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Pavlo LUKIANOV, Kateryna PAVLOVA

National Aviation University, Kyiv, Ukraine

NONLINEAR MODEL OF INTERACTION OF UNSTEADY FLUID FLOW WITH STRUCTURE IN HYDRAULIC SYSTEMS OF AIRCRAFT AND HELICOPTERS

The subject of this work *is the development of a nonlinear model of the interaction of an unsteady fluid flow with a structure and finding analytical solutions for the system of equations that correspond to the specified model. The convection effect of the fluid velocity field was already considered in the previous works of the authors of this paper. These studies are devoted to the water hammer without considering the "flow-structure" interaction. This work expands the possibilities of modeling and considers four equations instead of two equations of the theory of the water hammer (equations of conservation of mass and momentum), two of which relate to the motion of particles of a solid body (pipes or structures). The novelty of this work lies in the consideration of the model that describes the interaction of the flow with the structure, the convection in the velocity field, and the effect, together with the friction of the fluid against the solid wall, on the dynamics of the shock pulse propagation process both in the fluid and in the solid body. It should be noted that the solution of the nonlinear system of differential equations as a whole is carried out by an analytical method, which makes it possible to obtain an exact (rather than numerical) solution of the problem. Since the effects of various factors should be evaluated by comparison with the main components, dimensionless equations containing six parameters (dimensionless combinations) were obtained in this study. Two of these parameters were named after scientists – Darcy and Weisbach (steady friction) and Bruno (unsteady friction). Particular cases of the general (full) model were considered, and the effects of various factors on the dynamics of the interaction of the flow with the structure during the propagation of the shock pulse were determined.* ***Research methods*** *are purely theoretical. The concepts of a self-similar equation and a system of equations, balances of forces acting on particles of a fluid and a solid body, and a standard method of reducing a system of equations to a single equivalent equation are used.* ***Conclusions.*** *An extended model of the interaction between the unsteady fluid flow and the structure is proposed. The transition to a self-similar variable makes it possible to solve a nonlinear system of differential equations and obtain an analytical (exact) solution. The functions of longitudinal stress in a solid body, pressure disturbance, and velocity of motion of particles in a solid body (pipe) are linearly expressed by the velocity of shock pulse propagation in the fluid. It should also be noted that the results for the particular case of the linear model completely agree with the already known ones. The advantage of using a self-similar solution is that it is easy to obtain. The results of previous studies on the water hammer problem were qualitatively consistent. As the fluid viscosity increases, the shock pulse domain becomes more concentrative.*

Keywords: *aircraft; helicopter; incompressible (droplet) fluid; flow-structure interaction; water hammer; stress; surface deformation; fatigue.*

Introduction

The phenomenon of water hammer is closely related to such concepts as stress, deformations of the pipeline surface, which occur under the influence of these stresses and lead to fatigue and destruction of the material. This, in turn, affects flight safety. Modern works already take into account the two-dimensionality and turbulent nature of the flow [1], elastic deformations of the pipeline surface [2], and the phenomenon of cavitation, which arise in this case [3]. In the end, all this leads to the fatigue destruction of the pipeline surface [4], which is impermeable in the systems of aviation and rocket and space technology.

In the works of the classics on water hammer [5] and [6], the unsteady flow of fluid in an elastic shell is considered, therefore, only one-sided influence of the elasticity of the pipe on the propagation of the shock pulse in the fluid. The mutual propagation of shock pulses in a fluid and a solid is considered in [7, 8], where a system of four equations describing the interaction of the flow with the structure is derived. The author of [7, 8] found solutions corresponding to non-dispersive waves propagating with two different speeds – in a pipe and in a liquid. He did not find a general solution to the system of four equations.

The exact solution of the linear hyperbolic system of four equations was found in [9]. The author [9] managed to find four values of speed, two positive and two negative. At the same time, the well-known method of characteristics is used. In this case, we are talking only about the longitudinal motion of a rigid body, and solutions for lateral and torsional motions [10] were not considered in [9]. An analytical solution based on the same model, taking into account the Poisson coupling and junction coupling, was obtained in [11].

Works [12, 13] show the importance of taking into account convective acceleration and steady friction in the model of water hammer, as well as gas impurity in the drop liquid [14]. By the way, in [5] convective acceleration is taken into account as well.

When passing a shock impulse, and for unsteady flow in general, friction between the fluid and a solid wall, both steady [15, 16] and unsteady [17, 18], should be taken into account. Therefore, it makes sense to consider the nonlinear four-equation model and find out the effects of the convection of the velocity field and also friction on the process of propagation of shock pulse in the fluid and structure.

The above review of literary sources indicates that today unsteady fluid flow and its interaction with an elastic pipeline are described by the following system of differential equations [9, 10]:

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f}\frac{\partial p}{\partial z} = 0, \qquad (1)$$

$$\frac{\partial V}{\partial z} + \left(\frac{1}{K} + \left(1-\nu^2\right)\frac{2R}{Ee}\right)\frac{\partial p}{\partial t} = 2\nu\frac{\partial U}{\partial z}, \qquad (2)$$

$$\frac{\partial U}{\partial t} - \frac{1}{\rho_t}\frac{\partial \sigma_z}{\partial z} = 0, \qquad (3)$$

$$\frac{\partial U}{\partial z} - \frac{1}{E}\frac{\partial \sigma_z}{\partial t} = -\frac{\nu R}{Ee}\frac{\partial p}{\partial t}. \qquad (4)$$

In the system of equations (1) – (4) V, U, p, σ_z are the velocities of motion of the fluid and the solid body, pressure disturbance and longitudinal stress respectively, t, z are time and longitudinal coordinate, respectively; K, E, ν are volume stiffness of the fluid, Young's modulus and Poisson's ratio of the solid body respectively; ρ_f, ρ_t, R, e are the densities of the liquid and the solid body (pipe), the inner radius of the pipe and its thickness.

The system of linear equations (1) – (4) can be analytically solved without applying the method of characteristics, see section 2. As previous studies have shown, taking into account the convection of the velocity field in the fluid and the friction of the fluid against the pipe wall is essential. Therefore, the modern friction model is considered further in the work [17].

1. Problem formulation

Develop a non-linear model of the interaction of unsteady fluid flow with a flexible pipeline, which takes into account the convection of the fluid velocity field, the friction of the fluid against the wall (both stationary and unsteady).

Obtain a self-similar solution containing analytical expressions for the velocity fields of fluid and pipe, as well as pressure disturbances and longitudinal stresses in the pipe.

Based on the obtained solution, study:

– influence of convection of the fluid velocity field;

– the effect of steady friction;

– the effect of unsteady friction.

2. Self-similar solution of the linear problem of the interaction of an unsteady fluid flow with an elastic shell

The system of equations (1) – (4) is easily reduced to four self-similar equations relative to the self-similar variable

$$\eta(z,t) = z - ct\,. \qquad (5)$$

We have the following system of equations

$$-c\frac{dV}{d\eta} + \frac{1}{\rho_f}\frac{dp}{d\eta} = 0, \qquad (6)$$

$$\frac{dV}{d\eta} - \left(\frac{1}{K} + \left(1-\nu^2\right)\frac{2R}{Ee}\right)c\frac{dp}{d\eta} = 2\nu\frac{dU}{d\eta}, \qquad (7)$$

$$-c\frac{dU}{d\eta} - \frac{1}{\rho_t}\frac{d\sigma_z}{d\eta} = 0, \qquad (8)$$

$$\frac{dU}{d\eta} + \frac{c}{E}\frac{d\sigma_z}{d\eta} = \frac{c\nu R}{Ee}\frac{dp}{d\eta}. \qquad (9)$$

Let's exclude speed V from (6), (7) and stress σ_z from (8), (9). We obtain the following system of equations:

$$\frac{dU}{d\eta} + \frac{c}{E}\frac{d\sigma_z}{d\eta} = \frac{c\nu R}{Ee}\frac{dp}{d\eta}, \qquad (10)$$

$$\left(\frac{E}{c} - c\rho_t\right)\frac{dU}{d\eta} = \frac{\nu R}{e}\frac{dp}{d\eta}. \qquad (11)$$

Pressure can be excluded from the system of equations (10), (11) and then the following equation for the velocity of motion of pipe particles is obtained:

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$$\left[\frac{1}{\rho_f}-c^2\left(\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)\right]\frac{e}{\nu R}\left(\frac{E}{c}-c\rho_t\right)\frac{dU}{d\eta}=2\nu c\frac{dU}{d\eta}. \tag{12}$$

Equation (12) is interesting in that, on the one hand, it indicates the possibility of the existence of a constant speed of the propagation of disturbances inside a solid body (pipe), and on the other hand, it provides an opportunity to find these speeds. In mathematical language, (12) is equivalent to the following two independent equations:

$$\left[\frac{1}{\rho_f}-c^2\left(\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)\right]\frac{e}{\nu R}\left(\frac{E}{c}-c\rho_t\right)=2\nu c,$$
$$\frac{dU}{d\eta}=0. \tag{13}$$

If we analyze the set of solutions of system (13), it turns out that the pipe can move at any constant speed. This may be caused by the unsteady motion of the fluid or rather its speed. The propagation of the shock pulse causes the propagation of the pulse in the pipe. On the other hand, if the velocity of motion of the pipe particles corresponds to the solution of the first equation (13), then it can be different, that is, a function of the self-similar variable. But this is not possible, because the first equation gives only a constant value for the propagation of the disturbance wave. So, all that remains is to find these values.

The first equation (13) can be written in a more compact (convenient) form:

$$B\rho_t c^4-\left(\frac{\rho_t}{\rho_f}+BE+D\right)c^2-\frac{E}{\rho_f}=0, \tag{14}$$

where $B=\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}$, $D=\frac{2\nu^2R}{e}$.

Since the discriminant of the biquadratic equation (14) has the form

$$Dis=\left(\frac{\rho_t}{\rho_f}-BE\right)^2+2\left(\frac{\rho_t}{\rho_f}+BE\right)D+D^2>0,$$

then the square of the velocity c^2 must be real. The analysis of the values provides a basis for the assertion of the presence of four values of speed:

two positive ones

$$c_1=\left[\left(\left(\frac{\rho_t}{\rho_f}+BE+D\right)+\sqrt{Dis}\right)/2B\rho_t\right]^{1/2},$$

$$c_3=\left[\left(\left(\frac{\rho_t}{\rho_f}+BE+D\right)-\sqrt{Dis}\right)/2B\rho_t\right]^{1/2},$$

and two negative ones

$$c_2=-\left[\left(\left(\frac{\rho_t}{\rho_f}+BE+D\right)+\sqrt{Dis}\right)/2B\rho_t\right]^{1/2},$$

$$c_4=-\left[\left(\left(\frac{\rho_t}{\rho_f}+BE+D\right)-\sqrt{Dis}\right)/2B\rho_t\right]^{1/2}.$$

Therefore, the motion of the pipe wall can occur with two velocities in the direction of the pulse, as well as in the opposite direction (negative values of the velocity) - with the same absolute values of the velocities. This is entirely consistent with the theory of water hammer, starting with work [19], as well as modern works [9], where four different velocity values are also found. Let's show it.

If we rewrite equation (14) in a form identical to equation (24) [9], that is, leaving the factor 1 at the term c^4, we will have:

$$c^4-\left(\frac{\rho_t}{\rho_f}+BE+D\right)/B\rho_t c^2-\frac{E}{\rho_f B\rho_t}=0. \tag{15}$$

It is easy to verify that in equation (15) the coefficient at c^2 is equal to:

$$-\frac{\left(\left(\frac{\rho_t}{\rho_f}+\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)E+\frac{2\nu^2R}{e}\right)}{\left(\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)}=$$
$$=\frac{1+\frac{2\nu^2R}{e}\frac{\rho_f}{\rho t}}{\rho_f\left(\frac{\rho_t}{\rho_f}+\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)}+\frac{E}{\rho_t}. \tag{16}$$

The right part of the expression (16) entirely coincides with the expression (25) [9], except that $\rho_t=\rho_s$: the tube index is replaced by a more general – structure. There is also a complete coincidence for the factor with an exponent-free term:

$$\frac{E}{\rho_f B\rho_t}=\frac{E}{\rho_f\rho_t\left(\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{Ee}\right)}. \tag{17}$$

For this, it is sufficient to compare expression (17) with relations (23) and (24) [9]. So, it is clearly proven that the obtained equation and its solution coincide with the already known ones, but the method of obtaining it is much simpler.

Now let's consider a more general model, which makes it possible to study the influence of fluid convection and friction of the fluid against the wall on the interaction of the fluid with the structure (pipe).

3. Extended model of flow interaction with the structure: consideration of convection, steady and unsteady friction

Previous studies [12-14] showed that for short pipelines with a small longitudinal spatial scale, convection is not negligibly small, and the friction of the fluid against the wall can also change the flow pattern. To take into account the influence of one or another factor, it is convenient to use dimensionless quantities. At first, let's give the equation of conservation of momentum, which now has the form:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial z}+\frac{1}{\rho_f}\frac{\partial p}{\partial z}+\frac{\lambda}{4R}V|V|+ +kD\left(\frac{\partial V}{\partial t}+c_f\cdot \mathrm{sign}(V)\left|\frac{\partial V}{\partial z}\right|\right)=0. \quad (18)$$

The fourth term corresponds to the Weisbach-Darcy steady friction component [15, 16], and the fifth term is taken from the Vitkovsky-Bruno model [17]. According to this model,

$$f=f_q+\frac{kD}{V|V|}\left(\frac{\partial V}{\partial t}+c_f\cdot \mathrm{sign}(V)\left|\frac{\partial V}{\partial z}\right|\right). \quad (19)$$

In formula (19) D is an inner diameter of the pipe; R is a pipe radius; c is the speed of sound in a fluid; k is the coefficient, which is determined through the Vardy coefficient C^* [13]:

$$k=\frac{\sqrt{\tilde{N}^*}}{2}=\begin{cases}0.5\sqrt{0.00476}, & \text{laminar flow;}\\ 7.41\cdot \mathrm{Re}^{-\left(\log 14.3/\mathrm{Re}^{0.05}\right)}, & \text{turbulent flow.}\end{cases}$$

A comparison of (19) with (18) indicates that

$$f_q=\frac{\lambda L}{4R}.$$

The remaining equations, i.e. (2) – (4), are unchanged. To move to dimensionless quantities, it is necessary to highlight the balance terms in equations [12, 13]. Thus, in equation (19), the balance terms will be those that make up the simplified model, i.e., equation (1), without taking into account convection and friction. So,

$$\left[\frac{\partial V}{\partial t}\right]=\left[\frac{1}{\rho_f}\frac{\partial p}{\partial z}\right]. \quad (20)$$

In equation (20), and also below, the square brackets mean the magnitude scale. Equation (20) is equivalent

$$\frac{[V]}{[t]}=\frac{[V]^2}{[z]}=\frac{1}{\rho_f}\frac{[p]}{[z]}.$$

whence

$$[p]=\rho_f[V]^2. \quad (21)$$

Let's divide all the terms of equation (18) by the scale of the balance terms, that is, by $[V]^2/[z]$. We obtain the following equation:

$$\frac{\partial \bar{V}}{\partial \bar{t}}+\bar{V}\frac{\partial \bar{V}}{\partial \bar{z}}+\frac{\partial \bar{p}}{\partial \bar{z}}+DW\cdot \bar{V}|\bar{V}|+ +Br\left(\frac{\partial \bar{V}}{\partial \bar{t}}+\mathrm{sign}(\bar{V})\left|\frac{\partial \bar{V}}{\partial \bar{z}}\right|\right)=0. \quad (22)$$

In equation (22), a dash above a value means it is dimensionless. In honor of those who studied the corresponding processes, we introduce the Darcy-Weisbach (DW) and Bruno (Br) numbers:

$$DW=\frac{\lambda L}{4R},\quad Br=\frac{kD}{L}.$$

Let us now proceed to obtaining the dimensionless analogue of equation (2). There is a new unknown in it, the velocity of motion of the particles of the pipe U. We will consider the speed of sound in the pipe as the scale of this speed c_t. Taking into account what has just been said, equation (2) has the following dimensionless form:

$$\frac{\partial \bar{V}}{\partial \bar{z}}+Nu_1\frac{\partial \bar{p}}{\partial \bar{t}}=Nu_2\frac{\partial \bar{U}}{\partial \bar{z}}. \quad (23)$$

The following dimensionless parameters are used in equation (23):

$$Nu_1=\left[\frac{1}{K}+\left(1-\nu^2\right)\frac{2R}{eE}\right]\rho_f c_f^2,\ Nu_2=\frac{2\nu c_t}{c_f}.$$

The dimensionless analogue of equation (3) has the form:

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$$\frac{\partial \bar{U}}{\partial \bar{t}} - \frac{\partial \bar{\sigma}_z}{\partial \bar{z}} = 0. \qquad (24)$$

There are only two terms in equation (24), so they make a balance and have the same order of values. From these considerations, the scale of longitudinal stresses is obtained:

$$[\sigma_z] = \rho_t c_f c_t. \qquad (25)$$

If we use the expression for the scale of longitudinal stresses (25) and substitute, together with the above-mentioned scales, into equation (4), we obtain:

$$\frac{\partial \bar{U}}{\partial \bar{z}} - Nu_3 \frac{\partial \bar{\sigma}_z}{\partial \bar{t}} = -Nu_4 \frac{\partial \bar{p}}{\partial \bar{t}}. \qquad (26)$$

The dimensionless parameters in (26) have the following form:

$$Nu_3 = \frac{\rho_t c_f^2}{E}, \quad Nu_4 = \frac{\nu R}{e} \frac{c_f}{c_t} \frac{c_f^2 \rho_f}{E}.$$

Therefore, the system of dimensionless equations (22), (23), (24), (26) is obtained, which takes into account both the convection of the fluid velocity field and the friction of the fluid against the wall, steady together with unsteady.

4. Shock pulse formation: self-similar equations and their solutions

As already mentioned in the introduction, for the numerical solution of the problem of shock pulse propagation in domains with complex geometry and in simple domains, but taking into account the reflection and interaction of waves, it is extremely important to know the structure of the fields of all characteristics at the conventionally initial moment of time when a shock pulse was formed. In this case, it is convenient to use the self-similar variable, but now in dimensionless form:

$$\bar{\eta}(\bar{z}, \bar{t}) = \bar{z} - \bar{t}. \qquad (27)$$

Substituting expression (27) into equations (22), (23), (24), (26) turns them into the following system:

$$\begin{aligned} &-\frac{d\bar{V}}{d\bar{\eta}} + \bar{V}\frac{d\bar{V}}{d\bar{\eta}} + \frac{d\bar{p}}{d\bar{\eta}} + DW \cdot \bar{V}\left|\bar{V}\right| + \\ &+ Br\left(-\frac{d\bar{V}}{d\bar{\eta}} + \mathrm{sign}(\bar{V})\left|\frac{d\bar{V}}{d\bar{\eta}}\right|\right) = 0, \end{aligned} \qquad (28)$$

$$\frac{d\bar{V}}{d\bar{\eta}} - Nu_1 \frac{d\bar{p}}{d\bar{\eta}} = Nu_2 \frac{d\bar{U}}{d\bar{\eta}}. \qquad (29)$$

$$-\frac{d\bar{U}}{d\bar{\eta}} = \frac{d\bar{\sigma}_z}{d\bar{\eta}}, \qquad (30)$$

$$\frac{d\bar{U}}{d\bar{\eta}} + Nu_3 \frac{d\bar{\sigma}_z}{d\bar{\eta}} = Nu_4 \frac{d\bar{p}}{d\bar{\eta}}. \qquad (31)$$

System of equations (28) – (31) should be added by four more conditions regarding the unknown. It is convenient to use the values of the functions on the characteristics, that is, when the value of the self-similar variable is equal to zero. This will be done after finding the general solution of the system (28) – (31).

The solution method for system (28) – (31) is usual: exclusion of unknown functions and reduction of the system to one equation. The advantage of using a self-similar variable is that there is no need to rise the order of the system, that is, to additionally take partial derivatives of the equations.

The first step is to express pressure in terms of velocities from equation (29):

$$\frac{d\bar{p}}{d\bar{\eta}} = \frac{1}{Nu_1}\left(\frac{d\bar{V}}{d\bar{\eta}} - Nu_2 \frac{d\bar{U}}{d\bar{\eta}}\right). \qquad (32)$$

Substituting (32) into (28), we obtain:

$$\begin{aligned} &-\frac{d\bar{V}}{d\bar{\eta}} + \bar{V}\frac{d\bar{V}}{d\bar{\eta}} + \frac{1}{Nu_1}\left(\frac{d\bar{V}}{d\bar{\eta}} - Nu_2 \frac{d\bar{U}}{d\bar{\eta}}\right) + DW \cdot \bar{V}\left|\bar{V}\right| + \\ &+ Br\left(-\frac{d\bar{V}}{d\bar{\eta}} + \mathrm{sign}(\bar{V})\left|\frac{d\bar{V}}{d\bar{\eta}}\right|\right) = 0. \end{aligned} \qquad (33)$$

As a second step, it is convenient to exclude the longitudinal stresses in the pipe from equations (30), (31), together with the use of relation (32):

$$\frac{d\bar{U}}{d\bar{\eta}} - Nu_3 \frac{d\bar{U}}{d\bar{\eta}} = \frac{Nu_4}{Nu_1}\left(\frac{d\bar{V}}{d\bar{\eta}} - Nu_2 \frac{d\bar{U}}{d\bar{\eta}}\right),$$

or, aggregating terms with identical derivatives, we have:

$$\left(1 - Nu_3 + \frac{Nu_4 Nu_2}{Nu_1}\right)\frac{d\bar{U}}{d\bar{\eta}} = \frac{Nu_4}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}}. \qquad (34)$$

The system of equations (33), (34) already contains only two unknown functions, the velocities of fluid particles and the pipe ones. Since the process of wave propagation in the pipe is described by a linear equation, it is convenient to exclude the velocity of the pipe particles. In a compact form, one can write:

$$\frac{d\bar{U}}{d\bar{\eta}} = Nu^* \frac{d\bar{V}}{d\bar{\eta}}, \quad (35)$$

where $Nu^* = \frac{Nu_4}{Nu_1} / \left(1 - Nu_3 + \frac{Nu_4 Nu_2}{Nu_1}\right)$.

Combining equations (35) and (33), we have:

$$-\frac{d\bar{V}}{d\bar{\eta}} + \bar{V}\frac{d\bar{V}}{d\bar{\eta}} + \frac{\left(1 - Nu_2 Nu^*\right)}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}} + DW \cdot \bar{V}\left|\bar{V}\right| + \\ + Br\left(-\frac{d\bar{V}}{d\bar{\eta}} + sign(\bar{V})\left|\frac{d\bar{V}}{d\bar{\eta}}\right|\right) = 0. \quad (36)$$

Equation (36) already contains only one unknown function and is essentially equivalent to the system of equations (31) – (33). In order to finally obtain the "working" equation, let's add two "switches" of signs to (36), that are multipliers SW1 and SW2. So, we have:

$$\left(SW1 \cdot B^* + \bar{V}\right)\frac{d\bar{V}}{d\bar{\eta}} + SW2 \cdot DW \cdot \bar{V}^2 = 0. \quad (37)$$

Expressions for SW1 and SW2 are easily obtained by comparing (36) with (37). Their explicit form as follows:

$$SW1 \cdot B^* = \begin{cases} -1 + \frac{1 - Nu_2 Nu^*}{Nu_1}, & \text{for } \bar{V} \cdot \frac{d\bar{V}}{d\bar{\eta}} > 0; \\ -1 + \frac{1 - Nu_2 Nu^*}{Nu_1} - 2Br, & \text{for } \bar{V} \cdot \frac{d\bar{V}}{d\bar{\eta}} < 0 . \end{cases}$$

$$SW2 = sign(\bar{V}) = \begin{cases} 1, & \bar{V} > 0; \\ -1, & \bar{V} < 0. \end{cases}$$

The general solution of equation (37) is as follows:

$$\bar{V}(\bar{\eta}) = \exp\left[-DW \cdot SW2\left(C_1 + \bar{\eta}\right) + \right. \\ \left. + LambertW\left(SW1 \cdot B^* \exp\left[DW \cdot SW2\left(C_1 + \bar{\eta}\right)\right]\right)\right]. \quad (38)$$

The fact that the order of the differential equations has not been increased now makes it easy to find the rest of the unknown functions. They are:

$$\bar{U}(\bar{\eta}) = Nu^* \bar{V}(\bar{\eta}) + Const_1, \\ \bar{\sigma}_z(\bar{\eta}) = -Nu^* \bar{V}(\bar{\eta}) + Const_2, \quad (39) \\ \bar{p}(\bar{\eta}) = \frac{1}{Nu_1}(1 - Nu_2)\bar{V}(\bar{\eta}) + Const_3$$

with $Const_1$, $Const_2$, $Const_3$ to be integrating constants.

It is convenient to determine the integration constants in (38), (39) starting from particular cases, that is, simpler models. Therefore, the use of a self-similar variable made it possible to obtain the simplest form of solving the nonlinear problem of shock pulse propagation in an elastic shell with an incompressible fluid in unsteady motion (water hammer).

5. The effects of convection and friction on the interaction of the unsteady flow with the structure

5.1. Only convection is taken into account

In this case, the parameters in equation (28)

$$DW = 0, \quad Br = 0.$$

Equation (36), which is equivalent to the system of equations (28) – (31), is simplified to the following:

$$-\frac{d\bar{V}}{d\bar{\eta}} + \bar{V}\frac{d\bar{V}}{d\bar{\eta}} + \frac{\left(1 - Nu_2 Nu^*\right)}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}} = 0,$$

or to a form convenient for solving:

$$\left(\bar{V} - 1 + \frac{\left(1 - Nu_2 Nu^*\right)}{Nu_1}\right)\frac{d\bar{V}}{d\bar{\eta}} = 0. \quad (40)$$

The solution of equation (40) can only be a constant value of the shock pulse propagation speed in the fluid. It is obvious that

$$\bar{V} = 1 - \frac{\left(1 - Nu_2 Nu^*\right)}{Nu_1}. \quad (41)$$

As shown by formula (41), the speed of propagation of an impulse in a fluid differs, in dimensionless quantities, by a certain value that depends on three parameters, which are functions of the characteristics of the "fluid-pipe" system. The remaining characteristics of the system are found by formulas (39). As you can easily see, they also have constant values. Therefore, convection in the fluid velocity field only changes the value of the shock pulse propagation speed, but the functional character (constant value) does not.

Let us dwell on one more interesting question: does the velocity function, expression (41),

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$$\bar{V}=\bar{V}\left(Nu_1,Nu_2,Nu^*\right)$$

reach local or global extreme? According to the theory [20], the necessary conditions for the extreme of a function of several variables are:

$$\frac{\partial\bar{V}}{\partial Nu_1}=\frac{1-Nu_2Nu^*}{Nu_1^2}=0,\quad \frac{\partial\bar{V}}{\partial Nu_2}=\frac{Nu^*}{Nu_1}=0,\quad \frac{\partial\bar{V}}{\partial Nu^*}=\frac{Nu_2}{Nu_1}=0. \tag{42}$$

Since the third equation (42) is never met, this is enough to conclude that there is no extreme of the shock pulse propagation velocity function in the fluid.

5.2. Considering only unstedy friction

If the convective term and unsteady friction are not taken into account, then equation (36) will have the following form:

$$-\frac{d\bar{V}}{d\bar{\eta}}+\frac{\left(1-Nu_2Nu^*\right)}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}}+DW\cdot\bar{V}\left|\bar{V}\right|=0. \tag{43}$$

The solution of equation (43) is the following function:

$$\bar{V}(\bar{\eta})=\begin{cases}\exp\left[-fc\left(C_1+\bar{\eta}\right)+\right.\\ \left.+LW\left(\frac{\left(1-Nu_2Nu^*\right)}{Nu_1}\cdot\exp\left[fc\left(C_1+\bar{\eta}\right)\right]\right)\right], & \bar{V}\frac{d\bar{V}}{d\eta}>0;\\ \exp\left[-fc\left(C_1+\bar{\eta}\right)+\right.\\ \left.+LW\left(\frac{\left(1-Nu_2Nu^*-2Nu_1\right)}{Nu_1}\cdot\exp\left[fc\left(C_1+\bar{\eta}\right)\right]\right)\right], & V\frac{d\bar{V}}{d\eta}<0\ .\end{cases} \tag{44}$$

In (44) $fc=DW\cdot sign\left(\bar{V}\right)$, $LW=LambertW$.

5.3. Consideration of unsteady friction or convection and steady friction

If both steady and unsteady friction of the fluid against the pipe are taken into account at the same time, but the convective term is neglected, we have:

$$-\frac{d\bar{V}}{d\bar{\eta}}+\frac{\left(1-Nu_2Nu^*\right)}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}}+DW\cdot\bar{V}\left|\bar{V}\right|+ +Br\left(-\frac{d\bar{V}}{d\bar{\eta}}+c\cdot sign(\bar{V})\left|\frac{d\bar{V}}{d\bar{\eta}}\right|\right)=0. \tag{45}$$

Equation (40), which is more convenient to use now, will turn into the following:

$$SW1\cdot B^*\frac{d\bar{V}}{d\bar{\eta}}+SW2\cdot DW\cdot\bar{V}^2=0. \tag{46}$$

The general solution of equation (46) is:

$$\bar{V}(\bar{\eta})=\frac{1}{C_1+DW\cdot SW2/\left(SW1\cdot B^*\right)\cdot\bar{\eta}}.$$

Although this solution has an already known hyperbolic distribution (compare with (13) from work [8]), however, there is a problem with its use. If the condition

$$\bar{V}\left(\bar{\eta}=0\right)=1$$

is met then we obtain the explicit form:

$$\bar{V}(\bar{\eta})=\frac{1}{1+DW\cdot SW2/\left(SW1\cdot B^*\right)\cdot\bar{\eta}}. \tag{47}$$

With the calculated values of the parameters in formula (47), a negative value is obtained for the self-similar variable, which leads to a contradiction with the condition from which (47) was obtained: when the self-similar variable increases, the velocity does not decrease, as it corresponds to the general solution, and on the contrary the velocity grows. A simple conclusion can be drawn from this: it is impossible to discard convective acceleration and keep only friction in the model.

Finally, we will answer the question about the influence of unsteady friction. For this, we consider another partial case (36), when steady friction and convection are taken into account and compare the obtained results with the general solution. Equation (36) turns into the following in this case:

$$-\frac{d\bar{V}}{d\bar{\eta}}+\bar{V}\frac{d\bar{V}}{d\bar{\eta}}+\frac{\left(1-Nu_2Nu^*\right)}{Nu_1}\frac{d\bar{V}}{d\bar{\eta}}+ +DW\cdot\bar{V}\left|\bar{V}\right|=0. \tag{48}$$

The solution of equation (48) is a partial case of (38) with. The results calculated according to (48) are presented in Fig. 1.

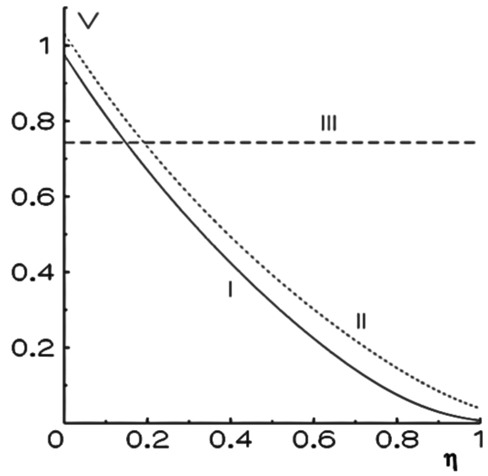

Fig. 1. Dimensionless velocity of shock pulse propagation in water: I – general solution (formula (38)); II – without taking into account unsteady friction (formula (39)); III – without taking into account friction (formula (41))

The following values were used for calculations:

$$DW = 1.25,\ SW \cdot B^{*} = -0.2543,\ SW \cdot B^{*} = -0.21,$$
$$Nu_1 = 1.0825,\ Nu_2 = 1.7233,\ Nu_3 = 0.10456,$$
$$Nu_4 = 0.096,\ Nu^{*} = 0.08462,\ C_1 = -0.18.$$

It can be seen that according to the model, which takes into account only convection and does not take into account friction, an underestimated constant value (curve III) is obtained for the speed of propagation of the shock pulse (water hammer shock). Neglecting the unsteady component of friction gives a slightly larger value (curve II) than compared to the complete model.

Since the lubricant itself is under high pressure in aviation equipment, calculations were also made for it. The following values of dimensionless parameters are obtained for AMG-10 aviation oil:

$$DW = 6.4;\ SW1 \cdot B^{*} = -0.210;\ C_1 = -0.033.$$

The functional dependence of the distribution of the speed of propagation of the shock pulse in the lubricant (see Fig. 2) indicates a significant, in comparison with water, concentration of the domain of non-zero values, approximately four times. Therefore, the combination of viscosity and convection in the case of an unsteady flow can lead to the concentration of energy in a finite region.

Discussion

The propagation of a water hammer, or more precisely a shock pulse, as an example of an unsteady flow can be considered within the framework of various models. For long pipelines, where the spatial scale is large, the convective term in the momentum conservation equation of fluid motion is insignificant. But in a technical devices, in particular in aviation equipment, the length of the pipeline is not significant. Therefore, consideration of convection is appropriate. In addition, the pipelines through which the fluid flows are not completely rigid and the pressure pulse causes the motion of the pipeline particles. From a physical point of view, we have the problem of the interaction of the flow with the structure. Available sources indicate the absence of models that simultaneously take into account the convection of the fluid velocity field and modern models for describing the friction of the fluid against the pipe. In addition, the approach used to solve the mathematical problem is based on additional differentiation of the equations, which leads (may lead) to the expansion of the set of solutions to the problem. These constraints may be unphysical, or rather, may not correspond to the original system of differential equations. An alternative to this approach is the use of self-similar equations. Although this use has a time limit (a time limit for the initial propagation of shock pulse until it encounters a boundary on it's way), the resulting functional dependences are very important for specifying conditionally initial distributions of all quantities for further numerical solution of the problem.

Conclusions

The paper considers the problem of the interaction of an unsteady flow (shock pulse) of a fluid with a structure (pipe). An analytical solution to this problem was obtained. The novelty is the simultaneous consideration of three factors: convection of the velocity field in the fluid, steady and unsteady friction of the fluid against a solid surface. If convection is not taken into account, then the disturbances of all fields propagate with constant velocities. The analytical solution of the problem by neglecting both convection and friction coincides with the already known one, but the method of obtaining it is simpler. Taking friction into account in the model, and accordingly in the solution of the problem, leads to a different from a constant value of the functional dependence of the velocity of propagation of the shock pulse, as well as other characteristics that are linear functions of the velocity in the fluid. At beginning (at $0 < \overline{\eta} < 0.5$), an approximately linear law of decreasing velocity is observed, which is consistent with previous studies [13, 14]. When approaching $\overline{\eta} \to 1$, the speed goes to zero according to a

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non-linear law. The effect of the unsteady friction component is that, without taking this component into account, higher values of velocity are obtained. In addition, with an increase in viscosity, the shock pulse domain is concentrated.

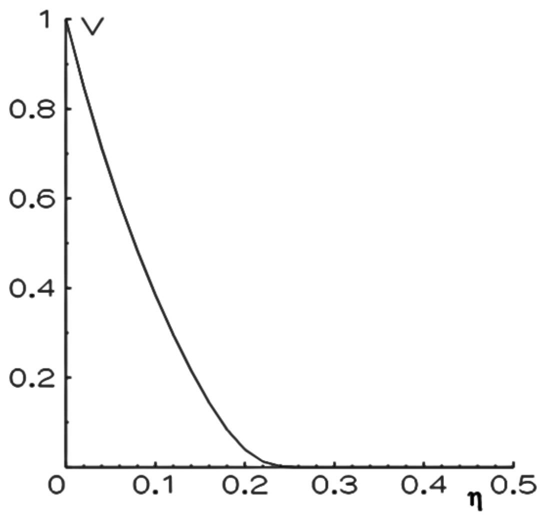

Fig. 2. Dimensionless propagation speed of shock pulse in aircraft lubricant

As further research, it is possible to use the obtained analytical solutions for numerical modeling of propagation processes, reflection of shock waves in pipelines, and more complex structures.

Contribution of authors: conceptualization – **Pavlo Lukianov**, **Katerina Pavlova**; formulation of task – **Pavlo Lukianov**; analysis – **Pavlo Lukianov**, **Katerina avlova**; software – **Katerina Pavlova**; development of mathematical model – **Pavlo Lukianov, Katerina-Pavlova**; analysis of results – **Pavlo Lukianov**, **Katerina Pavlova**.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

The Manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence methods while creating the presented work.

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НЕЛІНІЙНА МОДЕЛЬ ВЗАЄМОДІЇ НЕСТАЦІОНАРНОЇ ТЕЧІЇ РІДИНИ ЗІ СТРУКТУРОЮ В ГІДРАВЛІЧНИХ СИСТЕМАХ ЛІТАКІВ ТА ВЕРТОЛЬОТІВ

П. В. Лук'янов, К. С. Павлова

Предметом даної роботи є розробка нелінійної моделі взаємодії нестаціонарної течії рідини зі структурою та знаходження аналітичних розв'язків системи рівнянь, яка відповідає зазначеній моделі. Вплив конвекції поля швидкості рідини вже було враховано у попередніх роботах авторів цієї статті. Ці роботи присвячені гідравлічному ударові без урахування взаємодії «рідина-структура». Дана робота розширює можливості моделювання і розглядає вже замість двох рівнянь теорії гідравлічного удару (рівняння збереження маси та кількості руху) чотири рівняння, два з яких відносяться до руху частинок твердого тіла (труби або структури). Новизною даної роботи є саме урахування в моделі, що описує взаємодію течії із структурою, конвекції поля швидкості та вплив, разом із тертям рідини о тверду стінку, на динаміку процесу поширення ударного імпульсу як у рідині так і в твердому тілі. Слід особливо відзначити, що розв'язання в цілому нелінійної системи диференціальних рівнянь здійснюється аналітичним методом, що дає змогу отримати точний (а не чисельний) розв'язок задачі. Оскільки оцінку впливу різних чинників слід здійснювати шляхом порівняння із основними складовими, в роботі отримана система безрозмірних рівнянь, яка містить шість параметрів (безрозмірних комбінацій). Двоє з цих параметрів названо на честь вчених – Дарсі та Вейсбаха (стаціонарне тертя)і Бруно (нестаціонарне тертя). Розглянуто частинні випадки загальної (повної) моделі: визначено вплив різних чинників на динаміку взаємодії течії зі структурою під час поширення ударного імпульсу. **Методи досліджень** є суто теоретичними. Використовується поняття автомодельного рівняння та системи рівнянь, балансів сил, що діють на частинки рідини та твердого тіла, а також стандартний метод зведення системи рівнянь до одного – еквівалентного рівняння. **Висновки.** Запропонована узагальнена модель взаємодії нестаціонарної течії рідини зі структурою. Перехід до автомодельної змінної дозволив розв'язати нелінійну систему диференціальних рівнянь і отримати аналітичний (точний) розв'язок. Функції повздовжнього напруження в твердому тілі,

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збурення тиску та швидкості руху частинок твердого тіла (труби) лінійно виражаються за швидкістю поширення ударного імпульсу в рідині. Слід також відзначити, що результати за частинним випадком лінійної моделі повністю збігається з вже відомими. Але перевага використання автомодельного розв'язку очевидна – це простота його отримання. Отримано також якісний збіг із результатами попередніх досліджень задачі гідравлічного удару. Порівняння результатів, отриманих для води і авіаційного мастила, вказують на концентрацію енергії ударного імпульсу зі зростанням в'язкості рідини.

Ключові слова: літак; вертоліт; нестислива (крапільна) рідина; взаємодія течії зі структурою; гідравлічний удар; напруження; деформація поверхні; втома.

Лук'янов Павло Володимирович – канд. фіз.-мат. наук, старш. наук. співроб., доц. каф. гідрогазових систем, Національний авіаційний університет, Київ, Україна.

Павлова Катерина Сергіївна – магістр, асп. каф. гідрогазових систем, Національний авіаційний університет, Київ, Україна.

Pavlo Lukianov – Candidate of Physics and Mathematics Sciences, Senior Researcher, Associate Professor at the Hydro-Gas Systems Department, National Aviation University, Kyiv, Ukraine,
e-mail: Pavlo.Lukianov@npp.nau.edu.ua, ORCID: 0000-0002-5043-6182.

Kateryna Pavlova – Master of Technical Sciences, PhD Student of the Hydro-Gas Systems Department, National Aviation University, Kyiv, Ukraine,
e-mail: pavlovadazv@gmail.com, ORCID: 0000-0001-8818-4358.