UDK 621.735 R.U. Tsukanov

TRANSPORT CATEGORY AIRPLANE FLIGHT RANGE CALCULATION ACCOUNTING CENTER-OF-GRAVITY POSITION SHIFT WITH FUEL USAGE

Introduction

The main problem facing with world commercial aviation and practically determining competitiveness of transport category airplanes (TCA) is operating costs decreasing, main part of which is fuel expenses.

One of the ways to increase TCA fuel efficiency is fuel trim transfer (FTT) application, which allows to decrease airplane trim drag at cruise flight.

In the past during calculation of the airplane flight range, center-of-gravity (CG) position was usually assumed fixed, but with fuel usage, CG shifts within the definite range of CG positions. Until the FTT was not used in airplanes, the CG shift range was rather short, that allowed to use the specified assumption without any considerable mistakes.

In case of FTT utilization, CG shift can reach 15…20 % of mean aerodynamic chord (MAC), that requires to take into account the CG actual position during flight range calculation.

The aim of this publication is to develop the method of transport category airplane flight range calculation taking into account actual CG position with fuel using.

1. Statement of Research Problem

Let's consider steady level flight of an airplane. As it is known, within infinitesimal time interval dt an airplane moves horizontally on the value dL, km:

$$dL = 3.6 V\,dt\,, \tag{1}$$

where the speed V is measured in m/s [1].

Within the same time, mass of the airplane decreases by value of used fuel mass

$$dm = -P_{req}\,C_P\,dt\,, \tag{2}$$

where P_{req} – is the total required thrust of all airplane's engines, N; C_P – is the engine specific fuel consumption, kg/N·hour.

Expressing time interval dt from the equation (2) and substituting it in the equation (1), we get known expression for the flight range:

$$L = 3.6 \int_{m_2}^{m_1} \frac{V\,dm}{P_{req}\,C_P}\,. \tag{3}$$

where m_1 and m_2 – are the masses of airplane at the beginning and at the end of the cruise flight.

Specific fuel consumption, flight speed and altitude of airplane at cruise flight can be assumed constant and known. So, unknown in formula (3) is only the required thrust, which is equal to the airplane drag at the steady level flight.

The expression for the required engine thrust taking into account actual CG position was found in the publication [2]:

$$P_{req} = 0.7 p_H M^2 S C_{x0} + \frac{(mg)^2}{0.7 p_H M^2 S} \times$$

$$\times \left[A_{WHT} \left(\frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2 + A_{HT} \frac{S}{S_{HT}} \left(\frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2 \right], \quad (4)$$

where p_H – is the atmospheric pressure at the flight altitude; M – is the Mach flight number; S and S_{HT} – are the areas of wing and horizontal tail (HT), correspondingly; C_{x0} – is the airplane drag coefficient at zero lift; m – is the airplane current mass; A_{WHT} and A_{HT} – are the drag-due-to-lift factors of airplane without HT and separate HT; x_{CG} – is the airplane CG position relatively wing MAC leading edge; x_{pWHT} – is the center of pressure (CP) position of the airplane without HT relatively wing MAC leading edge; x_{pHT} – is the CP position of separate HT relatively HT MAC leading edge; ΔL – is the distance between MAC leading edges of wing and HT.

CG coordinate is a known function of the current mass and pitch angle υ [3] (which is equal to the airplane angle-of-attack α at cruise flight)

$$x_{CG} = f(m, \upsilon). \quad (5)$$

Airplane flight performance C_{x0}, A_{WHT} and A_{HT} are determined by the airplane shape and its flight mode (altitude and Mach number), and therefore they do not depend on airplane CG variation.

CP position of the airplane without HT and separate HT are determined by the airplane shape, flight mode and the angle-of-attack:

$$x_{pWHT} = b_a \left[\bar{x}_{FaWHT} + \frac{m_{z0WHT}}{C^{\alpha}_{yaWHT}(\alpha - \alpha_{0WHT})} \right], \quad (6)$$

$$x_{pHT} = b_{aHT} \left[\bar{x}_{FaHT} + \frac{m_{z0HT}}{C^{\alpha}_{yaHT}(\alpha - \alpha_{0HT})} \right], \quad (7)$$

where b_a, b_{aHT} – are the MAC of wing and HT correspondingly, $\bar{x}_{FaWHT}$, $\bar{x}_{FaHT}$ – are the coordinates of aerodynamic centers of the airplane without HT and separate HT correspondingly, m_{z0WHT}, m_{z0HT} – are the coefficients of pitching moment at zero lift of the airplane without HT and separate HT correspondingly, C^{α}_{yaWHT}, C^{α}_{yaHT} – are the derivatives of lift coefficient of the airplane without HT and separate HT correspondingly, α_{0WHT}, α_{0HT} – are the zero lift angles-of-attack of the airplane without HT and separate HT correspondingly.

Thus, to calculate the flight range, it is necessary to find the airplane angle-of-attack under the current flight mass and CG position. Formulas for the required lift coefficient of the airplane without HT and separate HT correspondingly were found in publication [2]

$$C_{yaWHT} = \frac{mg}{0.7\, p_H M^2 S}\left[\frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}}\right]. \tag{8}$$

$$C_{yaHT} = \frac{mg}{0.7\, p_H M^2 S_{HT}}\left[\frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}}\right]. \tag{9}$$

In the same time, the lift coefficient is related with the angle-of-attack by known formula

$$C_{yaWHT} = C^{\alpha}_{yaWHT}\left(\alpha - \alpha_{0WHT}\right),$$

whence the cruise flight angle-of-attack can be determined

$$\alpha = \alpha_{0WHT} + \frac{mg}{0.7\, p_H M^2 S C^{\alpha}_{yaWHT}}\left[\frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}}\right]. \tag{10}$$

To check the calculation correctness, the flight range can also be computed by known integral formula

$$L_{simp} = 3.6\frac{V K_{mean}}{C_P g} ln\left(\frac{m_1}{m_2}\right), \tag{11}$$

where K_{mean} – is the airplane mean lift-to-drag ratio, g – is the gravity acceleration.

The algorithm of airplane flight range calculation accounting actual CG position can be presented in the following form (Fig. 1).

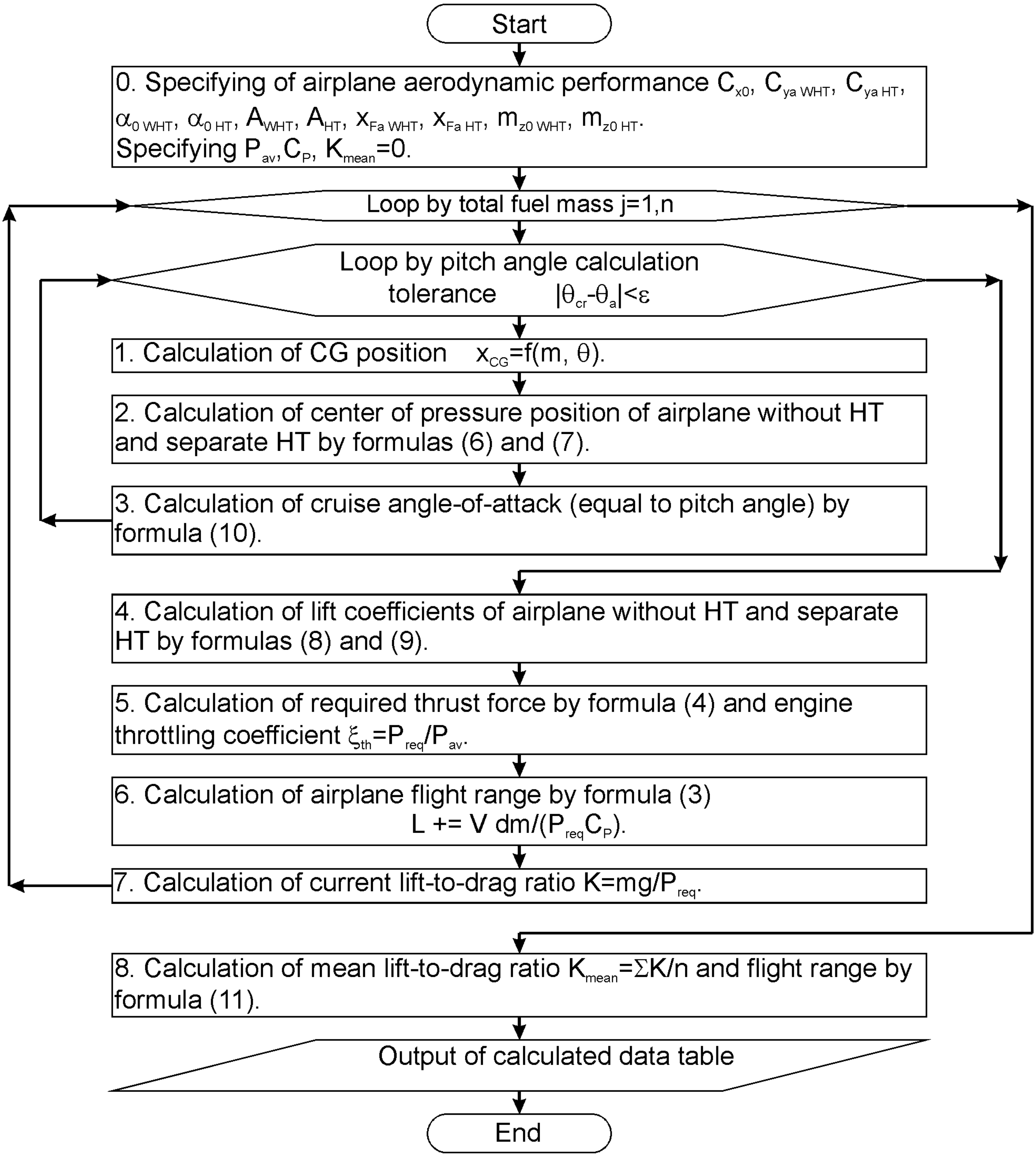

Figure 1 – Flight range calculation algorithm

2. Calculation Results

The considered algorithm is implemented by the author in calculation module of the Power Unit 11.7 software. Flight range calculations of the A-310-200 airplane were done as an example.

To calculate the airplane flight performance, the methodology of the Prof. V. I. Holiavko [4] was used as a basic one, which was updated to calculate wing with kinks at leading and trailing edges, and also with airfoil thickness ratio and its setting angle linearly varying spanwise.

In the process of aerodynamic performance calculation, we had found the value of derivative of the downwash angle near to HT $\varepsilon^{\alpha} = 0.1829$, that did not correspond to known experimental data [5]. So, the value $\varepsilon^{\alpha} = 0.5$ was assumed for the following calculation in accordance with the specified experimental data.

During computation of the wing aerodynamic center, the value $\bar{x}_{FaKp} = 0.6028$ had been found, that did not conform to known data about wing aerodynamic center position; thus its position was assumed equal to the airfoil aerodynamic center position $\bar{x}_{FaKp} = 0.2456$ for the following calculation.

When calculating the airplane drag coefficient at zero lift, the value $C_{x0} = 0.0297$ had been found, that did not correspond to actual values of the coefficient (Table 1), thus the value $C_{x0} = 0.0220$ was assumed for the following calculation.

Table 1 – Airplane drag coefficients at zero lift [6-11]

Airplane	Ил-62	Ил-86	Ту-154	Ту-204	Як-40	Як-42
C_{x0}	0.021	0.021	0.022	0.021	0.024	0.022

So, the following aerodynamic performance were assumed to calculate flight range of the A-310-200 airplane (Table 2).

Table 2 – Calculated aerodynamic performance of the A-310-200

C^{α}_{yaWHT}, 1/rad	4.9972	C^{α}_{yaHT}, 1/rad	2.5182
α_{0WHT}, rad	-0.0562	α_{0HT}, rad	-0.0447
A_{WHT}	0.0527	A_{HT}	0.1097
C_{x0}	0.0220	$\bar{x}_{Fa}$	0.4765
$\bar{x}_{FaWHT}$	0.0654	$\bar{x}_{FaHT}$	0.3258
m_{z0WHT}	-0.1887	m_{z0HT}	-0.1278

Information about flight range and payload of the A-310-200 airplane is shown in Table 3. The point 1 corresponds to flight with maximum fuel load and without any payload. The point 2 matches to flight with maximum fuel load and maximum takeoff mass. The point 3 corresponds to flight with maximum payload and maximum takeoff mass. Data in the top part of the table is taken from the source [12] (flight range taking into account typical international reserve 200 nautical miles). Results of calculation by the Power Unit 11.7 software are shown in the bottom part of the table. Calculated flight range after deduction of the reserve of 200 nautical miles L'_{calc} is introduced for adequate comparison. Real and calculating payload-range diagrams for the A-310-200 airplane are shown in Fig. 2.

Table 3 – Data to plot the payload-range diagram of A-310-200 airplane

Point number	1	2	3
m_{pl}, kg	0	17500	32917
m_f, kg	43113	43113	27695
m_0, kg	124500	142000	142000
ZFW, kg	81387	98887	114305
L_{310}, nm	4600	3900	2200
L_{310}, km	8527	7229	4078
L_{calc}, km	8922	8265	5549
L_{simp}, km	8889	8206	5520
ΔL_{simp}, %	0.37	0.71	0.52
L'_{calc}, km	8551	7894	5178
ΔL, %	0.29	9.21	26.98

Table 3 allows to make the following conclusions.

1. Range values L_{calc} obtained by formula (3) differ from the values L_{simp} obtained by formula (11) less than one percent, that indicates adequacy of formulas used in the algorithm.

2. Difference between the calculated flight range (L'_{calc}) and known from the document (L_{310}) makes $0.3...27$ %. At zero payload (when the airplane flies at low angles-of-attack) an error is less then one percent; that indicates correct set drag coefficient at zero lift.

The mistake sharply increases with payload increase. This dependence of an error can be explained by the following considerations. Firstly, because neither geometric no aerodynamic wing twisting were accounted in the polar calculation ($B = 0$). Secondly, because classic airfoils NACA-2212 and NACA-2210 were used in the calculation, instead of supercritical ones as is in real airplane. Thirdly, because additional wave drag caused by local shock waves was not accounted in the calculation.

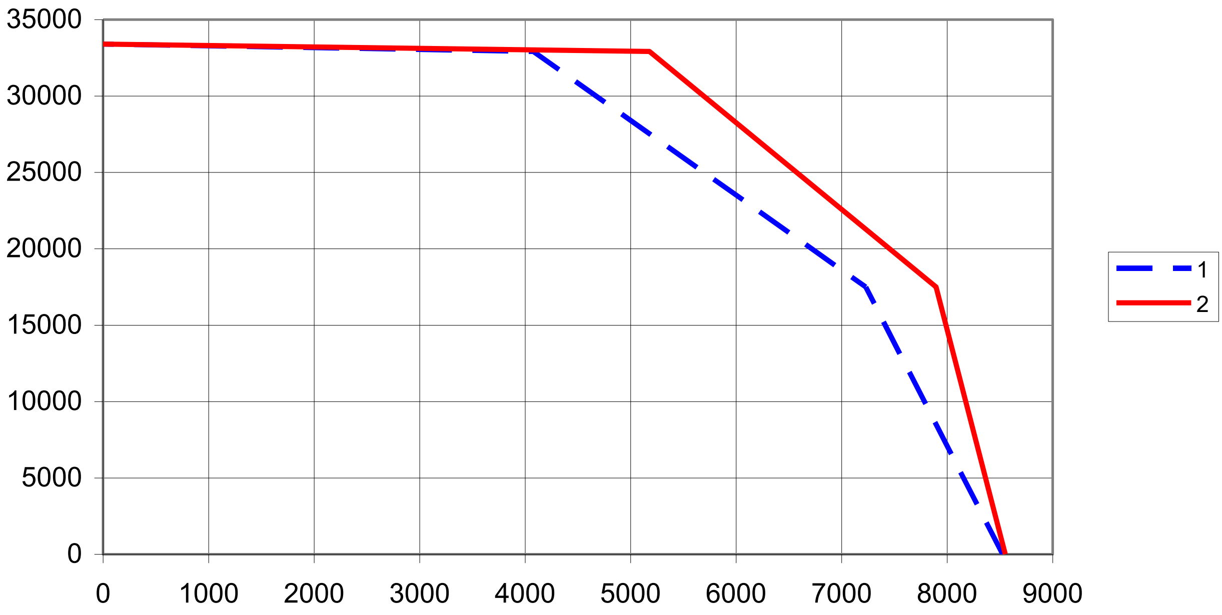

Figure 2 – Range-payload diagrams of A-310-200 airplane:
1 – from documents [12]; 2 – calculated

Conclusions

1. Mathematical model (algorithm and its program implementation using C language in Power Unit 11.7 system) has been developed to calculate airplane flight range accounting its actual center-of-gravity position shift with fuel usage.

2. Adequacy of the developed mathematical model is shown by means of comparison with known payload-range diagram of A-310-200 airplane.

List of Used Sources

1. Остославский, И. В. Динамика полёта. Траектории летательных аппаратов [Текст] / И. В. Остославский, И. В. Стражева. – М. : Машиностроение, 1969. – 500 с.

2. Tsukanov, R. U. Selection of Optimal Center-of-Gravity of Transport Category Airplane from Minimum Required Thrust Condition [Текст] / R. U. Tsukanov // Открытые информационные и компьютерные интегрированные технологии : сб. науч. тр. Нац. аэрокосм. ун-та им. Н. Е. Жуковского «Харьк. авиац. ин-т». – Вып. 76. – Х., 2017. – С. 23-30.

3. Tsukanov, R. U. Transport Category Airplane Center-Of-Gravity Shift Mathematical Simulation Accounting Fuel Trim Transfer [Text] / R. U. Tsukanov // Вопросы проектирования и производства конструкций

летательных аппаратов: сб. науч. тр. Нац. аэрокосм. ун-та им. Н. Е. Жуковского «ХАИ». – Вып. 3 (87). – Х., 2016. – С. 41-53.

4. Холявко, В. И. Расчёт аэродинамических характеристик самолёта [Текст] : учеб. пособие / В. И. Холявко. – Х. : Харьк. авиац. и-нт, 1991. – 72 с.

5. Петров, К. П. Аэродинамика элементов летательных аппаратов [Текст] / К. П. Петров. – М. : Машиностроение, 1985. – 272 с.

6. Стариков, Ю. Н. Практическая аэродинамика самолёта Ил-62М [Текст] / Ю. Н. Стариков. – Ульяновск : Центр ГА СЭВ, 1989. – 72 с.

7. Бехтир, В. П. Практическая аэродинамика самолёта Ил-86 [Текст] / В. П. Бехтир. – Ульяновск : Центр ГА СЭВ, 1991. – 135 с.

8. Бехтир, В. П. Практическая аэродинамика самолёта Ту-154М [Текст] / В. П. Бехтри, В. М. Ржевский, В. Г. Ципенко. – М. : Воздушный транспорт, 1997. – 288 с.

9. Стариков, Ю. Н. Практическая аэродинамика самолёта Ту-204 [Текст] : учеб. пособие / Ю. Н. Стариков, В. П. Иванченко. – Ульяновск : УВАУ ГА, 1996. – 51 с.

10. Практическая аэродинамика самолёта Як-40 [Текст] : учеб. пособие / А. М. Рогонов, В. П. Бехтир, В. Х. Копысов, Е. Н. Коврижных. – Ульяновск : УВАУ ГА, 2005. – 124 с.

11. Бехтир, В. П. Практическая аэродинамика самолёта Як-42 [Текст] / В. П. Бехтир. – М. : Транспорт, 1989. – 190 с.

12. A-310. Flight Deck and Systems Briefing for Pilots [Text] / Airbus Industries, 1993. — 188 p.

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Рецензент: д-р техн. наук, проф., А. В. Бетин, НИИ проблем физического моделирования полета самолетов, г. Харьков.