This paper formulates and solves a multivariate problem related to modeling the logistics of war reserve stockpiling for successful combat operations in the armed conflict area. The relevance of the study is related to a comprehensive solution to the problem of war reserve stockpiling to fulfill the objectives of a military operation, considering the capabilities of materiel suppliers, complex logistics of war materiel transportation to armed conflict areas, and military threats arising from the martial law in the country. The study creates a set of models that allow: prioritizing war materiel types for reserve stockpiling; formulating requirements for the size of reserves; selection of a rational structure of logistics supply chain, considering the risks of military threats; ensuring the success of combat operations in the armed conflict area. The article analyzes the existing problems of war reserve stockpiling under martial law, which arose due to: the variety of war materiel; small supply batches; different materiel manufacturers and suppliers located at a great distance from the armed conflict area; heterogeneous transport environment of supply; risks associated with military threats during war materiel reserve stockpiling. A method of presenting the importance of certain war materiel types for supply to the armed conflict area is proposed, considering the combat capability of individual weapons and assessments of military experts. The requirements for the amount of war materiel reserve in the interval representation, from the minimum to the maximum value of the inventory, have been formed. The minimum war materiel reserve to be used in the armed conflict area provides the necessary parity of military forces. Simultaneously, the nature of hostilities does not change, but the risks associated with the enemy's actions to destroy the war materiel reserve (war of reserves) may manifest themselves. The maximum war materiel reserve provides confidence in achieving the objectives of a military operation in the armed conflict area, but is difficult to implement due to the limited capabilities of weapons and materiel suppliers and existing military threats. An optimization model for the war reserve stockpiling was created, considering the limited capabilities of suppliers, permissible time for reserve stockpiling, and the risks that may arise from military threats and their impact on the logistics of war materiel supply (war of logistics). An optimization model for the war reserve stockpiling was developed, considering the combat capability of certain types of weapons, which makes it possible to create asymmetry in the military parity of forces due to the increased combat characteristics of modern weapons (quality over quantity). Due to the multitude of materiel manufacturers (possible suppliers) and various variants for the formation of supply chain, the task of enumeration of possible variants for the composition and structures of the logistics supply chain for war reserve stockpiling was formed and solved. The number of warehouse variants and logistics supply chain structures is estimated using the methods of enumeration theory, and a set of variants is formed for subsequent comparison and selection of a rational option. An optimization model for the formation of logistics links in the supply chain was created to form the necessary war reserve stocks in the armed conflict area. The example of HIMARS MLRS stockpiling illustrates the effectiveness of the proposed approach for successful combat operations in the armed conflict area, using both quantitative and qualitative assessments when comparing possible stockpiling variants. The scientific novelty of the study is related to the development of a set of original optimization models, models of the variant enumeration for the structures of the logistics supply chain, which allows scientifically sound formulation of requirements for the size of the war reserve stock for successful fulfillment of the objectives of a military operation in the armed conflict area, considering the capabilities of materiel suppliers, long supply chains in a heterogeneous transport environment, short delivery time, and the risks of military threats. It is advisable to use the results of the study for planning war reserve stock, for their effective use in the combat zone, justifying the composition of materiel manufacturers and suppliers, forming a logistics supply chain, which will ensure the success of combat operations in the armed conflict area.

Keywords: armed conflict area; war reserve stock; optimization of war reserve stock; logistics of war materiel supply; enumeration of supply variants; risks of military threats.
1. Introduction

The escalation of hostilities in the armed conflict area requires constant replenishment of materiel reserves, which ensures effective military operations against the aggressor [1-4]. The existing inventory theory, whose methods are used in the production of complex equipment, is associated with the planned replenishment of components, stocks, and raw materials to fulfill the orders of enterprises and ensure the continuity of the production cycle [5-8].

1.1. Motivation

Under martial law, the availability of materiel reserves is one of the key requirements for ensuring effective combat operations. The enemy's counteraction to the war reserve stockpiling (war of reserves) is a manifestation of the so-called hybrid warfare [9]. The lack of the necessary amount of materiel reserves can lead to the death of military personnel, disruption of defensive and engineering structures, and transition from offensive to defensive actions, etc. Therefore, the requirements for the war reserve stockpiling, replenishment, and materiel size differ from the requirements in peacetime [10]. Hence, the relevance of the topic of the proposed publication, which sets and solves the problem of stockpiling in the context of modern hybrid warfare, which uses the most advanced types of materiel aimed at destroying the stockpiling logistics (war of logistics and reserves).

1.2. State of the Art

A number of problems associated with the stockpiling and management of the logistics for the materiel supply are reflected in existing publications on this topic:

1. A large number of manufacturers (suppliers), who can supply weapons in small batches, form a war reserves. This complicates the management of logistics for the materiel supply. Existing publications consider the materiel supply in peacetime for the formation of the planned reserves in case of war, and not during military operations [11-13].

2. When supplying materiel to the armed conflict area, long logistics chains arise in a heterogeneous transport environment, which require a lot of time for supply and stockpiling. In existing publications, planning and stockpiling are often considered when developers located at a short distance from the armed conflict area supplied arms, which does not require long logistics chains [14, 15].

3. In long logistics supply chains, the war reserve stockpiling creates much possible vulnerability that can be triggered by military threats. Existing publications address logistical vulnerabilities that occur mainly in peacetime. Little attention has been paid to the study of the impact of military threats on logistical vulnerabilities in the arms supply to the armed conflict area [16, 17].

4. In the state of martial law, many supply risks arise due to military threats, which disrupt war reserve stockpiling volumes for use in the armed conflict area at the right time. The increase in the number and variety of risks associated with martial law has received little attention in existing publications [18, 19].

5. In the heterogeneous environment of supplying materiel to the armed conflict area, there is a large number of transshipment of military cargo, which leads to delays and increased time for the materiel supply. In existing publications, transshipment is considered for peacetime, without the influence of military threats that arise during martial law [20-22].

6. Reserve and planned stockpiling in peacetime differs from the war reserve stockpiling for use in the armed conflict area during martial law. In existing publications, reserve stocks are associated with peacetime arms production plans in accordance with the state's military doctrine [23-25]. The size of the reserve materiel stock in the armed conflict area and its replenishment in wartime depends on the situation at the front and the objectives of the military operation [26].

1.3. Objectives and methodology

A contradiction arises between the requirements for successful operational and tactical actions by the military in the armed conflict area, using existing materiel reserve and its operational replenishment and the imperfection of existing methods of planning and managing war reserve stockpiling, which requires the development of a set of models and applied information technology aimed at studying the reserve stockpiling and replenishment.

The study creates a set of models that will allow analyzing the logistics of stockpiling with regard to the required volume, justification of the logistics supply chain in a heterogeneous transport environment, in the context of military threats and martial law, to ensure the effectiveness of combat operations and the objectives of a military operation in the armed conflict area. In accordance with the research objective, the following tasks need to be addressed:

1. Analyze the priority of types of weapons in the materiel stockpiling (Section 2).

2. Develop an optimization model for the materiel stockpiling, considering the capabilities of manufacturers and suppliers (Section 3).
3. List the set and justify the structure of the logistics supply chain in the materiel stockpiling (Section 4).
4. Stockpile the necessary materiel in the armed conflict area, considering the combat capability of individual samples (Section 5).
5. Provide an example of modeling the logistics of materiel stockpiling, considering the risks of wartime (Section 6).

The following mathematical and modeling methods have been used: system analysis; methods of expert evaluation of variants; methods of the theory of planning experiments; integer (Boolean) optimization; methods of the enumeration theory of Pólya and De Bruijn; method of lexicographic arrangement of variants; method of qualitative evaluation of variants.

2. Analysis of the priority of types of weapons in the materiel stockpiling

Requirements for war reserve stockpiling related to the plans of operational and tactical actions of the military, as well as the current situation at the front. Therefore, the nomenclature of materiel, its number, and the ability to train military specialists for the use of weapons will be determined on each specific sector of the front, depending on the requirements for establishing military parity of forces [15].

Given that modern weapons systems with high combat capability (e.g., HIMARS, NASAMS, etc.) can be used in this area of the front by creating an asymmetry in military parity of forces (quality over quantity), it is possible to conduct successful combat operations with fewer weapons.

Therefore, it is important to identify the most effective samples for war reserve stockpiling for successful use in a particular combat zone.

Furthermore, the method of experiment planning will be used as it allows the identification of the most effective weapons for combat zone by using expert (military) assessments and conducting virtual experiments.

Table 1 presents the plan of a full factorial experiment (FFE), in which the factors are the types of weapons that can be used in the armed conflict area.

Table 1: Full factorial experiment to assess the combat capability of weapons types

<table>
<thead>
<tr>
<th>No.</th>
<th>Factors</th>
<th>Ratings (combat capability)</th>
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<tr>
<td></td>
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</tr>
<tr>
<td>8</td>
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</tbody>
</table>

As a result of the virtual experiment, using expert ratings, regression dependence is formed in the form (n=3):

\[
\begin{align*}
    y &= b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \\
    &+ b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3 + \\
    &+ b_{123} x_1 x_2 x_3.
\end{align*}
\]

Value of the coefficient \( b_j \), \( j = 1, n \) indicates the significance of factor \( x_j \). Further, the series in descending order of significance of the coefficient \( b_j \) is formed. Thus, it is possible to form the highest priority types of weapons to be used in a particular combat zone. For instance, assume the factor \( x_1 \) refers to HIMARS MLRS, \( x_2 \) refers to JAVELIN anti-tank system, \( x_3 \) refers to the CAESAR Self-Propelled Gun.

After the experts’ assessment, using the scores of possible weapons composition variants, the following simplified linear regression dependence can be obtained:

\[
\begin{align*}
    y &= b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 = \\
    &= 5.1 + 2.4 x_1 + 1.1 x_2 + 1.6 x_3.
\end{align*}
\]

After analyzing the obtained dependence, it can be concluded that the most important factor for use in the combat zone is \( x_1 \) (use of HIMARS), less important is \( x_3 \) (use of CAESAR), and the least important is factor \( x_2 \) (use of JAVELIN).

Thus, considering the received priorities of types of weapons, it is necessary to stockpile war reserve in the armed conflict area.
3. Optimization model for the materiel stockpiling, considering the capabilities of manufacturers and suppliers

In peacetime, the reserve stockpiling is carried out in accordance with the state order plans for the development of materiel. For the successful implementation of weapons production plans, stocks of components, materials and raw materials (CMRM) are formed in the Z range:

\[ Z_{\text{min}} \leq Z \leq Z_{\text{max}}, \]

where \( Z_{\text{min}} \) corresponds to the reserve stocks of the CMRM, the current value of \( Z \) corresponds to the planned CMRM stock, considering the reserve stocks, and \( Z_{\text{max}} \) corresponds to the stock, the value of which is associated with minimizing various risks (including logistics ones) that can lead to possible failures or stopping the production cycle.

In wartime, the materiel stockpiling is carried out in the context of military threats, in accordance with tactical plans and operational actions in the armed conflict area. Simultaneously, it is necessary to consider the limited capabilities of suppliers (manufacturers) of materiel (small batches of weapons produced, long production cycle, small production stocks, etc.), which leads to difficulties in supplying the required quantity in the required time frame. These difficulties can lead to losses in the form of deaths of military personnel, changes in hostilities, disruption of defense and engineering structures, etc. Therefore, it is important to solve the problem of the necessary level of materiel stockpiling, considering the requirements for types of weapons, volume of supplies, and delivery time. Given that for each \( j \)th type of materiel, when forming inventory, not one supplier but a set of \( n_j \) suppliers can be used, which is associated with their possibility of producing and supplying military products, the problem arises of choosing a rational composition of suppliers to form the desired level of Z stock, where \( Z_{\text{min}} \leq Z \leq Z_{\text{max}} \). Here, the value of the \( Z_{\text{min}} \) stock corresponds to the reserve stocks level during martial law, which guarantees the establishment of military parity of forces in the context of military threats. Stock value \( Z_{\text{max}} \) minimizes the risks associated with military threats and creates possible asymmetry in the military parity of forces, which ensures the successful implementation of the goals of the planned combat operation. When forming a \( Z_{\text{max}} \) stock, it is necessary to consider the limited capabilities of manufacturers and suppliers of weapons (small batches).

Possible variants of the composition of materiel suppliers can be formed by a complete search of N=2^n-1, where \( n \) is the possible number of materiel suppliers.

Enter the Boolean variable \( x_{ijk} \):

\[ x_{ijk} = \begin{cases} 1, & \text{if } k \text{th variant of supplier composition is used for } j \text{th materiel type;} \\ 0, & \text{in other case.} \end{cases} \]

The following indicators will be used in \( Z \) materiel stockpiling, \( Z_{\text{min}} \leq Z \leq Z_{\text{max}} \) considering the risks of wartime:

- \( Z \) is the value of materiel reserve in the armed conflict area;
- \( T \) is the total time spent on the war reserve stockpiling in the armed conflict area;
- \( R \) is the risks of military threats arising from the war reserve stockpiling in armed conflict area during martial law.

There may be not one, but several combat zones in the conflict zone. Therefore, the value of \( Z \) stock:

\[ Z = \sum_{i=1}^{M} z_i, \]

where \( z_i \) corresponds to the materiel reserve for the \( i \)th armed conflict area.

\( M \) is the number of combat zones in the armed conflict area.

Considering all the combat zones in the armed conflict area, as well as the types of weapons that come to the armed conflict area, the following expression is obtained:

\[ Z = \sum_{i=1}^{M} \sum_{j=1}^{n_j} \sum_{k=1}^{l_{ijk}} m_{ijk} x_{ijk}, \]

where \( Z_{\text{min}} \leq Z \leq Z_{\text{max}} \);

\( l_{ijk} \) is the size of the delivery batch, which is associated with the \( k \)th composition of suppliers for the \( j \)th type of materiel for reserve stockpiling in the \( i \)th combat zone;

\( m_{ijk} \) is the number of delivery batches, which is related to the limited capabilities of materiel suppliers for the \( k \)th warehouse of suppliers of the \( j \)th type of materiel for the \( i \)th combat zone.

Enter the variable \( x_{ijk} \):

\[ x_{ijk} = \begin{cases} 1, & \text{if } k \text{th } j \text{th materiel type } i \text{th combat zone;} \\ 0, & \text{in other case.} \end{cases} \]
Then:

Total time for war reserve stockpiling in the armed conflict area:

\[ T = \sum_{i=1}^{M} \sum_{j=1}^{m_i} \sum_{k=1}^{n_i} t_{ijk} x_{ijk}, \]

where \( t_{ijk} \) is the time spent on the delivery of one batch of materiel by the \( k^{th} \) warehouse of suppliers for the \( j^{th} \) type of materiel, for the \( i^{th} \) combat zone.

Risks of war reserve stockpiling associated with possible military threats to martial law:

\[ R = \sum_{i=1}^{M} \sum_{j=1}^{m_i} \sum_{k=1}^{n_i} r_{ijk} x_{ijk}, \]

where \( r_{ijk} \) is the risks of delivery of one batch of materiel by the \( k^{th} \) warehouse of suppliers for the \( j^{th} \) type of materiel, for reserve stockpiling in the \( i^{th} \) combat zone.

It is necessary to maximize the level of materiel reserves in the armed conflict area:

\[ \max Z, \quad Z = \sum_{i=1}^{M} \sum_{j=1}^{m_i} \sum_{k=1}^{n_i} l_{ijk} m_{ijk} x_{ijk}, \]

subject to the implementation of restrictions:

\[ Z_{\text{min}} \leq Z \leq Z_{\text{max}}, \]

\[ T \leq T', \quad T = \sum_{i=1}^{M} \sum_{j=1}^{m_i} \sum_{k=1}^{n_i} t_{ijk} x_{ijk}, \]

\[ R \leq R', \quad R = \sum_{i=1}^{M} \sum_{j=1}^{m_i} \sum_{k=1}^{n_i} r_{ijk} x_{ijk}, \]

where \( R' \) is the permissible total risk;

\( T' \) is the permissible time for the materiel delivery and stockpiling.

The problem can be solved by a complete search of all variants for the composition of the warehouse of the materiel suppliers, or, for a large dimensionality of the problem, by using one of the methods of branches and bounds in integer optimization (with Boolean variables).

4. Listing of the set and justification of the structure of logistics supply chain in the necessary war reserve stockpiling

4.1. Structural analysis of materiel supply chains for war reserves stockpiling in the armed conflict area

To form a sufficient level of materiel reserves in armed conflict area, \( Z_{\text{min}} \leq Z \leq Z_{\text{max}} \), possible structural logistics elements are used:

- materiel manufacturers and suppliers;
- vehicles;
- supplier warehouses;
- intermediate warehouses for temporary storage of materiel;
- transshipment centers;
- distribution centers;
- consolidation centers for the formation of arms batches;
- materiel warehouses in the armed conflict area;
- and others.

Given the long length of the front line and the war reserve stockpiling, there are many possible structures for organizing the logistics supply chain, which requires solving the combinatorial problem of enumeration of possible variants. To analyze the set of variants for logistics supply chain structures, the following actions are required:

1. Conduct a quantitative analysis of possible variants for creating logistics chain for the materiel supply to the armed conflict area.
2. Form (generate) variants for further analysis and comparison.

To solve this problem, the combinatorial method and enumeration theory are used [27].

In enumeration theory, variants arise when one set (e.g., \( A \)) is mapped to another (\( B \)). In our case, the set of logistics elements \( A \) is mapped to the set of vertices of the graph of the logistics structure of supply chain \( G \). When listing, it is necessary to form the so-called cycle indices (CI), which characterize the peculiarities of the composition of logistics elements \( A \) and the peculiarities of the structure of logistics supply chain \( G \) in the following form:

\[ Z(G) = \frac{1}{|G|} \sum_{g} (t_1^{c_1} t_2^{c_2} t_3^{c_3} ... t_n^{c_n}) g, \]

where \( c_i \) is the number of cycles of the \( i^{th} \) length that arise from mapping one set to another;

\( t_i \) is the auxiliary variable associated with \( c_i; \)

\( g \) is a separate component of CI.
The CI generation for the logistics supply chain structure graph is associated with the analysis of the structure of G graph. Substitution groups for vertices of G graph are used as analysis. The most commonly used substitution groups are the following:

- symmetric $S_n$;
- cyclic $C_n$;
- dihedral $D_n$;
- single $E_n$,

where $n$ is the number of vertices of the G graph.

The obtained CIs for the groups of substitutions of the vertices of G graph are used to quantitatively analyze possible variants of logistics supply chain structures. In this case, formulas for recalculating variants are used in the form of the results of the Pólya enumeration theorem and De Bruijn's theorem [27, 28].

For the simplest cases of enumeration, when possible variants of the composition are of interest, the problem is simplified and transformed into a separate case, in the form of a combinatorial analysis. For instance, assume three possible logistics elements for forming a supply chain for war reserve stockpiling:

- warehouse for temporary storage of military equipment;
- material distribution center;
- a point of consolidation of various types of material for further transportation to the armed conflict area.

Number of possible variants for compositing elements in the logistics supply chain:

$$K = L^M - 1,$$

where $L$ is the number of types of logistics elements ($L=3$);

$M$ is the number of possible supply chains (e.g., $M=2$).

In our case, the number of possible composition variants: $K = 3^3 - 1 = 8$.

The $L^n$ counter can be used to form (generate) variants of the composition of logistics chain for the supply of materiel to the armed conflict area.

For instance, this is a triple counter. Then the set of variants for the composition of logistics chain for the supply of materiel to the armed conflict area can be given in the form:

1. 0 1
2. 0 2
3. 1 0
4. 1 1
5. 1 2
6. 2 0
7. 2 1
8. 2 2.

When using G graph of logistics supply chain structure, it is necessary to consider possible types of structure topologies that are used quite often (Fig. 1).

Fig. 1. Typical system topology structures:

- a – radial structure;
- b – tree structure;
- c – iterative structure;
- d – radial-annular of the same type;
- e – radial-annular of the second type;
- f – radial-annular of the third type

In practice, mixed structures are often used, which are a combination of the types of topologies that have been discussed. For any considered type of topologies, it is possible to form a group of substitutions of the vertices of the G graph according to the type of structure, and then build a CI. The Cycle Index is the basis for enumerating logistics chain variants, considering the possible set of logistics elements. Here, one of the possible theorems of the enumeration theory is used [28].

As an example, it was determined the number of variants related to the mapping of possible logistics elements to the vertices of the radial structure of the logistics supply chain in G graph (Fig. 1, a).

The group of vertex substitutions of G graph can be represented as:
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H=E₁+S₂,

where E₁ is the unit group of substitutions associated with the vertex of the radial structure;
S₂ is the symmetric substitution group that is associated with two bottom vertices of the G graph.
The cyclic index of G graph consists of a composition of two cyclic indices:

\[ Z(H) = Z(E₁)Z(S₂). \]

Assume the number of types of logistics elements equals 3. Then:

\[ Z(H) = \frac{1}{2!} x₁(x₁^2 + x₂), \]

where the variables x₁, x₂ are related to the vertices of G graph.

To enumerate the variants of the structure of the logistics chain for the supply of materiel to the armed conflict area, the result of the second theorem of Pólya and De Bruijn will be used [28]:

\[ K = \left[ Z(G'; \frac{\partial}{\partial z₁}, \frac{\partial}{\partial z₂}, \ldots)Z(H, 1+z₁, 1+z₂, \ldots) \right]_{z₁=z₂=\ldots=0}, \]

where G’ is the vertex substitution group of G graph;
H is the substitution group associated with the composition of logistics supply element types;
\( \frac{\partial}{\partial zᵢ}, \frac{\partial}{\partial zⱼ}, \ldots \) is differential operators (partial derivatives) acting on the group of substitutions of the vertices of the G graph.

In this case, the condition is met that all auxiliary variables \( z₁ = z₂ = \ldots = 0 \).

For our example:

\[ K = 3 \cdot \frac{1}{2} (3^2 + 3) = \frac{3 \cdot 12}{2} = 18. \]

The obtained variants of the structures of logistics chain for the supply of materiel to the armed conflict area will be further evaluated and compared to select a rational option depending on the values of the selected logistics indicators.

4.2. Justification and selection of the rational composition of logistics links for the supply of materiel to the armed conflict area

It is assumed that each logistics supply chain is associated with a specific combat zone. It is necessary to form logistics supply chains considering the requirements of war reserve stockpiling in armed conflict area

\[ Z: Z_{\min} \leq Z \leq Z_{\max} \]

to successfully perform combat operations.

Individual links in the logistics chain are:
- transport sections of the highway;
- transport hubs;
- transport interchanges.

To solve the problem of choosing the rational composition of logistics links of weapons supply chain in the armed conflict area, to stockpile war reserve, integer programming with Boolean variables \( yₖ \) were used:

\[ yₖ = \begin{cases} 
1, \text{ if } k^{th} \text{ composition of links of logistics supply chain for war reserve stockpiling in the armed conflict area is selected} \\
0, \text{ in other case.} 
\end{cases} \]

where \( \sum_{k=1}^{L} yₖ = 1 \).

Then, the time spent on the materiel supply for reserve stockpiling in the armed conflict area:

\[ T = \sum_{i=1}^{L} \sum_{k=1}^{L} tᵢₖ yₖ, \]

where L is the number of combat zones in the armed conflict area;

\( l_i \) is the number of possible variants for the composition of the logistics chain for the materiel supply for reserve stockpiling in the \( i^{th} \) combat zone;
\( tᵢₖ \) is the time spent on the materiel delivery by the \( k^{th} \) warehouse of logistics links in the supply chain for the \( i^{th} \) combat zone.

Risks of military threats related to the materiel supply and stockpiling in the armed conflict area:

\[ R = \sum_{i=1}^{L} \sum_{k=1}^{L} rᵢₖ yₖ, \]

where \( rᵢₖ \) is the risks of military threats associated with the use of logistics supply links by the \( k^{th} \) warehouse for the \( i^{th} \) chain of materiel supply during war reserve stockpiling in the armed conflict area.

Logistics costs associated with the materiel supply for war reserve stockpiling in the armed conflict area:

\[ W = \sum_{i=1}^{L} \sum_{k=1}^{L} wᵢₖ yₖ, \]
where \( w_ik \) is logistics costs associated with the choice of the \( k^{th} \) variant of the composition of logistics links of the materiel supply chain in the \( i^{th} \) armed conflict area.

The following formulations of the problem related to the formation of logistics links in the materiel supply chains to the armed conflict area are possible:

1. Minimize the delivery time for war reserve stockpiling in the armed conflict area:

\[
\min T, T = \sum_{i=1}^{L} \sum_{k=1}^{1} t_{ik} y_{ik},
\]

subject to possible restrictions:

\[
Z_{\min} \leq Z \leq Z_{\max},
\]

\[
R \leq R', R = \sum_{i=1}^{L} \sum_{k=1}^{1} t_{ik} y_{ik},
\]

\[
W \leq W', W = \sum_{i=1}^{L} \sum_{k=1}^{1} w_{ik} y_{ik},
\]

where \( R', W' \) are acceptable values of risks and costs.

2. Minimize supply risks for war reserve stockpiling in the armed conflict area:

\[
\min R, R = \sum_{i=1}^{L} \sum_{k=1}^{1} r_{ik} y_{ik},
\]

subject to restrictions:

\[
Z_{\min} \leq Z \leq Z_{\max},
\]

\[
T \leq T', T = \sum_{i=1}^{L} \sum_{k=1}^{1} t_{ik} y_{ik},
\]

\[
W \leq W', W = \sum_{i=1}^{L} \sum_{k=1}^{1} w_{ik} y_{ik},
\]

3. Minimize logistics costs that are associated with delivery during war reserve stockpiling in the armed conflict area:

\[
\min W, W = \sum_{i=1}^{L} \sum_{k=1}^{1} w_{ik} y_{ik},
\]

subject to restrictions:

\[
Z_{\min} \leq Z \leq Z_{\max},
\]

5. Stockpiling of the necessary materiel in the armed conflict area, considering the combat capability of individual samples

Modern weapons have high combat capability (accuracy, range, zone of destruction, etc.), which allows for an asymmetry in the military parity of forces in a particular combat zone (quality over quantity) in the process of materiel stockpiling. Therefore, it is relevant to study war reserve stockpiling in the armed conflict area, considering the combat capability of weapons. Let’s consider the formulation and solution of the problem of distributing a batch of weapons in the armed conflict area, in which war reserve stockpiling is carried out considering the combat capability of individual types.

Assume \( N \) is the number of combat zones in the armed conflict area. When a materiel batch arrives at the armed conflict area, it is necessary, considering the types of weapons received, \( j (j = 1, P) \), to distribute them to individual combat zones, subject to their combat capability. Simultaneously, it is necessary to consider the total combat capability of all weapons in the combat zone, including the current one.

Enter an integer variable:

\[
x_{jk} = \begin{cases} 
1, & \text{if reserves are formed by } j^{th} \text{ materiel type} \\
0, & \text{in other case.}
\end{cases}
\]

where \( \sum_{k=1}^{n} x_{jk} = 1 \), which means that the materiel batch that has arrived will necessarily be used for war reserve stockpiling of the \( j^{th} \) materiel type in the armed conflict area. Then:

1. War reserves in the armed conflict area:

\[
W = \sum_{j=1}^{P} \sum_{k=1}^{n_j} W_{jk} x_{jk},
\]

where \( P \) is the quantity of materiel types;

\( n_j \) is the number of possible variants for the distribution of materiel around the combat zone for stockpiling of \( j^{th} \) type of weapons;
\( w_{jk} \) is the quantity of \( j^{th} \) materiel type used for war reserve stockpiling;
\( w_{jk}^{j} \) is the number of \( j^{th} \) type of weapons that are currently available in \( k^{th} \) warehouse of the combat zone, where war reserve stockpiling is taking place.

2. Combat capability of materiel in the armed conflict area:

\[
Q = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} q_{jk} x_{jk} + \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} q_{jk} x_{jk} ,
\]

where \( q_{jk} \) is the combat capability of a separate sample of the \( j^{th} \) materiel type.

3. Time spent on war reserve stockpiling in the armed conflict area:

\[
T = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} t_{jk} x_{jk} ,
\]

where \( t_{jk} \) is time spent on war reserve stockpiling of the \( j^{th} \) type for the \( k^{th} \) composition of war reserve in the armed conflict area.

4. Logistical risks of war reserve stockpiling in the armed conflict area, which are associated with wartime:

\[
R = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} r_{jk} x_{jk} ,
\]

where \( r_{jk} \) is the risk of military threats associated with the logistics of war reserve stockpiling of the \( j^{th} \) type of materiel for the \( k^{th} \) warehouse of the combat zone in the armed conflict area.

To create military parity of forces (as well as the possibility of creating asymmetry), it is necessary to maximize the combat capability of the use of materiel, considering the war reserves currently available in the armed conflict area:

\[
\text{max } Q = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} q_{jk} x_{jk} + \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} q_{jk} x_{jk} ,
\]

subject to restrictions:

\[
W_{\text{min}} \leq W \leq W_{\text{max}} , W = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} x_{jk} + \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} w_{jk} x_{jk} ,
\]

where \( W_{\text{min}} \) is the war reserves that ensure that the nature of combat operations in the armed conflict area is not violated, considering military threats;

\( W_{\text{max}} \) is the war reserves that will ensure the successful implementation of the goals of the military operation due to the possible creation of asymmetry in the military parity of forces.

\[
T \leq T', T = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} t_{jk} x_{jk} ,
\]

\[
R \leq R', R = \sum_{j=1}^{p} \sum_{k=1}^{n_{j}} r_{jk} x_{jk} ,
\]

where \( T', R' \) are acceptable values for the time and risk of war reserve stockpiling.

6. Example of modeling the logistics of materiel stockpiling, considering the risks of wartime

Furthermore, the logistics of war reserve stockpiling for conducting effective combat operations will be considered in an illustrated example of the supply of the HIMARS multiple launch rocket system using four possible suppliers (\( n=4 \)). Here, the number of possible variants for compositing suppliers for HIMARS MLRS stockpiling in the armed conflict area is \( N=2^{n-1}=2^{4-1}=15 \).

Table 2 shows the variants of compositing suppliers for HIMARS stockpiling. The following indicators are used to compare and eliminate unnecessary variants for composing possible suppliers:

- number of shipments of HIMARS supplies from the \( i^{th} \) supplier for war reserve stockpiling \( - n_{i} \);
- HIMARS delivery batch size for \( i^{th} \) supplier \( - l_{i} \);
- HIMARS MLRS stockpile size formed when using weapons of the \( k^{th} \) supplier warehouse in the supply of weapons \( - W_{ki} \);
- time spent on the supply of weapons of \( k^{th} \) suppliers of HIMARS MLRS \( - T_{ki} \);
- logistical risks of wartime related to the supply of HIMARS weapons to the \( k^{th} \) warehouse of suppliers \( - R_{ki} \).

Quantitative estimates are used to assess the HIMARS reserve formed in the armed conflict area, and, for the convenience of military experts, qualitative estimates will be used to assess time and risk [29]:

\[
T_{k} = \begin{cases} 
A = \text{minimum time}; \\
B = \text{satisfactory time}; \\
C = \text{permissible time}; \\
D = \text{maximum time}; 
\end{cases}
\]
A preliminary enumeration of variants for HIMARS MLRS stockpiling (see Table 2) has the following form:

1. 6, B, A  
2. 4, A, A  
3. 10, C, B  
4. 3, A, A  
5. 9, C, B  
6. 7, B, B  
7. 13, C, C  
8. 3, B, A  
9. 9, C, C  
10. 7, C, B  
11. 13, D, D  
12. 6, B, B  
13. 12, C, D  
14. 10, C, C  
15. 16, D, D.

After lexicographic ordering of variants for HIMARS MLRS stockpiling, we obtain:

15. 16, D, D  
7. 13, C, C  
11. 13, D, D  
13. 12, C, D  
3. 10, C, B  
14. 10, C, C  
5. 9, C, B  
9. 9, C, C  
6. 7, B, B  
10. 7, C, B  
1. 6, B, A  
12. 6, B, B  
2. 4, A, A  
4. 3, A, A  
8. 3, B, A.

As a result of the military experts' assessment of possible variants for HIMARS MLRS stockpiling, Table 2 was constructed.

To analyze the obtained results, it shall be assumed that the most important indicator of the reserves stockpiling is \( W \), \( W_{\min} \leq W \leq W_{\max} \), where \( W_{\min}=6 \), which corresponds to the reserve stocks of the HIMARS MLRS, and \( W_{\max}=20 \), which ensures the successful implementation of the goals of military operation due to the asymmetry in the military parity of forces. The values of \( W_{\min} \) and \( W_{\max} \) were determined by experts in the field of military logistics.

The next most important indicator is the time \( T \), which is necessary for the timely HIMARS MLRS stockpiling. Possible risks in the context of military threats in the HIMARS MLRS stockpiling are considered using the \( R \) indicator, which, according to military experts, will be the third most important stockpiling indicator.

To compare and select the best variants for HIMARS MLRS stockpiling, the ordering of the variants obtained in Table 2 will be used, considering the importance of indicators.
As a result of analyzing the list of variants and comparing them with each other, military experts concluded that the seventh variants for HIMARS MLRS stockpiling is the best. This variant corresponds to the HIMARS MLRS stockpiling in 13 units, is characterized by an acceptable value of the time and risk of stockpiling.

7. Conclusions

The conducted research is related to modeling the logistics process of war reserve stockpiling for the successful implementation of combat operations in the armed conflict area. The article analyzes the existing problems of war reserve stockpiling, which differ from stockpiling in peacetime: a large number of materiel suppliers located at a great distance from the armed conflict area; small batches of arms supplies; long supply chains in a heterogeneous transport environment; military threats and vulnerabilities; and martial law risks. The analysis of war reserve stockpiling showed that the value of stocks is formed in the interval \( Z_{\text{min}} \leq Z \leq Z_{\text{max}} \), where \( Z_{\text{min}} \) corresponds to the war reserve stocks, which ensures a non-violation of military operations in the context of military threats and logistical risks of the country's martial law, whereas \( Z_{\text{max}} \) ensures the successful implementation of the goals of a military operation by creating asymmetry in the military parity of forces. This paper analyzes the effectiveness of the use of certain types of weapons in the combat zone depending on military threats. For this purpose, the method of the theory of planning experiments is used, which forms a priority list of weapons to form the necessary reserve.

This paper proposes an optimization model for war reserve stockpiling, considering the limited capabilities of manufacturers and suppliers and wartime logistical risks. Simultaneously, small batches of weapons supplies, the time of delivery, and the possible composition of suppliers are considered. A systematic analysis of logistics supply chains is carried out, and variants of the composition and structure are enumerated using the methods of enumeration theory.

An optimization model for selecting logistics links in weapons supply chains has been created, considering the costs and risks of martial law. The paper provides an optimization of the size of war reserves considering the combat capability of individual types, which allows creating possible asymmetry in military parity of forces for the successful achievement of the goals of a military operation.

An illustrated example is given to confirm the effectiveness of the proposed approach. The example analyzes possible variants for compositing suppliers for HIMARS MLRS stockpiling. Simultaneously, both quantitative estimates of war reserves and qualitative estimates of the time and logistical risks of supply under martial law are used. To select the best variants for compositing suppliers, the composition of weapons suppliers, their capabilities and the amount of required reserve stocks are analyzed when forming the HIMARS stocks. For this purpose, a lexicographical ordering of variants for war reserve stockpiling is used.

The main contribution of the research is the developed set of original optimization models, models of the variant enumeration for the structures of the logistics supply chain, which allow scientifically sound formulation of requirements for the size of the war reserve stock.

The proposed approach makes it possible to consider the diversity of weapons, small batches of supplies, a large number of variants for compositing suppliers, limited capabilities of suppliers, size of reserve stocks, delivery time, and logistics of war reserve stockpiling in the context of military threats in the armed conflict area.

Authors’ contribution: system analysis, enumeration of variants for the composition of weapons suppliers – Oleg Fedorovich; analysis of combat capability characteristics of weapons – Mikhail Lukhanin; analysis of multiple suppliers for war reserve stockpiling – Oleksandr Prokhorov; assessment of logistics risks in war reserve stockpiling – Yurii Pronchakov; time optimization for war reserve stockpiling – Oleksandr Leshchenko; example of supplied war reserve stockpiling – Valeriy Fedorovich.

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Сформульована та вирішується задача багатоваріантного характеру, яка пов’язана з моделюванням логістики формування запасів озброєння та військової техніки (ОВТ) для успішного виконання бойових дій в зоні вогенного конфлікту (ЗВК). Актуальність дослідження пов’язана з комплексним рішенням проблеми формування запасів ОВТ для виконання цілей військової операції, з урахуванням можливостей постачальників, складної логістики транспортування ОВТ в ЗВК та військових загроз, які виникають з обмежених можливостей постачальників ОВТ та існуючих проблем. Створена оптимізаційна модель формування запасів ОВТ, яка враховує наступні риси:

- різноманіття ОВТ;
- малих партій постачання;
- різних виробників;
- військової операції в ЗВК;
- складної логістики транспортування ОВТ в ЗВК та військових загроз;
- можливостей постачальників ОВТ;
- ризику навігації в ЗВК;
- військових загроз;
- специфіки постачання ОВТ;
- військових загроз.

Метою дослідження є створення комплексу моделей, які дозволяють: визначити величини запасів ОВТ та військових загроз для успішного виконання бойових дій в ЗВК. Проаналізовані існуючі проблеми формування запасів ОВТ у вумовах вогенного гостиного стану, які виникли з обмежених можливостей постачальників ОВТ та існуючих проблем. Створена оптимізаційна модель формування запасів ОВТ, яка враховує наступні риси:

- різноманіття ОВТ;
- малих партій постачання;
- різних виробників;
- військової операції в ЗВК;
- складної логістики транспортування ОВТ в ЗВК та військових загроз;
- можливостей постачальників ОВТ;
- ризику навігації в ЗВК;
- військових загроз;
- специфіки постачання ОВТ;
- військових загроз.
рмування запасів ОВТ, з урахуванням бойової ефективності окремих видів озброєння, що дає можливість створення асеметрії у військовому паритеті сил з-за підвищених бойових характеристик сучасних видів озброєння (перевага якості над кількістю). З-за множини виробників (можливих постачальників) ОВТ та різних варіантів формування каналів постачання, сформована та вирішена задача переліку можливих варіантів складу та структур логістичних каналів постачання для формування запасів ОВТ. За допомогою методів теорії переліку оцінюється кількість варіантів складу та структур логістичних каналів постачання, формуються множини варіантів для наступного порівняння та вибору раціонального варіанту, Створена оптимізаційна модель формування логістичних каналів в каналах постачання для формування необхідних запасів ОВТ у ЗВК. Ефективність запропонованого підходу ілюструє на прикладі формування запасів РСЗВ HIMARS для використання успішних бойових дій в ЗВК, з використанням як кількісних, так і якісних оцінок при порівнянні можливих варіантів формування запасів. Наукова новизна дослідження пов’язана з розробкою комплексу оригінальних оптимізаційних моделей, моделей переліку варіантів структур логістичних каналів постачання, які дозволяю науково-обґрунтовано сформовувати вимоги до величини запасів ОВТ для успішного виконання цілей військової операції в ЗВК, з урахуванням можливостей постачальників ОВТ, довгих логістичних ланцюгів постачання в різнорідному транспортному середовищі, стислої терміну часу постачання, ризиків військових загроз. Результати дослідження доцільно використовувати для планування запасів озброєння та військової техніки, для ефективного використання їх в зоні військового конфлікту, обґрунтування складу виробників та постачальників ОВТ, формування логістичних каналів постачання, що забезпечить успішність бойових дій в зоні військового конфлікту.

Ключові слова: зона військового конфлікту; запаси озброєння та військової техніки; оптимізація запасів озброєння; логістика постачання озброєння; перелік варіантів постачання; ризики військових загроз.

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