ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

UDC 621.396

DOI: 10.32620/reks.2025.2.11

Eduard TSERNE[1], Yevhen HOLOVYNSKY[2], Olha ZHYLA[2]

[1] *National Aerospace University "Kharkiv Aviation Institute", Kharkiv, Ukraine*
[2] *Kharkiv National University of Radio Electronics, Kharkiv, Ukraine*

METHODOLOGY OF STATISTICAL SYNTHESIS AND ANALYSIS METHODS FOR STOCHASTIC SIGNAL PROCESSING IN MULTI-ANTENNA RADIO DIRECTION FINDERS

The subject *of this study is the development of a statistical synthesis methodology and analysis methods for processing stochastic signals in multi-antenna radio direction finders.* ***The aim*** *of this study is to improve the accuracy, stability, and adaptability of these systems under changing operating conditions, especially for applications involving unmanned aerial vehicles.* ***The objectives*** *of the study are: 1) formulation of analytical models of signals and noise with specified statistical characteristics; 2) determination of the criteria for the maximum likelihood function for solving optimisation problems; 3) creation of a basis for estimating marginal errors of measuring the angular positions of radio sources emitting stochastic signals; 4) development of a structural diagram of a radio direction finding system based on the obtained algorithm; 5) simulation modelling of the direction finder.* ***The methods*** *for solving the tasks are statistical theories of radio engineering systems, simulation modelling and optimisation in the spectral domain to solve the problems of designing radio direction finders capable of processing stochastic signals. This* ***approach*** *allows for the integration of amplitude and phase measurements for multiple antennas, ensuring compatibility with UAV-specific constraints, such as size, weight, and aerodynamic characteristics. The following* ***results*** *were obtained: 1) theoretical foundations for optimising radio direction finders for stochastic signals, validated by simulation and analytical modelling; 2) algorithms and block diagrams for a prototype single-antenna non-scanning radio direction finder, demonstrating the proposed methodology; 3) experimental verification confirming the feasibility of the proposed methods. This work provides a way for further development of multi-antenna direction-finding technologies, offering scalable solutions for use in UAVs and allowing accurate estimation of signal parameters under conditions of uncertainty. Future directions include the extension of the methodology to dual-antenna systems, hybrid configurations, and spatially distributed multi-antenna systems.*

Keywords: *statistical optimization; stochastic signal processing; radio direction finders; multi-antenna reception.*

1. Introduction

Motivation. Radio direction finders and radio direction finding systems [1] are widely used in radar [2, 3], radiometry [4, 5], radio navigation [6, 7], communication systems [8, 9], and radio control [10]. The main task of an individual radio direction finder is to measure the angular position of a radio source, and the main task of a group of direction finders is to measure the spatial position of such a source or group of sources. The existing theoretical and practical results on designing angular position determination systems [11, 12] are significant and provide a solid basis for further improving the structure of radio direction finders, which involves improving their accuracy, expanding their operational range, increasing their resistance to interference, and providing greater flexibility in adapting to changing operating conditions.

It is worth noting that most of the existing developments [13, 14] are focused on the use of radio direction finders in certain traditional conditions: from ground platforms, missiles, aircraft, helicopters and ships, where the dimensions, modes of movement and features of equipment installation are mostly stable and predictable. At the same time, the rapid proliferation of unmanned aerial vehicles (UAVs) is creating new challenges in the field of radio direction finding. In particular, the small size of UAVs, specific of their take-off and landing procedures, unique aerodynamic features, and a significantly different flight profiles make it impossible to directly transfer existing design solutions to a new platform. These factors necessitate a thorough rethinking of design approaches and improvement of radio direction finding systems to meet the requirements of modern operating conditions and new types of carriers. To update the design approach, it is advisable to develop generalized theoretical foundations for the synthesis radio direction finders with arbitrary configurations, which can subsequently be adapted to the characteristics of UAVs.

It is worth noting that in [15, 16], the authors investigated the features of optimal processing of functionally deterministic signals in multi-antenna radio direction finders. However, the assumption of full knowledge of the signal model limits the real scope of the obtained results because, under practical conditions, the task of direction finding of stochastic, a priori unknown signals, for which the optimal approach to processing and determining the parameters of received signals requires additional research and development [17].

The development of new algorithms for the direction finding of stochastic signals is also relevant due to the significant advancements in electronic components and computing technologies over recent decades [18, 19]. A major limitation in the implementation of algorithms for the direction finding of noise-like signals is typically their relatively wide frequency bandwidth. This previously complicated the digitization of such signals and required specialized computational devices for their processing, which could not always be implemented as compact onboard systems.

Currently, there are no significant technical obstacles to the implementation of compact multi-channel radio receiver devices capable of processing radio signals in real-time with bandwidths of several gigahertz. For example, multi-channel digitization of signals with sampling rates of up to 5.9 GSPS and their subsequent processing can be realized using a single system-on-chip from the Zynq UltraScale+ RFSoC family [20, 21]. For direct digitization of high-frequency signals within even broader frequency bands, modern multi-channel Mixed-Signal Front Ends such as AD9082 or promising AD9084 with sampling rates up to 20 GSPS, can be employed [22]. Moreover, all digital signal processing algorithms can be implemented on a single FPGA, making these systems exceptionally compact for onboard use.

In this regard, the statistical synthesis and analysis of methods for processing stochastic signals in multi-antenna radio direction finders becomes relevant, which will allow developing methodological foundations for adapting direction finding systems to real conditions of uncertainty, increasing the accuracy and reliability of determining signal parameters, and expanding the possibilities of using such systems in a wide range of modern and future operating scenarios.

State of the Art. Not many recent works have been devoted to the issues of statistical synthesis and analysis of signal processing algorithms in radio direction finders. The main results in this area have been obtained by solving signal detection problems when scanning a selected area of space with a directional pattern [23, 24]. The presented approaches develop the statistical theory of radio-direction-finder optimisation, allow to build optimal receivers and only allow to decide whether there is a signal at the receiver input or not [25]. The development based on this approach does not permit the assessment of the accuracy of direction finding by angular coordinates and its dependence on receiver parameters. It is also not possible to determine the range of unambiguous measurement bearings, because the simplest task of exceeding the threshold is solved.

A more efficient approach to building optimal radio-direction finder structures is the statistical synthesis of signal processing for signals with known shapes and parameters in the presence of Gaussian noise [26, 27]. The results obtained from this approach allow us to obtain the structure of the direction finder and estimate its potential accuracy [28, 29]. At the same time, such systems cannot perform their main functions when the signal shape is unknown or noisy. Interest in such systems is constantly growing, as the frequency spectrum is filled with new radio sources and the total number of interference sources increases [30, 31]. Some authors have tried to solve the problem of the direction finding of noise sources technically [32, 33], but such results are productive only for partial cases.

The authors of [34, 35] made a significant contribution to the development and application of algorithms for the direction of radiation sources for functionally determined signals and active interference-producer noise. The scientific novelty of the obtained results lies in the application of already known methods of spectral analysis of received signals by antenna arrays, such as MUSIC, ROOT-MUSIC, ESPRIT, MODE, and combinations of these methods, as well as measurement results statistical processing of groups of “super-resolution” methods [36]. This approach is effective, but it does not reveal methods of statistical synthesis and analysis of end-to-end signal processing, starting with field registration by each antenna and ending with potential variances of estimating the angular position of the noise source. The main contribution of the authors is the analysis of the correlation matrices of the observation equations at the output of the phased array antennas and the dependence of the estimation quality on the accuracy of the inverse matrix search. The authors paid less attention to the development of the mathematical apparatus for the synthesis of new methods of signal processing in multichannel reception.

The closest approach to the methodology of statistical synthesis and analysis of methods for processing stochastic signals in multi-antenna radio direction finders is that presented in [37]. The authors propose to perform statistical optimization of algorithms within the framework of the maximum likelihood function criterion and write the likelihood function in the spectral domain for signals with a stochastic spectrum. In other words, the waveform is not specified, but only the fre-

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

quency range and power spectral density of the noise signal are determined. This approach is effective and allows us to synthesize the best algorithms and meter structures under parametric uncertainty of the processes to be processed. The obtained results and methodology can be improved by taking into account the correlation properties of signals and noise and determining the likelihood function by considering the direct and inverse correlation matrix of processes observed in the receivers.

Problem statement. Based on the analysis of the theoretical research current state on the algorithms of radio direction finders and the relevance of issues optimising the radio direction finding systems structures, it is advisable to develop a methodology for statistical synthesis and analysis of methods for processing stochastic signals in multi-antenna radio direction finders. The methodology of such a synthesis should include the following components:

1) models of useful signals and noise and their statistical characteristics,

2) The maximum-likelihood function criterion for solving optimisation problems includes information on the inverse matrix of the correlation functions;

3) a methodology for estimating the marginal errors of measuring the angular radio source positions of stochastic signals.

The **paper structure** is as follows.

The first section discusses the motivation and relevance of the study. The results of the current research in the field of radio-direction-finding are reviewed. The general problem of optimizing the structure of direction-finding systems is developed, and the components of the methodology of such synthesis are defined.

Section 2 presents the geometry used for the direction finding problem of a radio jamming source using a multi-antenna direction finder. Mathematical models of useful and interfering signals are presented, and their statistical characteristics are determined.

In the third section, the maximum-likelihood function criterion is considered for solving problems of optimizing the structures of radio-electronic systems. A methodology for estimating the marginal errors of the angular position measurements of stochastic radio sources is presented herein.

The fourth section of the article is devoted to the synthesis and analysis of the method of processing stochastic signals in a single-antenna radio direction finder of a single radio source. In this section, we consider the elementary case of a single-antenna non-scanning radio direction finder to confirm the effectiveness of the proposed theoretical basis for the synthesis of optimal methods for measuring the angular position of stochastic signal sources.

On the basis of the synthesised mathematical model of the signal processing algorithm, the corresponding structural scheme of the single-antenna direction finder is developed in Chapter 5. Its simulation is carried out, as a result of which the marginal errors of bearing estimation at the radio source are determined and the results are analysed.

Section 6 summarises the conclusions of the research and determines its further direction.

2. Fundamentals of the statistical synthesis theory and analysis of methods for processing stochastic signals in multi-antenna radio direction finders

Radio direction finders are passive devices that receive signals $s_k(t)$ from radio radiation sources whose angular positions are to be determined. Receiving these signals by antennas spaced in space, it can be argued that spatio-temporal fields $s_{r,i}(t,\vec{r}',\theta_{s,k})$ are processed. The following notations are used: $s_k(t)$ is the signal generated by the k-th source, $k=\overline{1,K}$, $s_{r,i}(t,\vec{r}',\theta_{s,k})$ is the spatio-temporal field received by the i-th antenna, t is time, $\vec{r}'$ is spatial coordinates of the antenna surface, $\theta_{s,k}$ is bearing of each k-th source, $i=\overline{1,N}$. To simplify the problem, we will assume the following: the processing of the spatial field within each antenna has already been completed, each antenna is characterized by a radiation pattern $G_i(\theta-\theta_{0i})$, we receive stochastic signals from one radio radiation source by many antennas, $k=1$, the radiation source is a point source. The model of the stochastic signal of the emitter does not have a defined function, so we will represent it as a Fourier transform of the noise spectrum

$$s(t)=\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\dot{S}(2\pi f)e^{j2\pi ft}df\,, \qquad (1)$$

where $\dot{S}(2\pi f)$ is the complex stochastic spectral density of the emitted signal amplitude, $\dot{\Pi}(2\pi f)$ is a non-random complex frequency response of a stochastic signal transmitter. We will assume that the formation of a stochastic signal is limited in the spectrum of white Gaussian noise by the function $\dot{\Pi}(2\pi f)$. The density $\dot{S}(2\pi f)$ has the following characteristics:

$$\left\langle \dot{S}(2\pi f)\right\rangle=0\,,\left\langle \dot{S}(2\pi f_1)\dot{S}^*(2\pi f_1)\right\rangle=\frac{N_{0s}}{2}\delta(f_1-f_2)\,,$$

where $\langle\cdot\rangle$ is statistical averaging symbol. Using the model of scattered electromagnetic waves [38], it can be argued that the signal at the output of each receiving antenna has the form:

$$s_{r,i}(t,\theta_s)=$$
$$=K_{0,i}\int_{\Theta}G_i(\theta-\theta_{0,i})\delta(\theta-\theta_s)\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\dot{S}(2\pi f)\times$$
$$\times e^{j2\pi f(t-t_{del})}df\,d\theta=$$
$$=K_{0,i}\int_{\Theta}G_i(\theta-\theta_{0,i})\delta(\theta-\theta_s)\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\dot{S}(2\pi f)\times$$
$$\times e^{j2\pi f\,t}e^{-j\psi_i(f,\theta_s)}df\,d\theta \qquad (2)$$

where $K_{0,i}$ is the gain coefficient of the i-th antenna, $\delta(\theta-\theta_s)$ is delta function, which determines the angular position of the radio radiation point source, $\psi_i(f,\theta_s)=2\pi f\dfrac{\Delta r_i(\theta_s)}{c}$ is the phase shift of the received signals in each antenna relative to a certain phase center of the antenna array.

Fig. 1 shows the geometry of the electromagnetic field reception from the radiation source by the radio direction finder antennas, which corresponds to the defined signal models. It is assumed that the measurement of the sight bearing of a stochastic signal point source is performed in the far zone, i.e., the emitted electromagnetic wave front is flat in the area of their registration, and the beams from the source to each antenna are parallel. The antennas are presented in an arbitrary shape and have different orientations, spatial positions, and radiation patterns. Given the known antenna coordinates, frequency, and bearing θ_s, distances of delay or advance of the wave front $\dfrac{\Delta r_i(\theta_s)}{c}$ in each antenna relative to the array phase center can be easily calculated.

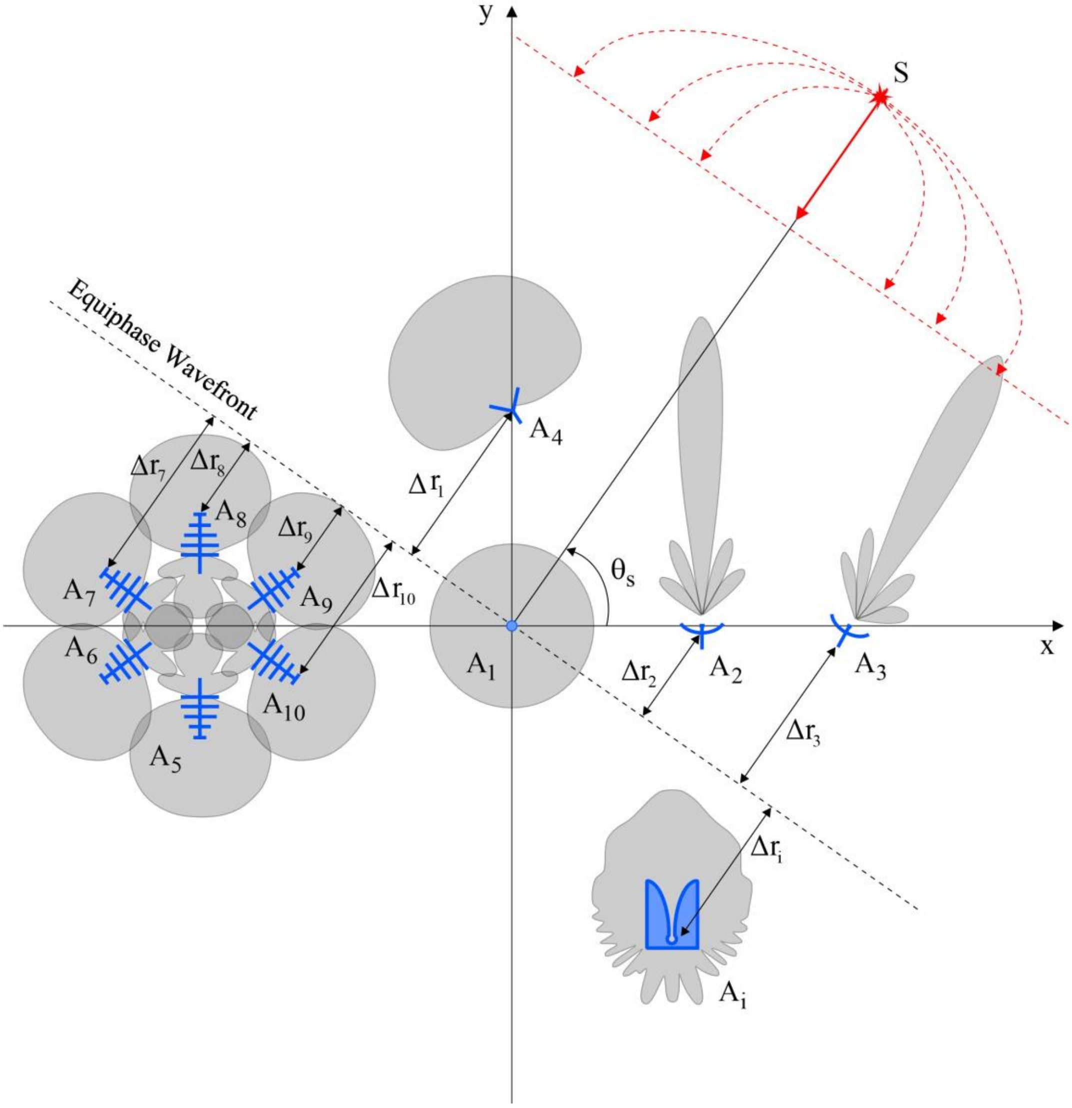

Fig. 1. Geometry of measuring the bearing θ_s in an antenna array with arbitrary placement of various antennas

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

The signals correlation function at the outputs of the i-th and j-th receivers is as follows:

$$\begin{aligned}R_{s,ij}(t_1-t_2,\theta_s)&=\left\langle s_{r\,i}(t_1,\theta_s)s_{r\,j}(t_2,\theta_s)\right\rangle=\\&=K_{0,i}K_{0,j}G_i(\theta_s-\theta_{0,i})G_j(\theta_s-\theta_{0,j})\times\\&\times\frac{N_{0s}}{2}\int_{-\infty}^{\infty}\left|\dot{\Pi}(2\pi f)\right|^2 e^{j2\pi f(t_1-t_2)}\times.\\&\times e^{-j2\pi f[\Delta r_i(\theta_s)-\Delta r_j(\theta_s)]c^{-1}}df\,.\end{aligned}\qquad(3)$$

The useful signal correlation function in each i-th receiver is equal to

$$\begin{aligned}R_{s,ii}(t_1-t_2,\theta_s)&=\left\langle s_{r,i}(t_1,\theta_s)s_{r,i}(t_2,\theta_s)\right\rangle=\\&=K_{0,i}^2G_i^2(\theta_s-\theta_{0,i})\frac{N_{0s}}{2}\int_{-\infty}^{\infty}\left|\dot{\Pi}(2\pi f)\right|^2 e^{j2\pi f(t_1-t_2)}df=\\&=K_{0,i}^2G_i^2(\theta_s-\theta_{0,i})\frac{N_{0s}}{2}R_h(t_1-t_2)\,,\end{aligned}\qquad(4)$$

where

$$R_h(t_1-t_2)=\int_{-\infty}^{\infty}\left|\dot{\Pi}(2\pi f)\right|^2 e^{j2\pi f(t_1-t_2)}df\,,\qquad(5)$$

is the pulse response autocorrelation function of the stochastic signal transmitter output path.

The mathematical expectation of the received signals is defined as follows

$$\begin{aligned}m_{s,i}(t)&=\left\langle s_{r,i}(t,\theta_s)\right\rangle=\\&=\left\langle K_{0,i}\int_{\Theta}G_i(\theta-\theta_{0,i})\delta(\theta-\theta_s)\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\times\right.\\&\left.\times\dot{S}(2\pi f)e^{j2\pi f\,t}e^{-j\psi_i(f,\theta_s)}df\;d\theta\right\rangle=\\&=K_{0,i}\int_{\Theta}G_i(\theta-\theta_{0,i})\delta(\theta-\theta_s)\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\times\\&\times\left\langle\dot{S}(2\pi f)\right\rangle e^{j2\pi f\,t}e^{-j\psi_i(f,\theta_s)}df\,d\theta=0\end{aligned}\qquad(6)$$

In each receiver, internal noises $n_i(t)$ are added to the registered useful signals, which are approximated by white Gaussian noise with the following statistical characteristics:

$$m_{n,i}(t)=\left\langle n_i(t)\right\rangle=0,\qquad(7)$$

is the mathematical expectation of the internal noise;

$$P[\vec{u}(t)\,|\,\lambda=\theta_s]=R_{n,ij}(t_1-t_2)=\left\langle n_i(t_1)n_j(t_2)\right\rangle=0,\;(8)$$

is the mutual correlation function of internal noise in different channels;

$$R_{n,i}(t_1-t_2)=\left\langle n_i(t_1)n_i(t_2)\right\rangle=\frac{N_{0ni}}{2}\delta(t_1-t_2)\qquad(9)$$

is the autocorrelation function of the noise in each i-th channel.

To simplify the calculations, we assume that the power spectral densities $N_{0ni}/2$ in each channel are the same and equal to $N_{0n}/2$.

The oscillations that are subject to further optimal processing are called observation equations. The observation equation system for the presented geometry and measurement conditions is as follows:

$$u_i(t)=s_{r,i}(t,\theta_s)+n_i(t).\qquad(10)$$

The correlation function of observations (10) at the outputs of the i-th and j-th channels is equal to

$$\begin{cases}R_{u,ij}(t_1-t_2)\big|_{i\neq j}=R_{s,ij}(t_1-t_2,\theta_s);\\R_{u,ij}(t_1-t_2)\big|_{i=j}=R_{s,ii}(t_1-t_2,\theta_s)+\dfrac{N_{0ni}}{2}\delta(t_1-t_2).\end{cases}\qquad(11)$$

The mathematical expectations of the received signals in each meter will be zero, because

$$\begin{aligned}m_{u,i}(t)&=\left\langle u_i(t)\right\rangle=\left\langle s_{r,i}(t,\theta_s)\right\rangle+\\&+\left\langle 2s_{r,i}(t,\theta_s)n_i(t)\right\rangle+\left\langle n_i(t)\right\rangle=0\end{aligned}\qquad(12)$$

3. Maximum likelihood function criterion for solving optimisation problems.

We optimised the processing of the observation equation (10) using the maximum likelihood functional method [39]. This method involves determining the optimal parameter λ by finding the maximum of the likelihood functional $P(\vec{u}(t)\,|\,\lambda)$, where $\vec{u}(t)=\left\|u_1(t),...,u_i(t)\right\|$. The likelihood functional $P(\vec{u}(t)\,|\,\lambda)$ is the conditional probability density function of registering random oscillations $u_i(t)$ at a fixed value of λ. For the given geometry and problem of direction finding for stochastic signal sources, random oscillations correspond to the observation observation equations in each of the antennas (10), and the parameter to be estimated is the bearing $\lambda=\theta_s$.

Examples of plausibility functions for remote sensing and radar tasks are given in [39, 40]. Many studies have used the plausibility functional criterion; however, each task requires its refinements and limitations. Using the methodology for constructing a wide class of plausibility functions for active and passive radio-measurement devices, we write $P(\vec{u}(t)\,|\,\lambda)$ for the specified measurement conditions as follows:

$$P(\vec{u}(t)\,|\,\lambda=\theta_s)=k(\theta_s)\times$$
$$\times\exp\left\{-\frac{1}{2}\int_0^T\int_0^T\left[\vec{u}^T(t_1)-m_u(t_1)\right]\times\right.$$
$$\left.\times\underline{W}(t_1,t_2,\theta_s)\left[\vec{u}(t_2)-m_u(t_2)\right]dt_1\,dt_2\right\}, \qquad (13)$$

where $\underline{W}(t_1,t_2,\theta_s)$ is the inverse matrix of the inverse correlation functions (11), T is observation time, $k(\lambda)$ is a normalizing coefficient with no physical meaning, but its derivative has a physical meaning. This coefficient arises during the limiting transition from the mathematical representation of the probability density in a discrete form to a continuous one.

The function $\underline{W}(t_1,t_2,\theta_s)$ is found from the integral equation

$$\int_0^T \underline{R}_u(t_1,t_2,\theta_s)\underline{W}_u(t_2,t_3,\theta_s)dt_2=\underline{I}\delta(t_1-t_3), \quad (14)$$

where $\underline{R}_u(t_1-t_2,\theta_s)=\left\langle\vec{u}(t_1)\vec{u}^T(t_2)\right\rangle$ is the matrix of the correlation functions, $\delta(t_1-t_3)$ is a delta function, $\underline{I}$ is a unit matrix.

The maximum of functional (13) can be found by differentiating it and equating it to zero

$$\frac{dP(\vec{u}(t)\,|\,\theta_s)}{d\theta_s}=0. \qquad (15)$$

More often, when solving such problems, it is not the likelihood function that is differentiated but its logarithm, as the logarithm is a monotonic function that does not change the position of the maximum of the function $P(\vec{u}(t)\,|\,\theta_s$. In this case, we substitute (12) into (13) and find the derivative of the logarithm $P(\vec{u}(t)\,|\,\theta_s$ in (15)

$$\frac{d\ln P(\vec{u}(t)\,|\,\theta_s)}{d\theta_s}=0, \qquad (16)$$

we obtain the likelihood equation

$$\mathrm{spur}\int_0^T\int_0^T\frac{d\underline{R}_u(t_1,t_2,\theta_s)}{d\theta_s}\underline{W}_u(t_1,t_2,\theta_s)dt_1dt_2=$$
$$=-\int_0^T\int_0^T\vec{u}^T(t_1)\frac{d\underline{W}_u(t_1,t_2,\theta_s)}{d\theta_s}\vec{u}(t_2)dt_1dt_2. \qquad (17)$$

Further specification of the correlation function and its inverse function will allow us to obtain the optimal processing method $u_i(t)$ for estimating the angular position of the source of the stochastic signals.

Finding the analytical expression $\underline{W}_u(t_1,t_2,\theta_s)$ from (14) is quite difficult. To simplify the calculations, it is advisable to represent expression (14) and the likelihood equation (17) in the spectral domain. After performing all calculations in the frequency domain, it is possible to present the processing in time at the final stage. Let us consider this approach in more detail.

Assuming that the observation equations are stationary on the interval $(0,T)$ and the observation time is much longer than the correlation time, we proceed to the difference arguments $\tau=t_1-t_2$ and infinite integration limits in (14)

$$\int_{-\infty}^{\infty}\underline{R}_u(t_1-t_2,\theta_s)\underline{W}_u(t_2-t_3,\theta_s)dt_2=\underline{I}\delta(t_1-t_3), \quad (18)$$

The Fourier transform of the right-hand side is equal to a unit matrix, and we perform the following on the left-hand side

$$\int_0^T\left[\int_{-\infty}^{\infty}\underline{G}_R(f,\theta_s)\exp(j2\pi f(t_1-t_2))\,df\right]\times$$
$$\times\underline{W}_u(t_2-t_3,\theta_s)dt_2=$$
$$=\int_{-\infty}^{\infty}\underline{G}_R(f,\theta_s)\left[\int_0^T\underline{W}_u(t_2-t_3,\theta_s)\exp(-j2\pi f\,t_2)\,dt_2\right]\times$$
$$\times\exp(j2\pi f\,t_1)df=\begin{vmatrix}t_2-t_3=t'\\ t_2=t'+t_3\\ dt_2=dt'\end{vmatrix}=$$
$$=\int_{-\infty}^{\infty}\underline{G}_R(f,\theta_s)\left[\int_0^T\underline{W}_u(t',\theta_s)\exp(-j2\pi f\,(t'+t_3))\,dt'\right]\times$$
$$\times\exp(j2\pi f\,t_1)df=$$
$$=\int_{-\infty}^{\infty}\underline{G}_R(f,\theta_s)\left[\int_0^T\underline{W}_u(t',\theta_s)\exp(-j2\pi f\,t')\,dt'\right]\times$$
$$\times\exp(-j2\pi f\,t_3)\exp(j2\pi f\,t_1)df=$$
$$=\int_{-\infty}^{\infty}\underline{G}_R(f,\theta_s)\underline{G}_W(f,\theta_s)\exp(j2\pi f(t_1-t_3))df=$$
$$=F^{-1}\left\{\underline{G}_R(f,\theta_s)\underline{G}_W(f,\theta_s)\right\}, \qquad (19)$$

Where

$$\underline{G}_R(f,\theta_s)=F\left\{\underline{R}_u(t_1-t_2,\theta_s)\right\} \qquad (20)$$

is the power spectral density matrix of the received oscillations,

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

$$\underline{G}_W(f,\theta_s) = \underline{G}^{-1}{}_R(f,\theta_s) = F\{\underline{W}_u(t_1 - t_2,\theta_s)\} \quad (21)$$

is the inverse matrix to $\underline{G}_R(f,\theta_s)$, $F\{\cdot\}$ is the Fourier transform operator, $F^{-1}\{\cdot\}$ is the inverse Fourier transform operator.

From the calculations obtained, it follows that the Fourier transform of the right and left sides of equation (14) can be used to obtain the following expression

$$\underline{G}_R(f,\theta_s)\underline{G}_W(f,\theta_s) = \underline{I}. \quad (22)$$

The Fourier transform from the likelihood equation (17) is given by

$$T\,\mathrm{spur}\int_{-\infty}^{\infty}\frac{d\underline{G}_R(f,\theta_s)}{d\theta_s}\underline{G}_R^{-1}(f,\theta_s)df = $$
$$= -\int_{-\infty}^{\infty}\vec{\dot{U}}_T^{+}(j2\pi f)\frac{d\underline{G}_R^{-1}(f,\theta_s)}{d\theta_s}\vec{\dot{U}}_T(j2\pi f)df, \quad (23)$$

where

$$\vec{\dot{U}}_T(j2\pi f) = \int_0^T \vec{u}(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\vec{u}_T(t)e^{-j2\pi ft}dt \quad (24)$$

is the spectrum of the vector process realization $\vec{u}(t)$ shortened by the observation interval T, and $(\cdot)^+$ is the sign of the Ermita conjugation.

Thus, formulas (19)-(24) give expressions for solving the optimization problem in the spectral domain and obtaining a method for processing the observed spectra $\vec{\dot{U}}_T(j2\pi f)$.

Marginal errors in the angular position estimation. The statistical synthesis theory and analysis of signal processing methods defines the estimation boundary errors $\hat{\lambda}$ as the lower bound of the Kramer-Rao inequality:

$$\left\langle(\hat{\lambda}-\lambda)^2\,|\,\lambda\right\rangle\Big|_{\lambda=\lambda_{true}} \ge -\left\langle\frac{d^2\ln P(\vec{u}(t)\,|\,\lambda)}{d\lambda^2}\right\rangle^{-1}\Bigg|_{\lambda=\lambda_{true}}, \quad (25)$$

where λ_{true} is the true value of the parameter to be evaluated.

In this case, the dispersion of the error in estimating the bearing will have the following expression

$$\sigma_{\theta_s}^2 = -\frac{1}{\left\langle\dfrac{d^2\ln P(\vec{u}(t)\,|\,\theta_s)}{d\theta_s^2}\right\rangle}\Bigg|_{\theta_s=\theta_{s\,true}}. \quad (26)$$

The analysis of the analytical expressions of the dispersion $\sigma_{\theta_s}^2$ is of considerable relevance at the stage of planning experiments, in particular, to determine the optimal antenna array geometry and the radiation patterns spatial orientation each of its elements. Optimality is understood as the achievement of minimal errors in estimating the bearing θ_s.

4. An example of synthesis and analysis for processing stochastic signals method in a single-antenna radio direction finder of a single radio radiation source

To confirm the effectiveness of the proposed theoretical foundation for the synthesis of optimal methods to measure the bearing of stochastic signal sources, we consider the elementary case of a single-antenna non-scanning radio direction finder [41].

Formulation of the optimisation problem. According to the criterion of the maximum likelihood functional, it is necessary to estimate the angular position of a radio source optimally θ_s over a time $t \in [0,T]$ interval based on the results obtained from receiving a useful signal $s_r(t,\theta_s)$ in a single-antenna radio direction finder against the background of internal noise $n(t)$.

For the specified measurement conditions, the observation equation can be expressed as follows:

$$u(t) = s_r(t,\theta_s) + n(t), \quad (27)$$

where

$$s_r(t,\theta_s) = K_0\int_{\Theta} G(\theta-\theta_0)\delta(\theta-\theta_s)\times$$
$$\times\int_{-\infty}^{\infty}\dot{\Pi}(2\pi f)\dot{S}(2\pi f)e^{j2\pi f\,t}df\,d\theta. \quad (28)$$

Unlike the general signal model (2), the obtained expression (28) does not contain information about the phase shift, which is proportional to the angular position of the radio radiation source. Therefore, in a single-antenna radio direction finder, all information concerning the angular position of the radio radiation source is contained in the amplitude of the received oscillations.

The correlation function of the observation equation is equal to

$$R_u(t_1-t_2,\theta_s) =$$
$$= K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}R_h(t_1-t_2) + \frac{N_{0n}}{2}\delta(t_1-t_2). \quad (29)$$

Having calculated the Fourier transform from (29), we obtain the power spectral density of the received oscillations as

$$G_R(f,\theta_s)=\int_{-\infty}^{\infty}R_u(\tau,\theta_s)e^{-j2\pi f\tau}d\tau=$$
$$=K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}\int_{-\infty}^{\infty}R_h(\tau)e^{-j2\pi f\tau}d\tau+$$
$$+\frac{N_{0n}}{2}\int_{-\infty}^{\infty}\delta(\tau)e^{-j2\pi f\tau}d\tau=$$
$$=K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}\left|\dot{\Pi}(2\pi f)\right|^2+\frac{N_{0n}}{2}. \qquad (30)$$

The derivative of the power spectral density given by (23) is calculated as follows

$$\frac{d\underline{G}_R(f,\theta_s)}{d\theta_s}=K_0^2\frac{dG^2(\theta_s-\theta_0)}{d\theta_s}\frac{N_{0s}}{2}\left|\dot{\Pi}(2\pi f)\right|^2. \qquad (31)$$

The expression for the matrix $\underline{G}_W(f,\theta_s)$ in the case of optimising the structure of a single-antenna direction finder becomes a function and is equal to $G_W(f,\theta_s)=G_R^{-1}(f,\theta_s)$, and the derivative of this function is given by

$$\frac{dG_R^{-1}(f,\theta_s)}{d\theta_s}=-\frac{1}{G_R^{-2}(f,\theta_s)}\frac{dG_R(f,\theta_s)}{d\theta_s}. \qquad (32)$$

Substituting (30) and (31) into (23), we solve the likelihood equation for the case of registering signals in one antenna

$$\int_{-\infty}^{\infty}\frac{dG_R(f,\theta_s)}{d\theta_s}G_R^{-1}(f,\theta_s)df=$$
$$=\frac{1}{T}\int_{-\infty}^{\infty}\left|\dot{U}_T(j2\pi f)\right|^2\frac{1}{G_R^{-2}(f,\theta_s)}\frac{dG_R(f,\theta_s)}{d\theta_s}df, \qquad (33)$$

in the following form

$$\int_{-\infty}^{\infty}\frac{\left|\dot{\Pi}(2\pi f)\right|^2}{\left[K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}\left|\dot{\Pi}(2\pi f)\right|^2+\frac{N_{0n}}{2}\right]}df=$$
$$=\frac{1}{T}\int_{-\infty}^{\infty}\left|\dot{U}_T(j2\pi f)\right|^2\times$$
$$\times\frac{\left|\dot{\Pi}(2\pi f)\right|^2}{\left[K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}\left|\dot{\Pi}(2\pi f)\right|^2+\frac{N_{0n}}{2}\right]^2}df \qquad (34)$$

To determine θ_s, we multiply and divide the sub-integral expression on the left side of equation (34) by the denominator, put it outside the brackets $K_0^2G^2(\theta_s-\theta_0)\frac{N_{0s}}{2}$, and solve the resulting equation with respect to $G^2(\theta_s-\theta_0)$

$$G^2(\theta_s-\theta_0)\int_{-\infty}^{\infty}\frac{\left[\left|\dot{\Pi}(2\pi f)\right|^2+\frac{1}{q}\right]\left|\dot{\Pi}(2\pi f)\right|^2}{\left[\left|\dot{\Pi}(2\pi f)\right|^2+\frac{1}{q}\right]^2}df=$$
$$=\frac{1}{K_0^2\frac{N_{0s}}{2}}\frac{1}{T}\int_{-\infty}^{\infty}\left|\dot{U}_T(j2\pi f)\right|^2\frac{\left|\dot{\Pi}(2\pi f)\right|^2}{\left[\left|\dot{\Pi}(2\pi f)\right|^2+\frac{1}{q}\right]^2}df, \qquad (35)$$

where $q=\left(K_0^2G^2(\theta_s-\theta_0)N_{0s}\right)/N_{0n}$ is signal-to-noise ratio of the received oscillations $u(t)$.

The right-hand side of (35) represents the processing of the received oscillations in the spectral domain. First, a Fourier periodogram $\left|\dot{U}_T(j2\pi f)\right|^2$ must be generated from the data $u(t)$. Then the periodogram must be passed through a matched filter $\left|\dot{\Pi}(2\pi f)\right|^2$ and a decorrelation filter $1/\left[\left|\dot{\Pi}(2\pi f)\right|^2+\frac{1}{q}\right]^2$. The matched filter should completely repeat the frequency response of the stochastic signal transmitter for optimal filtering of the receiver's internal noise. The decorrelation filter expands the resulting frequency spectrum of the received oscillations and increases the number of uncorrelated samples when estimating the bearing θ_s. The degree of decorrelation depends on the ratio q, the higher the signal level above the noise, the better the suppressed components of the informative spectrum are distinguished. After filtering the periodogram in the specified filters, it is necessary to perform frequency averaging and normalization by T to estimate the power of the received oscillations.

In the left-hand side of (35), in the numerator of the fraction under the integral, there is also a value of $1/q$, which can be neglected, because in practice it is advisable to choose $q>100$. In this case, we have

$$\int_{-\infty}^{\infty}\left|\dot{\Pi}(2\pi f)\right|^2\frac{\left|\dot{\Pi}(2\pi f)\right|^2}{\left[\left|\dot{\Pi}(2\pi f)\right|^2+\frac{1}{q}\right]^2}df=2\Delta F, \qquad (36)$$

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

where ΔF is the resulting spectral width of the received oscillations after coordinated filtering and decimation in the positive and negative frequency domain. Considering the above operations, we obtain the following

$$G^2(\theta_s - \theta_0)\, 2\Delta F = \frac{1}{K_0^2 \frac{N_{0s}}{2}} \hat{P}_{s,T}, \tag{37}$$

where $\hat{(\cdot)}$ is the sign of the value estimate, and $\hat{P}_{s,T}$ is the estimate of the power of the useful signal on the interval $(0,T)$. If we assume that the noise from the received oscillations is completely filtered out or its influence is negligible, then we obtain

$$\hat{P}_{s,T} \approx K_0^2 G^2(\theta_s - \theta_0) N_{0s} \Delta F. \tag{38}$$

Dividing the left and right parts of (37) by $2\Delta F$, we obtain an estimate of the square of the radiation beam pattern toward the radiation source

$$\hat{G}^2(\theta_s - \theta_0) = \frac{1}{K_0^2 N_{0s} \Delta F} \hat{P}_{s,T} =$$

$$= \frac{1}{K_0^2 N_{0s} \Delta F T} \int_{-\infty}^{\infty} \left| \dot{U}_{TW}(j2\pi f) \right|^2 df, \tag{39}$$

where

$$\dot{U}_{TW}(j2\pi f) = \dot{U}_T(j2\pi f) \left[\frac{\dot{\Pi}(2\pi f)}{\left| \dot{\Pi}(2\pi f) \right|^2 + q^{-1}} \right]. \tag{40}$$

According to the Parceval-Laplace theorem

$$\int_{-\infty}^{\infty} \left| \dot{U}_{TW}(j2\pi f) \right|^2 df = \int_0^T u_{TW}^2(t) dt, \tag{41}$$

where

$$u_{TW}(t) = \int_{-\infty}^{\infty} \dot{U}_{TW}(j2\pi f) e^{j2\pi f t} df \tag{42}$$

is the time-decorrelated observation equation. In this case, the optimal method for estimating the angular position of a stochastic signal source in the time domain is

$$\hat{G}^2(\theta_s - \theta_0) = \frac{1}{K_0^2 N_{0s} \Delta F} \hat{P}_{s,T} =$$

$$= \frac{1}{K_0^2 N_{0s} \Delta F} \frac{1}{T} \int_0^T u_{TW}^2(t) dt. \tag{43}$$

The resulting estimate in (39) and (43) is not an estimate of the bearing θ_s, since this bearing is the argument of the square of the radiation pattern or some function $L[\theta_s] = \hat{G}^2(\theta_s - \theta_0)$. It is desirable that the function $L[\theta_s]$ be linear, in which case the estimate θ_s will be

$$\hat{\theta}_s = L^{-1} \left[\frac{1}{K_0^2 N_{0s} \Delta F} \hat{P}_{s,T} \right] + \theta_0. \tag{44}$$

Note that the angle θ_0 must be known, because it is the reference direction from which the angular position of the radio radiation source is determined. Most practical radiation patterns, such as Gaussian or sinc functions, are nonlinear. In this case, the estimation $\hat{\theta}_s$ is ambiguous and it is necessary to additionally apply restrictions on the change in the antenna position or to select only a linear region of the radiation pattern for measurement.

5. Development of a single-antenna radio direction finder structural scheme, marginal error analysis of its operation and results discussion

Structural scheme of the radio direction finder. From the analysis of the estimation method (44), it follows that the optimal direction finder for a stochastic signal source should be constructed according to the scheme shown in Fig. 2.

The operation of the scheme is shown in Fig. 2 is expressed as follows. An antenna oriented in the direction θ_0 receives electromagnetic oscillations from a source of stochastic signals from the bearing θ_s. The signals at the antenna output are sequentially processed in a matched filter with a frequency response $\dot{\Pi}(2\pi f)$ and in a decorrelation filter $\dot{\Pi}_W(2\pi f) = 1 / \left[\left| \dot{\Pi}(2\pi f) \right|^2 + q^{-1} \right]$.

We perform statistical synthesis of the signal processing method in a radio-direction finder that met contradictory requirements. It should have a small weight and dimensions, ergonomic placement on board, high accuracy, several unambiguous measurement bearings, simple implementation, and low cost. To meet these requirements, we propose using a combination of the following three radio direction finders: an amplitude radio direction finder with wide-directional antennas, an amplitude radio direction finder with narrow-directional antennas, and a phase radio direction finder with omni-directional antennas. The measurement geometry is shown in Fig. 2.

After filtering, the signal is sent to a quadratic detector and averaged over a low-pass filter. The result of the received oscillation power estimation is then normalised in the amplifier to the required level, which can be digitised in an analogue-to-digital converter and subsequently processed in a digital processor. The digital processor performs normalization of the obtained signals using the value $K_0^2 N_{0s} \Delta F$ and solves the inverse problem of extracting bearing θ_s. In addition to implementing various mathematical calculations for θ_s based on the obtained power estimates, it is also possible to impose certain conditions on the measurement or to scan the entire range of bearings with the radiation pattern. For example, it is possible to define the condition that the bearing estimate θ_s when varying the reference angle θ_0 will have an optimal value $\hat{\theta}_s = \theta_0$ when the right side reaches the maximum value, because real radiation patterns reach their maximum at $\theta_s - \theta_0 = 0$.

Marginal error of the radio direction finder. To determine the analytical expressions for the marginal variance of the stochastic signal source angular position estimate, we write the second derivative of the logarithm of the likelihood function in expression (26) as follows

$$\frac{d^2 \ln P(\vec{u}(t) \mid \theta_s)}{d\theta_s^2} =$$
$$= -\frac{1}{2}\frac{d}{d\theta_s}\int_0^T\int_0^T \frac{dR_u(t_1,t_2,\theta_s)}{d\theta_s} W_u(t_1,t_2,\theta_s)dt_1dt_2 -$$
$$-\frac{1}{2}\frac{d}{d\theta_s}\int_0^T\int_0^T u(t_1)\frac{dW_u(t_1,t_2,\theta_s)}{d\theta_s}u(t_2)dt_1dt_2 =$$
$$= -\frac{1}{2}\int_0^T\int_0^T \frac{d^2R_u(t_1,t_2,\theta_s)}{d\theta_s^2} W_u(t_1,t_2,\theta_s)dt_1dt_2 -$$
$$-\frac{1}{2}\int_0^T\int_0^T \frac{dR_u(t_1,t_2,\theta_s)}{d\theta_s}\frac{dW_u(t_1,t_2,\theta_s)}{d\theta_s}dt_1dt_2 -$$
$$-\frac{1}{2}\int_0^T\int_0^T \frac{d^2W_u(t_1,t_2,\theta_s)}{d\theta_s^2}u(t_1)u(t_2)dt_1dt_2. \quad (45)$$

Substituting (45) into (26), we obtain the following dispersion

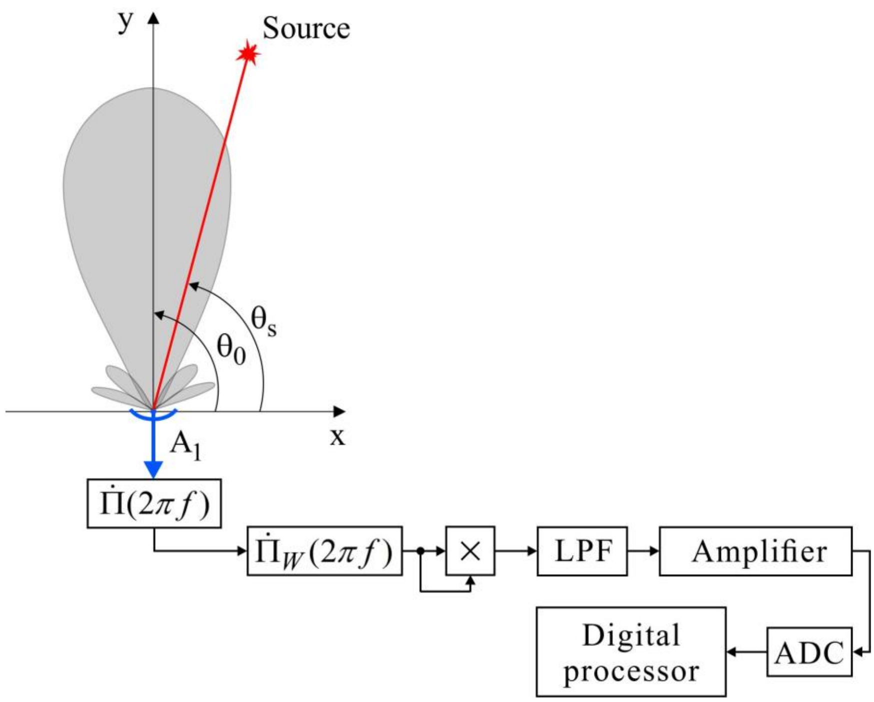

Fig. 2. Structural scheme of an optimal single-antenna radio direction finder for a stochastic signal source

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

$$\sigma^2_{\theta_s} = -1 \Bigg/ \Bigg\langle -\frac{1}{2}\int_0^T\int_0^T \frac{d^2R_u(t_1,t_2,\theta_s)}{d\theta_s^2} W_u(t_1,t_2,\theta_s)dt_1dt_2 - -\frac{1}{2}\int_0^T\int_0^T \frac{dR_u(t_1,t_2,\theta_s)}{d\theta_s}\frac{dW_u(t_1,t_2,\theta_s)}{d\theta_s}dt_1dt_2 - -\frac{1}{2}\int_0^T\int_0^T \frac{d^2W_u(t_1,t_2,\theta_s)}{d\theta_s^2}u(t_1)u(t_2)dt_1\,dt_2\Bigg\rangle\Bigg|_{\theta_s=\theta_{s\ true}} =$$

$$=\Bigg[\frac{1}{2}\int_0^T\int_0^T \frac{d^2R_u(t_1,t_2,\theta_s)}{d\theta_s^2} W_u(t_1,t_2,\theta_s)dt_1dt_2 + +\frac{1}{2}\int_0^T\int_0^T \frac{dR_u(t_1,t_2,\theta_s)}{d\theta_s}\frac{dW_u(t_1,t_2,\theta_s)}{d\theta_s}dt_1dt_2 + +\frac{1}{2}\int_0^T\int_0^T \frac{d^2W_u(t_1,t_2,\theta_s)}{d\theta_s^2}\langle u(t_1)u(t_2)\rangle dt_1d\,t_2\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}} =$$

$$=\Bigg[-\frac{1}{2}\int_0^T\int_0^T \frac{dR_u(t_1,t_2,\theta_s)}{d\theta_s}\times \times\frac{dW_u(t_1,t_2,\theta_s)}{d\theta_s}dt_1dt_2\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}}. \qquad (46)$$

To concretise (46), we rewrite the expression in the spectral domain. To do so, we assume that the observation equations are stationary and that it is possible to proceed to the difference argument $\tau = t_1 - t_2$ in the expressions

$$\sigma^2_{\theta_s} = \Bigg[-\frac{1}{2}\int_0^T\int_\tau^T \frac{dR_u(\tau,\theta_s)}{d\theta_s}\frac{dW_u(\tau,\theta_s)}{d\theta_s}dt_1d\tau - -\frac{1}{2}\int_{-T}^0\int_0^{\tau+T} \frac{dR_u(\tau,\theta_s)}{d\theta_s}\frac{dW_u(\tau,\theta_s)}{d\theta_s}dt_2d\tau\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}} =$$

$$= -\frac{T}{2}\int_{-T}^T\left(1-\frac{|\tau|}{T}\right)\frac{dR_u(\tau,\theta_s)}{d\theta_s}\times \times\frac{dW_u(\tau,\theta_s)}{d\theta_s}d\tau\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}}. \qquad (47)$$

Typically, when using stochastic sensing signals, the correlation time is much shorter than the observation time, so we can assume $|\tau| << T$ and $\left(1-|\tau|/T\right)\approx 1$. Taking into account the above assumptions, we rewrite expression (47) in the spectral domain

$$\sigma^2_{\theta_s} = \Bigg[-\frac{T}{2}\int_{-\infty}^{\infty}\frac{dG_R(-f,\theta_s)}{d\theta_s}\frac{dG_R^{-1}(f,\theta_s)}{d\theta_s}df\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}} =$$

$$=\Bigg[\frac{T}{2}\int_{-\infty}^{\infty}\frac{dG_R(-f,\theta_s)}{d\theta_s}\times \times\frac{dG_R(f,\theta_s)}{d\theta_s}\frac{1}{G_R^2(f,\theta_s)}df\Bigg]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}}. \qquad (48)$$

Substituting (31) into (49), we specify the expression for the marginal errors of the bearing estimates as

$$\sigma^2_{\theta_s} = \left[\frac{T}{2}\frac{\left(\dfrac{dG^2(\theta_s-\theta_0)}{d\theta_s}\right)^2}{G^4(\theta_s-\theta_0)}\times \times\int_{-\infty}^{\infty}\frac{\left|\dot{\Pi}^*(2\pi f)\right|^2\left|\dot{\Pi}(2\pi f)\right|^2}{\left[\left|\dot{\Pi}(2\pi f)\right|^2+\dfrac{1}{q}\right]^2}df\right]^{-1}\Bigg|_{\theta_s=\theta_{s\ true}} =$$

$$= \frac{G^4(\theta_s-\theta_0)}{T\Delta F\left(\dfrac{dG^2(\theta_s-\theta_0)}{d\theta_s}\right)^2}\Bigg|_{\theta_s=\theta_{s\ true}} \qquad (49)$$

From the obtained expression (50), it follows that the marginal dispersion of the error in estimating the source stochastic signal bearing is inversely proportional to the observation time, the frequency spectrum width of the received signals after filtering in the matched filter and the decorrelation filter, and the square normalised slope of the radiation pattern at the true direction point to the radiation source. The dependence of the maximum errors on the normalised slope $G^2(\theta_s-\theta_0)$ follows from the need to solve the inverse problem of determining θ_s from Eq $L[\theta_s] = \hat{G}^2(\theta_s-\theta_0)$.

Let us consider an example of modelling the analytical expression (50) for the case $T\Delta F = 10^6$ and the radiation pattern in the form of the sinc-function, which is described by the following expression

$$G(\theta_s-\theta_0) = a\,\mathrm{sinc}\left\{\pi\frac{\theta_s-\theta_0}{\Delta\theta}\right\}, \qquad (50)$$

where a is the amplitude equal to 1, $\Delta\theta$ is the width of a function $G(\theta_s-\theta_0)$, which is chosen to be equal to 5º. The results of calculating the maximum root mean square error of the bearing estimation and the radiation pattern normalized square are shown in Fig. 3. At the maximum points of the radiation pattern, a discontinuity of the 2nd kind is observed in the errors due to the zero value of the derivatives at these points. The minimum errors were observed in the zeros of the radiation patterns.

Fig. 4 shows the error values and the radiation pattern in the amplitude range 0–1. The minimum error approaches zero at the zero points of the sinc function and is limited only by the computational power of the engineering calculation package.

If you choose a point to recover from the equation on a linear section of the square of the radiation pattern, for example, at 2.5º, the error will be 12.5º. An enlarged graph of these value ranges is shown in Fig. 5.

From the calculations obtained, it follows that for the case of the angular position single-antenna measurement of a source stochastic signal, it is desirable to carry out all measurements close to the zero value of the radiation pattern. At the same time, due to the large number of zeros in the radiation pattern, which are periodically repeated through the value of $\Delta\theta$, ambiguity of measurements arises. The bearing range for accurately determining the source position is very narrow and unacceptable for practical use. It is also not possible to use the maximum for measurement θ_s, because the error in the maximum of the radiation pattern tends to infinity. The best option would be to create a radiation pattern with a single zero value and organise the scanning of the environment using such a pattern. In this case, the minimum value at the output of the meter after one scanning cycle corresponds to the direction to the radiation source, and the accuracy of such measurements is the highest.

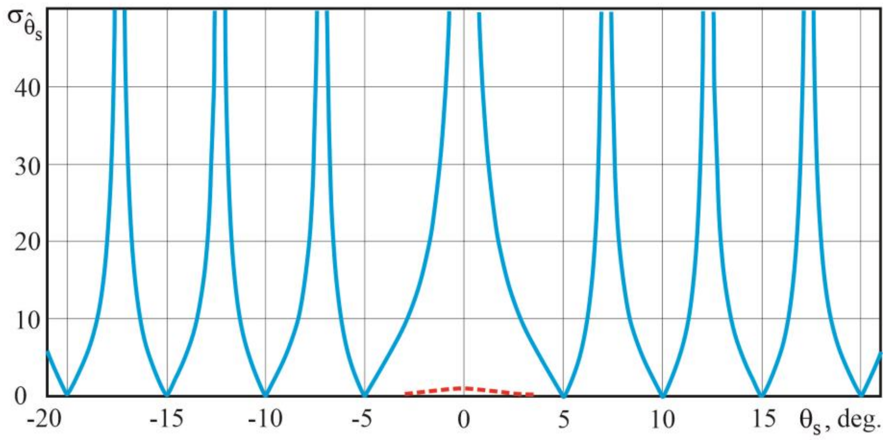

Fig. 3. Marginal average squared errors of bearing measurement σ_{θ_s} for different values θ_s (solid blue line) and normalised squares of the radiation pattern (red dashed line)

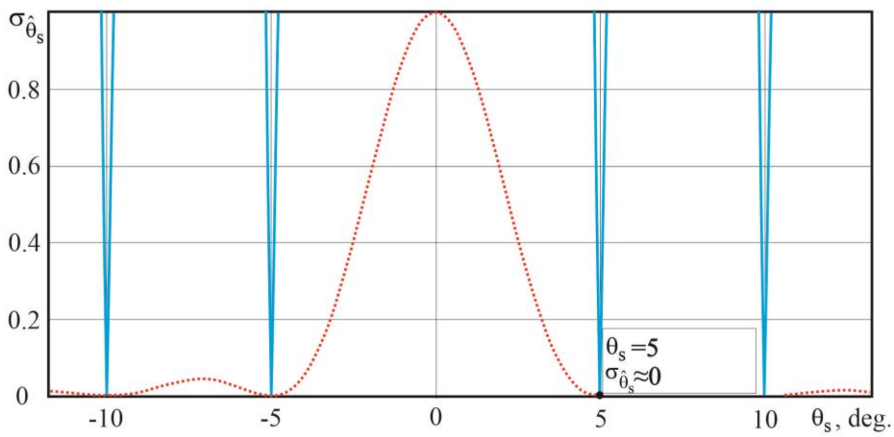

Fig. 4. Marginal average squared errors of bearing measurement σ_{θ_s} for different values θ_s (solid blue line) and normalised squares of the radiation pattern (red dashed line)

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

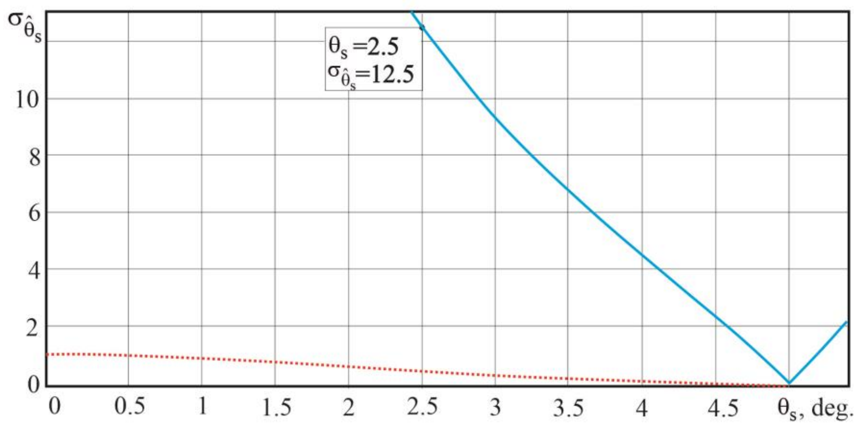

Fig. 5. Marginal average squared errors of bearing measurement σ_{θ_s} for different values θ_s (solid blue line) and normalised squares of the radiation pattern (red dashed line)

6. Features of practical implementation of the synthesized single-antenna direction finding system

The direction-finding system, whose structural diagram is shown in Fig. 2, can be practically implemented. In general, two main implementation approaches using modern electronic components can be identified.

The first approach involves processing the high-frequency signals received by the antenna and calculating their power in analogue form. A possible structural diagram of the direction finder obtained using this approach is shown in Fig. 6.

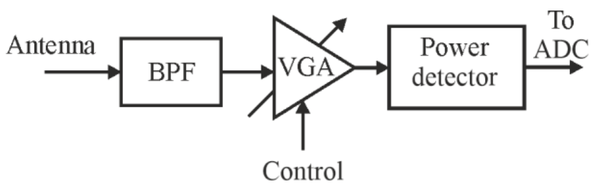

Fig. 6. Structural diagram of a single-antenna direction finder via analogue processing of high-frequency signals

The operation of the proposed scheme is illustrated in Fig. 4 is expressed as follows. The signal from the antenna output is fed to a band pass filter (BPF), which passes only those oscillations that correspond within the operating frequency band of the direction finder. The filtered signal then goes to a variable-gain amplifier, which amplifies it to a level sufficient for further detection. Next, a detector forms a low-frequency envelope that is proportional to the power of the received signal. This low-frequency signal is digitized by an analogue-to-digital converter (ADC) and then processed by a computing device, which can be implemented using a microcontroller.

Surface acoustic wave–based (SAW) microchips are recommended for use as band-pass filters because at high frequencies, such filters provide low insertion loss within the passband and a faster transition of the frequency response from the passband to the stopband [42]. Logarithmic detectors are preferable to power detectors because they typically offer a wider dynamic range than quadratic detectors. The practical implementation of such a scheme is compact and cost-effective and can be installed onboard a small drone. However, the main drawback of the proposed method is its inability to change the operating frequency of the direction finder. A possible solution to this limitation is the implementation of a direction finder using Software-Defined Radio (SDR) technology.

A possible structural diagram of a single-antenna direction finder using SDR technology is shown in Fig. 7.

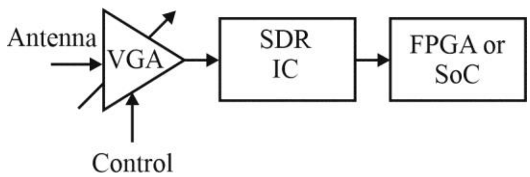

Fig. 7. Structural diagram of a single-antenna direction finder with SDR-based signal processing

The scheme shown in Figure 7 operates as follows. The signal received by the antenna was amplified to a level sufficient for further processing by the SDR chip. The amplified signal is then fed into a highly integrated SDR chip, which performs signal amplification, downconversion to an intermediate frequency, filtering, and digitization. The digital data are transferred to a processing block based on an FPGA or SoC, where the received signal power is estimated and a possible bearing to the radio emission source is determined according to the implemented algorithm. Examples of highly integrated SDR chips include the AD9363 and LMS6002D [43, 44]. The main advantage of SDR-based implementation is the ability to quickly and easily retune the operating frequency of the direction finder, as well as the flexibility to process intermediate-frequency signals in digital form. This enables the use of more advanced signal processing and direction-finding algorithms.

At the same time, it should be noted that the considered single-antenna direction finder is the simplest example, and it is intended primarily to demonstrate the overall feasibility of the proposed methodology. The main drawback in the practical implementation of such a system is the need to incorporate a scanning mechanism capable of changing the angular position of the antenna system. In other words, this type of system does not allow determining the bearing to a radio emission source from a single measurement, and its integration on a UAV is not considered practical. Currently, multi-antenna direction-finding systems are more effective and will be the focus of further research in this area.

7. Conclusions

This paper presents the theoretical foundations the synthesis of signal processing algorithms for radio direction finders with arbitrary configurations, which include differences in the shapes of directional patterns. Models of signals and noise and their statistical characteristics are provided. It is shown that the criterion of the maximum-likelihood functional allows solving the optimisation problems of radio-direction finder synthesis, and the representation of the likelihood equation in the spectral domain allows determining the physical content of the inverse correlation matrices in optimal processing. A significant advantage of the proposed end-to-end new methods synthesis according to the presented theory is the ability to determine the maximum achievable measurement errors.

The effectiveness of the presented theoretical foundation was confirmed by solving a simplified problem of measuring the angular position of a radio source using a single-antenna non-scanning radio direction finder. An interesting finding is that the angle-measurement accuracy near the zero-value region of the radiation pattern is better than that at its maximum values.

Future research directions. Further research will focus on applying the proposed methodology to optimizing scanning radio direction finders, dual-antenna radio direction finders, combinations of radio direction finders with different types of antennas, and multi-antenna spatially dispersed radio direction finders. Further research will include the study of various types of noise-like signals and the challenges that may arise during their processing in multichannel systems.

Contribution of authors: conceptualization, methodology – **Eduard Tserne**; formulation of tasks, analysis – **Eduard Tserne**, **Olha Zhyla**; synthesis of the optimal signal processing algorithm – **Olha Zhyla**; simulation modelling, visualization – **Yevhen Holovynskyi**; writing original draft – **Eduard Tserne**, writing-review and editing – **Olha Zhyla**.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The work has been funded by the Ministry of Education and Science of Ukraine. The state registration number of the projects are 0123U102002 and 0123U102000.

Projects information

The project 0123U102002 is aimed at experimentally designing and manufacturing an experimental a radio imaging system model with high spatial resolution of the underlying surface from an unmanned aerial vehicle (UAV) or helicopter. This system will be relatively lightweight and affordable for production in Ukraine, as it will be assembled mainly using domestic radio-engineering units and components or foreign ones that are not and will not be subject to export control. To generate high-resolution azimuthal radar images of the surface, the radar will be designed using the latest advances in the theory of statistical synthesis of radio systems, and the technology for synthesizing an artificial antenna aperture will be improved by using continuous signals with a wide spectrum. Such signals, using an advanced spatiotemporal processing method, will also ensure the achievement of high spatial resolution in range.

The project 0123U102000 aims at developing an optimal method for detecting UAVs in a multi-frequency radiometric complex with a noise-protected target illumination signal. The authors of the project also aim to create a working model of a radiometric complex for detecting different types and classes of UAVs under different tactical, background, and meteorological conditions, which will meet the contradictory requirements for radio engineering measurements: high spatial resolution, high fluctuation sensitivity, and all-weather capability. The development of a working model of the radiometric complex will combine the advantages of existing methods and supplement them with high-precision measurements to determine the contrasts of different types and classes of UAVs in active and passive noise location modes.

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

Data availability

The manuscript contains no associated data.

Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence methods while creating the presented work.

All the authors have read and agreed to the published version of this manuscript.

References

1. Zeng, L., & Li, C. Recent Advances in Intermodulation and Harmonic Radar Sensing and Tracking. *Proceedings of the 2023 Asia-Pacific Microwave Conference*, Taipei, Taiwan, IEEE, 2023, pp. 846-848. DOI: 10.1109/APMC57107.2023.10439965.

2. Pavlikov, V., Volosyuk, V., Zhyla, S., Van, H. N., & Van, K. N. A new method of multi-frequency active aperture synthesis for imaging of SAR blind zone under aerospace vehicle. *Proceedings of the 14th International Conference The Experience of Designing and Application of CAD Systems in Microelectronics*, Lviv, Ukraine, IEEE, 2017, pp. 118-120. DOI: 10.1109/CADSM.2017.7916099.

3. Wu, J., Wen, F., & Shi, J. Direction Finding in Bistatic MIMO Radar With Direction-Dependent Mutual Coupling. *IEEE Communications Letters*, 2021, vol. 25, no. 7, pp. 2231-2234. DOI: 10.1109/LCOMM.2021.3072310.

4. Pavlikov, V., Zhyla, S., & Odokienko, O. Structural optimization of Dicke-type radiometer. *Proceedings of the 2016 2nd International Young Scientists Forum on Applied Physics and Engineering*, 2016, Kharkiv, Ukraine, IEEE, pp. 171–174. DOI:10.1109/YSF.2016.7753829.

5. Pavlikov, V., Zhyla, S., Odokienko, A., & Antonov, M. Radiometer with signal energies ratio. *Proceedings of the IEEE Radar Methods and Systems Workshop*, Kiev, Ukraine, IEEE, 2016, pp. 99-102. DOI: 10.1109/RMSW.2016.7778560.

6. Krzysztof, C., Rajchowski, P., & Sadowski, J. Wideband Radio Direction Finder Implemented in Software Defined Radio Technology. *Applied Mechanics and Materials. Trans Tech Publications*, 2016, vol. 817, pp. 348-355. DOI: 10.4028/www.scientific.net/amm.817.348.

7. Ziółkowski, C., & Kelner, J. M. Radio Bearing of Sources with Directional Antennas in Urban Environment. *International Journal of Microwave and Wireless Technologies*, 2018, vol. 10, iss. 7, pp. 759-771. DOI: 10.1017/S1759078718000648

8. Gil-Martínez, A., Poveda-García, M., Cañete-Rebenaque, D., & Gómez-Tornero, J. L. Frequency-Scanned Monopulse Antenna for RSSI-Based Direction Finding of UHF RFID Tags. *IEEE Antennas and Wireless Propagation Letters*, 2022, vol. 21, no. 1, pp. 158-162. DOI: 10.1109/LAWP.2021.3122233.

9. Bi, J., Zhao, M., Zheng, G., Chen, Cao, H., Yao, G., Su, F., Wang, T., Li, W., & Zhang, G. Exploiting high-precision AoA estimation method using CSI from a single WiFi station. *Signal Processing*, 2025, vol. 228, p. 109750. DOI: 10.1016/j.sigpro.2024.109750.

10. Cook, H. A., Kahn, M. T. E., & Balyan, V. Radio Direction-Finding Techniques for an Unmanned Aerial Vehicle. *Proceedings of the 2019 International Conference on Management, Education Technology and Economics*, Fuzhou, China, 2020, pp. 1-10. DOI: 10.1007/978-981-15-2329-8_1.

11. Pavlikov, V., Volosyuk, V., Zhyla, S., & van Huu, N. Active Aperture Synthesis Radar for High Spatial Resolution Imaging. *Proceedings of the 9th International Conference on Ultrawideband and Ultrashort Impulse Signals*, Odessa, Ukraine, IEEE, 2018, pp. 252-256. DOI: 10.1109/UWBUSIS.2018.8520021.

12. Pavlikov, V., Volosyuk, V., Zhyla, S., Van, H., & Van, K. N. UWB active aperture synthesis radar the operating principle and development of the radar block diagram. *Proceedings of the IEEE Microwaves, Radar and Remote Sensing Symposium*, Kiev, Ukraine, IEEE, 2017, pp. 27-30. DOI: 10.1109/MRRS.2017.8075018.

13. González-Coma, J. P., Nocelo López, R., Núñez-Ortuño, J. M., & Troncoso-Pastoriza, F. Beacon-Based Phased Array Antenna Calibration for Passive Radar. *Remote Sening*, 2025, vol. 17, iss.3, p. 490. DOI: 10.3390/rs17030490.

14. Ilčev, D. S. New Aspects of Progress in the Modernization of the Maritime Radio Direction Finders (RDF). *Transactions on Maritime Science*, 2021, vol. 10, no. 1, pp. 68-83. DOI: 10.7225/toms.v10.n01.005.

15. Pavlikov, V. V., Zhyla, S. S., van Kiem, N., & Odokienko, O. V. Optimal signal processing for radiometric imaging with multi-antenna & multi-band passive radars. *Proceedings of the International Conference on Antenna Theory and Techniques*, Kharkiv, Ukraine 2015, pp 1-3. DOI: 10.1109/ICATT.2015.7136821.

16. Zhyla, S., Tserne, E., Volkov, Y., Shevchuk, S., Gribsky, O., Vlasenko, D., Kosharskyi, V., & Kovalchuk D. Statistical Synthesis and Analysis of Functionally Deterministic Signal Processing Techniques for Multi-Antenna Direction Finder Operation. *Computation*, 2024, vol. 12, iss.9, pp. 170-191. DOI: 10.3390/computation12090170.

17. Volosyuk, V., & Zhyla, S. Statistical Theory of Optimal Functionally Deterministic Signals Processing in Multichannel Aerospace Imaging Radar Systems. *Computation*, 2022, vol. 10, iss. 12, pp. 213-233. DOI: 10.3390/computation10120213.

18. Markulić, N., Nguyen, J., Benites, J. L., Martens, E., & Craninckx, J. A 10GS/s Hierarchical Time-Interleaved ADC for RF-Sampling Applications.

Proceedings of the 2024 *IEEE Symposium on VLSI Technology and Circuits*, Honolulu, USA, 2024, pp. 1–2. DOI: 10.1109/VLSITechnologyandCir46783. 2024.10631521.

19. Moreira, M. B., Lapuyade, H., Rivet, F., & Deval, Y. A Wide-Band High-Speed Sample and Hold in 0.35μm CMOS Technology. *Proceedings of the 2023 IEEE 14th Latin America Symposium on Circuits and Systems*, Quito, Ecuador, 2023, pp. 1–4. DOI: 10.1109/LASCAS56464.2023.10108219.

20. Dusari, N. R., & Rawat, M. Efficient Implementation of Direction of Arrival Using RFSoC Platform. *Proceedings of the 2023 IEEE Microwaves, Antennas, and Propagation Conference*, Ahmedabad, India, 2023, pp. 1–6. DOI: 10.1109/MAPCON58678. 2023.10464165.

21. Shahul, H. V., & Chinni, G. P. Design and Realization of RFSoC FPGA-Based Multi-Channel Data Acquisition & Digital Beam Former System. *Proceedings of the 2024 IEEE Space, Aerospace and Defence Conference*, Bangalore, India, 2024, pp. 182–185. DOI: 10.1109/SPACE63117.2024. 10667814

22. Analog Devices. AD9084 Datasheet and Product Info. Available at: https://www.analog.com/en/ products/ad9084.html (accessed 25 Febuary 2025).

23. Zhenjia, C., & Zhang, Y. Cooperative Energy Detection Algorithm Based on Background Noise and Direction Finding Error. *AEU - International Journal of Electronics and Communications*, 2018, vol. 95, pp. 326–341. DOI: 10.1016/j.aeue.2018.08.029.

24. Zhou, L., Ye, K., & Zhang, X. Two-Dimensional Direction Finding for L-Shaped Coprime Array via Minimization of the Ratio of the Nuclear Norm and the Frobenius Norm, *Remote Sensing*, 2024, vol. 16, iss. 18, p. 3543. DOI: 10.3390/rs16183543.

25. Wu, Q., Ding, G., Wang, D. J., & Yao, Y. -D. Spatial-Temporal Opportunity Detection for Spectrum-Heterogeneous Cognitive Radio Networks: Two-Dimensional Sensing. *IEEE Transactions on Wireless Communications*, 2013, vol. 12, no. 2, pp. 516-526. DOI: 10.1109/TWC.2012.122212.111638.

26. Riabukha, V., Semeniaka, A., Katiushyn, Y., & Atamanskiy, D. Selection of Number of Compensatory Channels and Placement of Receivers with Non-Identical Frequency Response at Phased-Array Radar under Gaussian Noise Jamming. *Radioelectronics and Communications Systems*, 2022, vol. 65, pp. 1–10. DOI: 10.3103/S0735272722010010.

27. Riabukha, V., Semeniaka, A., & Katiushyn Y. Analysis of protection efficiency of phased-array radars against the Gaussian noise jamming. *Telecommunications and Radio Engineering*, 2021, vol. 80, iss. 2, pp. 1-6. DOI: 10.1615/TelecomRadEng. 2021036556.

28. Holland, R. L. *The statistical theory of radio direction finding. Technical Memorandum AccNr ADA104156*. USA, US Defense Technical Information Center Publ., 1981. 47 p.

29. Voitko, V. V., Ilnyckij, A. I., Iliashov, O. A., Steiskal, A. B., & Marchenko, A. O. Statistical Estimates of the Accuracy of Mono-Pulse Direction Finding of Sources of Radioemission by Dual-Channel Phase Devices. *Visnyk NTUU "KPI". Seriya Radiotekhnika, Radioaparatobuduvannya – Bulletin of NTUU KPI. Series Radiotechnique, Radioapparatus Building*, 2020, no. 81, pp. 30–37. DOI: 10.20535/ RADAP.2020.81.30-37.

30. Fabirovskyy, S., Storozh, V., & Solomko A. Research and development of a high-precision direction finder for radio emission sources detecting. *Information and Communication Technologies, Electronic Engineering*, 2023, vol. 3, no. 2, pp. 158–169. DOI: 10.23939/ictee2023.02.158.

31. Henault, S., Antar, Y., Rajan, S., Inkol, R., & Wang, S. Impact of mutual coupling on wideband adcock direction finders. *Proceedings of the Canadian Conference on Electrical and Computer Engineering*, Niagara Falls, ON, Canada, IEEE, 2008, pp. 1327-1332. DOI: 10.1109/CCECE.2008.4564755.

32. Purwanto, K. N. J., Yahya, A., Khamis, N. H., Nor, N. M., Shaari M. R., & Sidek, A. R. M. Development of Radio Direction Finder using 6 Log Periodic Dipole Array Antennas. *Proceedings of the 5th International Conference on Information Technology, Computer, and Electrical Engineering*, Semarang, Indonesia, 2018, pp. 157–160. DOI: 10.1109/ ICITACEE.2018.8576965.

33. Polikarovskykh, O., & Hula, I. Implementing the Search Algorithm of the Correlation Interferometer Direction Finder through the GNU Radio Software Platform. *Security of Infocommunication Systems and Internet of Things*, 2023, vol. 1, no. 2, article no. 2006. DOI: 10.31861/sisiot2023.2.02006.

34. Quanhua, L. , Zhang, K., Zheng, Z., & Liang, Z. Applying auxiliary arrays for mainlobe blanking to counter mainlobe repeater jamming. *Signal Processing*, 2024, vol. 225, article no. 109599. DOI: 10.1016/j.sigpro.2024.109599.

35. Atamanskiy, D. V. Noise emissions sources direction-finding in the process of their background air threats detection in radars with phased antenna array. *Radioelectronics and Communications Systems*, 2017, vol 60, pp. 303–311. DOI: 10.3103/S07352727 17070032.

36. Sharma, U., & Agrawa, M. High-resolution direction-finding algorithm using Qth−order cumulants tensor: Q−TMUSIC. *Digital Signal Processing*, 2024, vol. 155, article no. 104735. DOI: 10.1016/j.dsp.2024.104735.

37. Kim, S. -A., Chung, H., & Kim, Y. -H. Direction Finding Method for Fast Beam Alignment in Wireless Communication Systems. *Proceedings of the 2023 14th International Conference on Information and Communication Technology Convergence*, Jeju Island, Korea, IEEE, 2023, pp. 1133-1136. DOI: 10.1109/ ICTC58733.2023.10393492

38. Volosyuk, V., Pavlikov, V., & Zhyla, S. Phenomenological Description of the Electromagnetic

Field and Coherent Images in Radio Engineering and Optical Systems. *Proceedings of the International Conference on Mathematical Methods in Electromagnetic Theory*, Kyiv, Ukraine, IEEE, 2018, pp. 302–305. DOI: 10.1109/MMET.2018.8460321.

39. Volosyuk, V. K., & Kravchenko, V. F. *Statisticheskaya teoriya radiotekhnicheskikh sistem distantsionnogo zondirovaniya i radiolokatsii* [Statistical Theory of Radio-Engineering Systems of Remote Sensing and Radar]. Moscow, Fizmatlit Publ., 2008. 704 p. (In Russian)

40. Volosyuk, V. K., Gulyaev, Yu. V., Kravchenko, V. F., Kutuza, B. G., Pavlikov, V. V., & Pustovoit, V. I. Modern methods for optimal spatio-temporal signal processing in active, passive, and combined active-passive radio-engineering systems. *Journal of Communications Technology and Electronics*, 2014, vol. 59, pp. 97–118. DOI: 10.1134/S1064226914020090.

41. Lie, J. P., Blu, T., & See, C. M. S. Single Antenna Power Measurements Based Direction Finding. *IEEE Transactions on Signal Processing*, 2010, vol.58, no. 11, pp. 5682-5692. DOI: 10.1109/TSP.2010.2065227.

42. Lu, Z., Tian, H., & Dong, Y. A Highly Selective and Compact SAW Filter With Cascaded DMSs. *Proceedings of the 2024 IEEE MTT-S International Wireless Symposium*, Beijing, China, 2024, pp. 1–3. DOI: 10.1109/IWS61525.2024.10713780.

43. Analog Devices. AD9363 Datasheet and Product Info. Available at: https://www.analog.com/en/products/ad9363.html (accessed 20 March 2025).

Received 05.02.2025, Accepted 20.05.2025

МЕТОДОЛОГІЯ СТАТИСТИЧНОГО СИНТЕЗУ І АНАЛІЗУ МЕТОДІВ ОБРОБКИ СТОХАСТИЧНИХ СИГНАЛІВ У БАГАТОАНТЕННИХ РАДІОПЕЛЕНГАТОРАХ

Е. О. Церне, Є. А. Головинський, О. В. Жила

Предметом цього дослідження є розроблення методології статистичного синтезу та методів аналізу для обробки стохастичних сигналів у багатоантенних радіопеленгаторах. **Мета роботи** полягає в підвищення точності, стійкості та адаптивності цих систем в умовах експлуатації, що змінюються, особливо для додатків, пов'язаних із безпілотними літальними апаратами (БПЛА). **Завдання** на вирішення яких спрямовано дослідження: 1) формулювання аналітичних моделей сигналів і шумів із заданими статистичними характеристиками; 2) визначення критеріїв функції максимальної правдоподібності для розв'язування задач оптимізації; 3) створення основи для оцінювання граничних помилок вимірювання кутового положення стохастичних джерел сигналів; 4) розроблення структурної схеми радіопеленгаційної системи на основі розробленого алгоритма; 5) імітаційне моделювання пеленгатора. **Методами** розв'язання поставлених задач є статистичні теорії радіотехнічних систем, імітаційне моделювання та оптимізація в спектральній області для розв'язання задач проєктування радіопеленгаторів, здатних обробляти стохастичні сигнали. Цей підхід дає змогу інтегрувати результати амплітудних і фазових вимірювань для кількох антен, забезпечуючи сумісність з обмеженнями, характерними для БПЛА, такими як розмір, вага й аеродинамічні характеристики. Було отримано такі **результати**: 1) теоретичні засади оптимізації радіопеленгаторів для стохастичних сигналів, підтверджені за допомогою імітаційного та аналітичного моделювання; 2) розроблено алгоритми та блок-схеми прототипу одноантенного нескануючого радіопеленгатора, що демонструє запропоновану методику; 3) експериментальна перевірка, що підтверджує здійсненність запропонованих методів. Дана робота відкриває шлях для подальшого розвитку технологій багатоантенної пеленгації, пропонуючи масштабовані рішення для застосування в БПЛА і даючи змогу точно оцінювати параметри сигналу в умовах невизначеності. Майбутні напрямки включають розширення методології на двоантенні системи, гібридні конфігурації та просторово розподілені багатоантенні системи.

Ключові слова: статистична оптимізація; стохастична обробка сигналів; радіопеленгатори; багатоантенний прийом.

Церне Едуард Олексійович – д-р філос., доц. каф. аерокосмічних радіоелектронних систем, Національний аерокосмічний університет "Харківський авіаційний інститут", Харків, Україна.

Головинський Євген Андрійович – асп. каф. аерокосмічних радіоелектронних систем, Національний аерокосмічний університет "Харківський авіаційний інститут", Харків, Україна.

Жила Ольга Володимирівна – канд. фіз.-мат. наук, доц. каф. вищої математики, Харківський національний університет радіоелектроніки, Харків, Україна.

Eduard Tserne – PhD in Telecommunications and Radio Engineering, Associate Professor at the Aerospace Radio-Electronic Systems Department, National Aerospace University "Kharkiv Aviation Institute", Kharkiv, Ukraine, e-mail: e.tserne@khai.edu, ORCID: 0000-0003-0709-2238.

Yevhen Holovynskyi – PhD Student at the Aerospace Radio-Electronic Systems Department, National Aerospace University "Kharkiv Aviation Institute", Kharkiv, Ukraine,
e-mail: y.a.holovynskyi@khai.edu, ORCID: 0009-0004-7342-4207.

Olha Zhyla – Candidate of Physical and Mathematical Sciences, Associate Professor at the Department of Higher Mathematics, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine,
e-mail: olha.kuryzheva@nure.ua, ORCID: 0000-0002-6888-8953.