a closed-form analytical expression for the stiffener moment of inertia has been derived. The formulation follows directly from the NACA-based stability solution and does not rely on empirical coefficients at the derivation stage. This provides a clear physical interpretation of how stiffener rigidity affects the global shear buckling resistance of the beam web.

It is shown that the resulting analytical relationship reproduces the characteristic power law with an exponent of 4/3 between the stiffener moment of inertia and the applied shear force, which is widely used in engineering practice. This confirms that commonly used empirical formulas have a theoretical foundation within the classical stability theory of orthotropic plates, and that their numerical coefficients can be interpreted through the stiffness parameters of the panel.

Unlike simplified design approaches, the proposed expressions explicitly account for the web stiffness contribution, material properties of both stiffeners and skin, Poisson’s ratio, panel geometry, and stiffener spacing. This significantly extends the applicability of the method, particularly for structures made of dissimilar materials or with non-standard reinforcement layouts.

The derived analytical relationships allow investigation of the limiting behavior of the “web-stiffener” system. It is demonstrated that for low stiffener rigidity, the panel response approaches that of an isotropic plate, whereas for large stiffener moments of inertia, the shear stability is governed predominantly by the stiffening system. This result is essential for understanding load transfer mechanisms and for making rational design decisions.

The condition of simultaneous global and local shear buckling has also been examined, and an analytical expression has been obtained for the minimum stiffener moment of inertia required to satisfy this condition. The proposed formulation is structurally consistent with existing engineering expressions, while providing a more physically grounded assessment of the influence of geometric and material parameters.

Comparison with well-known empirical relations shows that discrepancies increase with increasing web thickness, indicating the limitations of simplified formulas in which web stiffness is often neglected. The proposed analytical expressions eliminate this limitation and provide more physically consistent predictions.

Overall, the results may serve as a theoretically justified basis for preliminary design and optimization of thin-walled beam elements in aerospace structures subjected to shear loading. Future research should focus on incorporating post-buckling behavior, nonlinear material effects, and validation of the proposed relationships through numerical simulations and experimental studies.

Unlike envelope-type design formulas, the expressions derived in this work are applicable to arbitrary combinations of materials and stiffness ratios, since no structural component was neglected in the formulation.

The buckling of the stiffeners themselves was not considered, as typical stiffeners in such configurations are not directly load-carrying members. The design criteria for load-bearing stiffeners differ in nature and are governed primarily by strength requirements rather than by web shear stability.

A comparison of test data for actual beams with the required moment of inertia shows that the results of this study can be used as a minimum size, which will subsequently be increased in the event of rib loading or rounding of dimensions to standard values.

Conflict of Interest

The author declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The research was conducted without financial support.

Data availability

The work has no associated data.

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

Use of Artificial Intelligence

The author confirms that he did not use artificial intelligence methods while creating the presented work.

References

1. Fomychov, P. O., Zaruts'kyy, A. V., Mandzyuk, S. F. *Rozrakhunok na mitsnist' litaka. Chastyna 1* [Aircraft strength analysis. Part 1]. Kharkiv, National Aerospace University, 2017. 137 p. (In Ukrainian).

2. Seydel, E. *NACA Technical Memorandum No. 705. The critical shear load of rectangular plates*. Washington National Advisory Committee for Aeronautics, 1933. 27 p.

3. Bruhn, E. F. *Analysis and design of flight vehicle structures*. Cincinnati, Tri-State Offset Co., 1965. 880 p.

4. Kyrpikin, A. O. *Rozrakhunok na statychnu mitsnist' pererizu kryla velykoho podovzhennya. Chastyna 1–3* [Static strength analysis of high-aspect-ratio wing section. Parts 1–3]. Kharkiv, National Aerospace University, 2023. 88 p. (In Ukrainian).

5. Niu, M. *Airframe stress analysis & sizing*. Hong Kong, Technical Book Co., 2011. 500 p.

6. Bleich, F. *Buckling strength of metal structures*. New York, McGraw-Hill, 1952. 508 p.

7. Galambos, T. V. *Guide to stability design criteria for metal structures*. Wiley & Sons, 2008. 944 p.

8. Timoshenko, S. *Theory of elastic stability*. *2nd ed.* New York, McGraw-Hill, 1961. 541 p.

9. Chemerys, O. M., Kolodezhnyy, V. A., Trubachov, S. I. *Budivel'na mekhanika mashyn* [Structural mechanics of machines]. Kyiv, NTUU “Igor Sikorsky KPI”, 2017. 258 p. (In Ukrainian).

10. Gerard, G., Becker, H. *NACA Technical Note No. 3781. Handbook of structural stability. Part 1: Buckling of flat plates*. Washington, National Advisory Committee for Aeronautics, 1957. 106 p.

11. Abbott, R. *Analysis and design of composite and metallic flight vehicle structures*. Georgetown, Abbott Aerospace SEZC Ltd., 2019. 299 p.

12. Niu, M. *Airframe structural design*. Hong Kong, Technical Book Co., 1988. 612 p.

13. *Leichtbau: Bauelemente, Bemessungen und Konstruktionen von Flugzeugen und anderen Leichtbauwerken*. Berlin, Heidelberg, Springer, 1960. 769 p.

14. Coburn, T. *Aerospace Strength Handbook*. Pomona, Aircraft University, 2021, Vol. 1–2. 356 p.

15. Bougoffa, S., Durif, S., Mezghanni, O., Bouchaïr A., Daoud, A. Analysis of beam web panels with full length transverse stiffener in compression. *Engineering Structures*, 2023, vol. 297. 21 p. DOI: https://doi.org/10.1016/j.engstruct.2023.116996.

16. Javidinejad, A. *Standard Methods for Aerospace Stress Analysis*. Viejo, Wiley, 2025. 261 p.

17. Couture, S. Becoming an Aircraft Stress Engineer: A guide that bridges university with the aerospace industry. Canada, Aircraft Stress Co., 2025. 560 p.

18. Bui, H. C., Trinh, D. K., Nguyen, M. T. Effect of web stiffeners on lateral buckling of I-section simply supported beam. Steel Construction, 2022, vol. 15, iss. 2, pp. 121-130. DOI: https://doi.org/10.1002/stco.202100007.

19. Bougoffa, S., Mezghanni, O., Durif, S., Bouchair, A., Daoud, A. Full length transverse stiffener under compression. *ce/papers*, 2022, vol. 5, iss. 4, pp. 967–973. DOI: https://doi.org/10.1002/cepa.1841.

20. Nitawaki, M., Ishii, D. The plastic deformation capacity of the structural system for thin-web steel beam reinforced with stiffener plates. *ce/papers*, 2023, vol. 6, iss. 3–4, pp. 1603–1607. DOI: https://doi.org/10.1002/cepa.2409.

21. Mastan, S., Sekar, A., Nachair, S. S. Performance of composite steel-concrete beam with stiffener next to web opening. *Engineering Research Express*, 2024, vol. 6, no. 3. 15 p. DOI: https://doi.org/10.1088/2631-8695/ad6e56.

Received 21.04.2026, Received in revised form 07.06.2026
Accepted date 15.06.2026, Published date 17.06.2026

ПІДБІР МОМЕНТУ ІНЕРЦІЇ ЕЛЕМЕНТУ ЖОРСТКОСТІ СТІНКИ БАЛКИ

С. С. Савінов

Предметом вивчення в статті є поведінка зсувної втрати стійкості стінки тонкостінної балки в авіаційній конструкції, підкріпленій поздовжніми стрингерами, при дії поперечного зсувного навантаження. Особливу увагу приділено впливу жорсткості стрингерів на критичні зсувні напруження та несучу здатність стінки. **Метою** статті є розроблення аналітичного підходу для визначення необхідного моменту інерції поздовжніх стрингерів на основі умов глобальної та локальної зсувної втрати стійкості стінки балки, а також надання теоретичного обґрунтування широко використовуваних емпіричних розрахункових залежностей. **Завдання**: сформулювати механічну модель підкріпленої стінки балки, представленої у вигляді ортотропної пластини з еквівалентними характеристиками жорсткості; вивести аналітичні вирази для критичних зсувних напружень за умов глобальної втрати стійкості; розв’язати зворотну задачу стійкості для визначення необхідного моменту інерції стрингерів; проаналізувати локальну зсувну втрату стійкості елементарних панелей стінки, обмежених стрингерами; встановити умову одночасного виникнення глобальної та локальної втрати стійкості; порівняти отримані аналітичні результати з відомими емпіричними формулами, що використовуються в авіаційній інженерній практиці. **Методи** дослідження базуються на класичній теорії стійкості тонкостінних конструкцій, зокрема теорії зсувної втрати стійкості ортотропних пластин, розроблений у фундаментальних роботах NACA, а також на аналітичних методах будівельної механіки та аналізу стійкості. **Отримані результати**: розроблено аналітичну модель, що описує зсувну стійкість підкріпленої стінки балки; отримано замкнені аналітичні вирази для визначення необхідного моменту інерції стрингерів за умов глобальної зсувної втрати стійкості; встановлено співвідношення, яке забезпечує одночасне виконання критеріїв глобальної та локальної втрати стійкості; показано, що отримані рішення мають чітку фізичну інтерпретацію впливу геометричних та матеріальних параметрів на стійкість конструкції; продемонстровано, що виведені аналітичні залежності відтворюють характерну степеневу залежність між жорсткістю стрингерів і прикладеним зсувним навантаженням, яка спостерігається в інженерній практиці. **Висновки**. Розроблений аналітичний підхід дозволяє безпосередньо оцінювати необхідну жорсткість поздовжніх стрингерів на етапі попереднього проєктування авіаційних конструкцій з урахуванням як глобальних, так і локальних форм втрати стійкості при зсуві. Отримані результати можуть бути ефективно використані для раціонального проєктування та оптимізації тонкостінних балкових елементів. **Наукова новизна** отриманих результатів полягає в тому, що запропоновано аналітичне розв’язання оберненої задачі визначення моменту інерції стрингерів на основі умов глобальної зсувної втрати стійкості; сформульовано єдиний критерій для одночасного врахування глобальної та локальної втрати стійкості; а також надано теоретичне обґрунтування емпіричних розрахункових залежностей, широко застосовуваних в авіаційній інженерії, на основі класичної теорії стійкості.

Ключові слова: зсувна втрата стійкості; ортотропна панель; тонкостінна балка; ребра жорсткості; момент інерції ребра; циліндрична жорсткість.

Савінов Сергій Сергійович – аспірант, кафедра міцності літальних апаратів, Національний аерокосмічний університет «Харківський авіаційний інститут», Харків, Україна.

Savinov Serhii – Postgraduate Student of the Department of Aircraft Strength, National Aerospace University “Kharkiv Aviation Institute”, Kharkiv, Ukraine,
e-mail: s.s.savinov@student.khai.edu, ORCID: 0009-0008-8260-3900.