Radioelectronic and Computer Systems, 2024, no. 3(111)
ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

UDC 623.746.-519
doi: 10.32620/reks.2024.3.06

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FORMATION OF A HETEROGENEOUS GROUP OF UAVS WITH A REASONABLE NUMBER OF FALSE AND REAL DRONES

The subject *of scientific research is the joint use of false and real unmanned aerial vehicles (UAVs) as part of a heterogeneous group of UAVs to perform military tasks. This article* ***aims*** *to determine the appropriate number of false and real drones (UAVs) as part of a heterogeneous group of UAVs to ensure that a certain number of real UAVs can fly to targets with the aim of further reliable target destruction.* ***The scientific task*** *is to develop a methodology for determining the appropriate number of false drones in a heterogeneous group of UAVs, considering the diversity of UAVs included in the UAV group. To achieve the goals of scientific research, partial scientific tasks were solved. The joint use of false drones as part of a UAV group to defeat targets with a given degree of damage was formalized. The formalization was carried out taking into account two possible cases of use: a) when the enemy has a sufficient number of means to destroy the entire group of UAVs; b) when the enemy has an insufficient number of means to destroy the entire UAV group. A mathematical model for determining the optimal composition of false and real drones (UAVs) as parts of a heterogeneous group of UAVs has been developed, which will allow to fulfill the task of defeating enemy targets with the desired degree of reliability. A program code has been developed that simplifies the mathematical calculations in the presented mathematical model and allows it to be used in the process of making an appropriate military decision. An algorithm to find the numbers of real and false UAVs in a heterogeneous group of UAVs is proposed. The obtained formulas and algorithms were verified by computer simulation using the Monte Carlo method.* ***Methods****. The mathematical model is based on combinatorial methods of probability theory. Programming for calculating analytical formulas and computer modeling of the Monte Carlo method was carried out based on the R computer language. The following* ***results*** *were obtained. A multifunctional algorithm is presented: on one hand, its application makes it possible to determine the optimal number of false UAVs in a heterogeneous group of UAVs to ensure that the required number of real UAVs reach the target, and on the other hand, to determine the predicted loss level of real UAVs in a heterogeneous group of UAVs when using a certain number of false drones.* ***Conclusions****. The availability of the developed mathematical model, algorithm, and program code makes it possible to predict the possible results of the combat use of heterogeneous groups of UAVs based on the initial parameters and to substantiate recommendations for a possible composition of such groups.*

Keywords: *unmanned aerial vehicle; drone; decoy; heterogeneous group.*

1. Introduction

Unmanned aerial vehicles (UAVs, drones), among other unmanned aerial systems, are increasingly being used for various purposes, including military purposes [1, 2]. There are several explanations for this:

The use of drones (UAVs) is less expensive than using manned aircraft. Modern UAVs are capable of performing the same tasks as manned aircraft, but the launch of one UAV is much cheaper than the launch of a single fighter or other type of aircraft. In addition, the use of UAVs requires fewer maintenance personnel than aviation;

the use of drones (UAVs) significantly reduces the potential losses of troops. Although the drone operator is in the risk zone (can be detected and hit by the enemy), the probability of his defeat is much lower than that of the pilot of a military aircraft in flight;

the possibility of using UAVs to perform reconnaissance and strike missions in areas that are reliably covered by enemy air defense systems [3];

use of UAVs as decoys for enemy air defense. The existing variety of UAVs makes it possible to use them to expose enemy air defense by launching UAV waves. The first UAV wave can be used to expose an enemy air defense system, and the second and subsequent waves can be used to expose and destroy important enemy targets.

1.1. Motivation

It should be noted that the development of UAVs has also led to the development of methods of their use.

For the effective use of UAVs, a sufficiently large number of optimal flight trajectories and methods for calculating the trajectories have been developed [4].

In addition, methods for the simultaneous use of different types of UAVs, namely reusable or disposable reconnaissance UAVs and strike UAVs, have been developed and are being effectively used. These methods make it possible to simultaneously search for important enemy targets in areas and destroy them using the results of such detection. This approach is extremely relevant because the modern battlefield is characterized by high mobility of enemy objects [5].

However, one of the most common ways to use UAVs for various tasks is to use them in groups (swarms) of UAVs [6]. This challenge motivated our study, which focused on a heterogeneous group of UAVs.

1.2. State of the art

Numerous scientific studies have been conducted on the use of UAVs and their swarms (groups).

Study [7] presents an improved Ex-MADDPG algorithm based on MADDPG to solve the task assignment problem in a target attack scenario using a swarm of UAVs. This algorithm uses average simulation observations and swarm synchronization mechanisms for deploy in systems of arbitrary scale by training only few agents. The algorithm can maintain its performance during the expansion process and achieve an arbitrary expansion of the number of UAVs. In addition, the proposed method proved to be feasible and effective in scenarios of attack by an entire group of UAVs in both simulations and practical experiments.

Study [8] was devoted to creating obstacles for false targets, which is an effective approach for reducing the ability or the probability of UAV detection.

In study [9], the authors investigated the use of UAV onboard phased array radar (MMPAR) to detect, track, and classify malicious UAVs in a group. The simulations demonstrate that cognitive adjustment of MMPAR parameters and the position of an airborne platform using RL helps overcome anomalies in detecting, tracking, and classifying UAVs in a group.

Paper [10] considered an ensemble of drones moving in a two-dimensional domain, each of which is carrying a communication device, and investigated the problem of information transfer in a swarm when the transmission capability is short-ranged. The problem is discussed under the framework of temporal networks, and special attention is given to the analysis of the transmission time of messages transported within the swarm.

In [11], the impact of including a certain number of false UAVs (drones) in a group of real UAVs was studied to increase the probability of fulfilling the main task of the UAV group by dispersing (distracting) the efforts of enemy air defense assets by false UAVs. The study proposed a mathematical model, the use of which makes it possible to propose recommendations on the specific numbers of real and false UAVs that should be included in a UAV group to ensure the fulfillment of the group's task. However, one of the limitations of this mathematical model is that the probabilities of detection and defeat by enemy for real and false UAVs are equal, and the enemy is assumed to have a limited number of means of destruction. This suggests the use of the same type of UAVs, both real and false, to form a homogeneous group of UAVs. However, it is extremely difficult to achieve such conditions since it is better to use either outdated UAVs or other types of lower-cost UAVs instead of real UAVs as false UAVs.

In contrast to such approaches concentrated on physical mechanisms, the model proposed in the paper can be considered a simplified description of a swarm after averaging such mechanisms and effects (e.g. such as a pursuit-evasion optimization problem in [12]) while preserving the multi-staged decision-making process for both an adversary and a planning center. As such, it aims to achieve sufficient simplicity for the sake of statistical analysis and inference during a volatile military situation. Such a task has also been considered in [13, 14, 15] from different perspectives. In particular, the level of operation planning in the case of a solitary swarm was considered in [15], and a model of the decision-making process of planning an attack with a swarm of UAVs was proposed in [16]. A probabilistic Markov chain-based multi-stage confrontation process was also considered [17].

As a result, a new study is presented, the content of which is to substantiate statistically inferred recommendations for the optimal number of real and false UAVs that should be included in a heterogeneous group of UAVs. This necessitates a new study, which aims to substantiate recommendations on the optimal number of real and false UAVs that should be included in a heterogeneous group of UAVs. Moreover, the probabilities of detection and defeat for real and false UAVs will differ, and it is also necessary to consider cases where the enemy may have sufficient and insufficient means of destruction to destroy a heterogeneous group of UAVs. In contrast to the approaches described above, the proposed model can be seen as a simplified swarm description after averaging the underlying mechanisms and effects while preserving the multistage decision-making process for both the adversary and a planning center. As such, it aims to achieve sufficient simplicity for the sake of statistical analysis and inference during a volatile military situation.

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

1.3. Objectives and methodology

The objective of this article is to determine the appropriate number of false and real drones (UAVs) as part of a heterogeneous group of UAVs to ensure that a certain number of real UAVs can fly to targets with the aim of further reliable target destruction. The scientific task is to develop a methodology for determining the appropriate number of false drones in a heterogeneous group of UAVs, considering the diversity of UAVs included in the group.

The structure of this paper is as follows:

1. The joint use of false drones as part of a UAV group to defeat targets with a given degree of damage was formalized (section 2).
2. A mathematical model for determining the optimal composition of false and real drones (UAVs) as part of a heterogeneous group of UAVs has been developed, the use of which will allow to fulfill the task of defeating enemy targets with the desired degree of damage (section 2).
3. A program code has been developed that simplifies the mathematical calculations of the presented mathematical model and allows it to be used in the process of making appropriate military decisions.
4. An algorithm to find the numbers of real and false UAVs in a heterogeneous group of UAVs is proposed (section 2).
5. The results of the practical example obtained using an analytical solution coincide with the results obtained using computer modeling (section 3).

The availability of the developed mathematical model, algorithm, and program code makes it possible to predict the possible results of the combat use of heterogeneous groups of UAVs based on the initial parameters and to substantiate recommendations for a possible composition of such groups.

2. The model

To create a heterogeneous group of UAVs, a decision is made at the planning stage to include a certain number of real UAVs n_1 and a certain number of false UAVs n_2. It is advisable to use cheaper drones as false UAVs because their main purpose is to deplete the enemy air defense system. After creating a heterogeneous group of real and false (n_1+n_2) UAVs, they are launched into the area of the mission.

To protect important objects, the enemy creates an air defense system in the area in which such important objects are located.

For the purposes of this article, it is assumed that the launch of the entire heterogeneous group of real and false UAVs, as well as its flight to the long-range detection limit of enemy air defense systems, was successful because this process is not the subject of this article. Thus, the enemy's air defense systems begin to detect a group of UAVs starting from the long-range detection limit, which is determined by the technical characteristics of the air defense systems. Here, we denote the probability of detecting a real UAV by p_1 and a false UAV by p_2. These probabilities may differ because real and false UAVs are different in type and are therefore displayed differently on the screen of the detection means of the enemy. After the UAV is detected by the enemy air defense systems, a decision is made to fire at it with missiles. The probability of a real detected UAV being hit is denoted by q_1, and by q_2 for a false UAV. However, two cases (I and II) are possible when air targets are being fired at.

The meaning of the first case (I) is that the enemy has enough missiles to attack all detected UAVs (real and false)

$$\xi_1+\xi_2 \le d \ , \tag{1}$$

where ξ_1 is the number of real UAVs detected by enemy air defense systems;

ξ_2 is the number of false UAVs detected by enemy air defense systems;

d is the available number of enemy air defense missiles.

The meaning of the second case (II) is that the enemy does not have enough missiles to hit all detected UAVs (real and false)

$$\xi_1+\xi_2 > d \ . \tag{2}$$

The complexity of the calculations lies in the fact that the party launching the UAV to perform a specific task does not know for sure how many false or real UAVs the enemy can detect and therefore defeat.

Based on the above, the purpose of this study was to determine the probability of reaching the target using a certain number of real drones. The resulting probability distribution makes it possible to form a heterogeneous group of UAVs with the optimal number of false and real drones, the use of which will ensure the fulfillment of the task with the required degree of probability.

The results of the mathematical formalization provide analytical formulas for the desired probabilities and matrix representations for the corresponding distributions. The latter is convenient for writing efficient computer code, and snippets of such code are included in the following. Since the model is inherently statistical, the programming language R, which is known for its simplicity in statistical analysis and ease of reliable prototyping, was chosen.

In the initial stage, we determine the probability distribution of the number of detected (false and real) UAVs. We denote the corresponding distribution by

$$F^{(j)} = \left\| P(\xi_j = k) \right\|_{k=\overline{0,n_j}}, \quad (3)$$

where j is the type of UAV: real ($j=1$) or false ($j=2$);

k is the number of UAVs detected by the enemy (false and real together).

For clarity of notation, let us agree that $F^{(j)}$ is column vectors.

Recall that p_j is the probability of detecting a UAV of a certain type (false or real) and n_j is the number of UAVs of the corresponding type, $j=1,2$.

The actions of detecting different UAVs are independent events, and therefore ξ_1 and ξ_2 are independent binomial random variables. So,

$$F_k^{(j)} = P(\xi_j = k) = C_{n_j}^k p_j^k (1-p_j)^{n_j - k} \times k = \overline{0,n_j}, j=1,2, \quad (4)$$

The joint distribution is the product of the distributions as follows:

$$P(\xi_1 = k, \xi_2 = l) = P(\xi_1 = k) P(\xi_2 = l)$$

and, is given in matrix form as follows

$$P = F^{(j)} \times T(F^{(j)}), \quad (5)$$

where $F^{(1)}$ is the distribution of probabilities of defeating real UAVs;

$F^{(2)}$ is the distribution of probabilities of defeating false UAVs;

$T(F^{(j)})$ is a transposed matrix (in this case, a row vector).

The algorithm (3)...(5) can be represented as a code, figure 1.

After detecting UAVs, the enemy decides to destroy them. Let us consider two cases (I and II) according to whether the enemy has enough missiles d to attack all the detected targets or not. In this regard, it is convenient to represent the matrix P (the distribution of detected UAVs) as the sum of two matrices

$$P = P^{(I)} + P^{(II)}, \quad (6)$$

where

$$P_{m_1,m_2}^{(I)} := P_{m_1,m_2} 1\{m_1 + m_2 \le d\}, \quad (7)$$

$$P_{m_1,m_2}^{(II)} := P_{m_1,m_2} 1\{m_1 + m_2 > d\}, \quad (8)$$

and, 1 is the indicator function of the condition.

The algorithm (6)...(8) can be represented as follows:

```
flag = row(p_detection) + col(p_detection) - 2 <= d
```

In the first case, there are enough missiles to attack all the targets detected, so $\xi_1 + \xi_2 \le d$. Thus, we assume that each UAV is attacked by a single missile. Denote the number of affected (defeated) UAVs of the type j by η_j and the corresponding probability distribution by

$$V^{I,j} = \left\| P(\eta_j = k, \xi_1 + \xi_2 \le d \right\|_{k=\overline{0,n_j}}, j=1,2. \quad (9)$$

The conditional distribution of n_j given that m_j UAVs are detected is binomial with the probability of success (the defeat of a drone) q_j . The probability distribution is given via the matrices $S^{(j)}$,

$$\begin{aligned} S_{k,m_j}^{(j)} &= P(\eta_j = k / \xi_1 + \xi_2 \le d, \xi_j = m_j) = \\ &= S_{m_j}^k q_j^k (1-q_j)^{m_1 - k}, 0 \le k \le m_j \le n_j, \end{aligned} \quad (10)$$

where it is assumed that the matrix element is zero if the condition $0 \le k \le m_j \le n_j$ is not fulfilled.

```
p_detection_drones  = dbinom(x  = 0:n[1],
                             size = n[1],
                             prob  = p[1])
p_detection_decoys  = dbinom(x  = 0:n[2],
                             size = n[2],
                             prob  = p[2])
p_detection  = p_detection_drones  %*% t(p_detection_decoys)
```

Figure 1. Code for algorithm (3)...(5)

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

On the other hand, the probability of defeating k real UAVs in a raid (Case I is still being considered so there are enough missiles to attack all detected UAVs) can be expressed as follows

$$V_k^{I,1} = \sum_{m_1 \ge k, m_1 \le d} S_{k,m_1}^{(1)} A_{m_1}^{(1)}, \tag{11}$$

where $A_{m_1}^{(1)}$ is the probability of detecting m_1 real UAVs assuming there are enough missiles to attack all detected real and false UAVs. In matrix form, the formula is given as follows:

$$V^{I,1} = S^{(1)} \times A^{(1)}. \tag{12}$$

The probability $A_{m_1}^{(1)}$ can be calculated using the following formula

$$A_{m_1}^{(1)} = \sum_{m_2 = \overline{0,n_2}} 1\{m_1 + m_2 \le d\} P_{m_1,m_2} = \\ = \text{rowSums}(P^{(I)}), \quad m_1 = \overline{0,n_1} \tag{13}$$

The algorithm (9)...(13) can be represented as a code, figure 2.

Now consider the second case, in which there are not enough missiles to hit all the targets detected so we have $\xi_1 + \xi_2 > d$. In this case, the enemy randomly selects to hit d UAVs out of $\xi_1 + \xi_2$, where the enemy is assumed to not know which targets are false and which are real; thus, all options are equally likely.

The probability of selecting i real UAVs for defeat, in the case that m_1 real and m_2 false UAVs were detected, can be described by a hypergeometric distribution

$$\frac{C_{m_1}^{i} C_{m_2}^{d-i}}{C_{m_1+m_2}^{d}}. \tag{14}$$

Therefore, the total probability of selecting i real UAVs for destruction (case II) is given by

$$\frac{C_{m_1}^{i} C_{m_2}^{d-i}}{C_{m_1+m_2}^{d}} P_{m_1,m_2}^{(II)} = \frac{C_{m_1}^{i} C_{m_2}^{d-i}}{C_{m_1+m_2}^{d}} 1\{m_1 + m_2 > d\} P_{m_1,m_2}. \tag{15}$$

The probability distribution of the number of defeated UAVs of a type j provided that i UAVs are selected for defeat, is binomial with the number of experiments i and the probability of success (defeat) q_j.

This distribution is given by the matrix $S^{(j)}$ defined in the previous step.

The probability of defeating k UAV of the type j in a raid in case II is given by

$$V^{II,j} = \left\| P(\eta_j = k, \xi_1 + \xi_2 > d) \right\|_{k=\overline{0,n_j}}, j = 1,2. \tag{16}$$

Then, using the formula of total probability and the above considerations, we obtain:

$$V_k^{II,1} = \sum_{i \ge k} S_{k,i}^{(1)} \times \sum_{m_1 = \overline{0,n_1}, m_2 = \overline{0,n_2}} \frac{C_{m_1}^{i} C_{m_2}^{d-i}}{C_{m_1+m_2}^{d}} \times \\ 1\{m_1 + m_2 > d\} P_{m_1,m_2}, k = \overline{0,n_1}, \tag{17}$$

or in matrix form,

$$V^{II,1} = S^{(1)} \times B^{(1)}, \tag{18}$$

$$B_i^{(1)} = \sum_{m_1 = \overline{0,n_1}, m_2 = \overline{0,n_2}} \frac{C_{m_1}^{i} C_{m_2}^{d-i}}{C_{m_1+m_2}^{d}} P_{m_1,m_2}^{(2)}, \quad i = \overline{0,n_1}, \tag{19}$$

The algorithm (16)...(19) can be represented as a code, figure 3.

```
f = sapply(0:n[1],
        function(x) dbinom(0:n[1], x, p=q[1]))
if(d < n[1]){
  f[, (d+2):(n[1]+1)] = 0
}
a_1 = p_detection
a_1[!flag] = 0
a_1 = rowSums(a_1)
f_1 = f %*% a_1
     f_1 = drop(f_1)
```

Figure 2. Code for the algorithm (11)...(15)

```
p_2 = p_detection
p_2[flag]  = 0
a_2 = sapply(0:n[1],
          function(i){
            tmp = outer(0:n[1],
                      0:n[2],
                      Vectorize(function(x,y)  dhyper(i,  x, y, min(d,  x + y))))
            tmp = tmp * p_2
            sum(tmp)
          })
f_2 = f %*% a_2
f_2 = drop(f_2)
```

Figure 3. Code for the algorithm (16)...(19)

The total probability of defeating k real UAVs is the sum of probabilities for the two cases, and the corresponding distribution is given by the following matrix

$$V^{I,1}+V^{II,1} . \quad (20)$$

Note that the probability that k real UAVs will be hit coincides with the probability that $(n-k)$ real UAVs will reach the target. Algorithm (20) can be represented as a code:

```
p_drones = f_1[length(f_1):1]  + f_2[length(f_2):1].
```

In conclusion, this subsection describes an algorithm for finding probability distributions for the numbers of affected real and false UAVs in two different cases considered in this paper.

2.1. Monte Carlo simulations

In the previous section, analytical formulas for the probabilities of defeating real and false UAVs in the two cases studied in this paper were derived. At the same time, the process of using the obtained formulas involves many calculations by hand, which can ultimately lead to errors in the calculations and to a not entirely accurate research result. The above requires the use of a numerical algorithm that can help mitigate predicted risks. The Monte Carlo algorithm also allows us to calculate complex statistics and non-trivial probabilities for which explicit formulas are unknown or cumbersome. The Monte Carlo method also allows the combination of models of different natures and thus serves as a basis for further research. Thus, it is natural to use the Monte Carlo method to solve the problem of modeling a large number of acts of using a heterogeneous group of UAVs to accomplish a task in the face of active enemy air defense systems.

Let us denote the number of computer simulations by N and the number of real UAVs that survived in the jth simulation by $\eta_{1,j}$. Then, according to the law of large numbers, the probability that k real UAVs survive will be approximately equal to

$$\frac{\sum_{j=1}^{N} 1(\eta_{1,j}=k)}{N} . \quad (21)$$

Each simulation follows the following sequence of steps:

1. For each drone (real or false UAV), we use a random number generator to determine whether it is detected.

2. Whether the enemy has enough missiles to defeat all detected real and false UAVs (according to cases I and II) is determined.

3. The number of real UAVs among those detected by the enemy that were attacked by missiles is found. In case II, we use the random number generator again to determine this number.

4. The number of real UAVs that were hit is found.

5. The amount of real UAVs that survived is found.

6. Steps 1-5 are repeated the required number of times, N.

The result: a sample $H=(\eta_{1,1},...,\eta_{1,N})$, where N is number of repetitions. The main algorithm can be represented as a code, figure 4.

The direct use of the Monte Carlo method provides a point estimate of the desired probability. Since it is based on random simulations of the process, the estimate may differ from the true value calculated in the previous section. If the number of simulations tends to infinity, then according to the law of large numbers, the Monte Carlo estimate will converge to the true value of the desired probability. Of course, we cannot run an

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

```
## auxiliary functions
# out: matrix n x N, logical
detectObjects = function(n, p, N){
  runif(n = n * N) %>%
    sapply(FUN = function(x) x <= p) %>%
    matrix(ncol = n,
           nrow = N) %>%
    apply(1, sum)
}
# out: number of hits
determineHits = function(n, p){
  runif(n) %>%
    {. <= p} %>%
    sum %>%
    return
}
## main part
generateMC = function(n, d, p, q, N = 1e4){
  # generate detection events for all N
  detection_drones = detectObjects(n[1], p[1], N)
  detection_decoys = detectObjects(n[2], p[2], N)
  # generate launches for all N
  # and determine which ones target drones
  launches_drones = map2_int(detection_drones,
                             detection_decoys,
                             .f = function(m_1, m_2){
                               if(d >= m_1 + m_2){
                                 return(m_1)
                               }
                               sample(m_1 + m_2,
                                      size = d,
                                      replace = FALSE) %>%
                                 {. <= m_1} %>%
                                 sum %>%
                                 return
                             })
  launches_decoys = pmin(detection_drones + detection_decoys, d) -
    launches_drones
  # determine hits
  hit_drones = sapply(launches_drones,
                      FUN = function(x) determineHits(x, p=q[1]))
  hit_decoys = sapply(launches_decoys,
                      FUN = function(x) determineHits(x, p=q[2]))
  return(list('drones' = n[1] - hit_drones,
              'decoys' = n[2] - hit_decoys))
}
```

Figure 4. Illustration of an algorithm for determining the amount of real UAVs that survived

infinite number of simulations; however, for applied problems, it is sufficient to answer the following questions:

(a) to find the confidence interval for the desired probability for a given number of simulations, that is, to determine the accuracy and reliability of the conclusions based on the Monte Carlo method;

(b) to find a sufficient number of simulations to calculate the desired probability with predetermined accuracy and reliability.

Such tasks are well known problems in mathematical statistics. Let us recall the relevant calculations. Suppose that p is the probability of success in one experiment and $\hat{p}_n$ is the relative success rate in n independent experiments (in our case, in n computer simulations of a successful UAV raid). By the central limit theorem, we have the convergence of the normalized error

$$\frac{\sqrt{n}(\hat{p}_n - p)}{\sqrt{\hat{p}_n(1-\hat{p}_n)}} \quad (22)$$

to the standard normal distribution $N(0,1)$. Given the confidence level δ one can use statistical tables (computer programs) to find a constant c such that

$$P(|N(0,1)|<c)=\delta . \quad (23)$$

Then for large n we have the approximate value of the initial probability. Thus, the confidence interval for the probability p is a segment. For example, if $\delta=0,95$, than $c=1,96$.

Note that the function $x(1-x)$ reaches the global maximum of 0.25 at $x=0.5$. Therefore the value of $c\frac{\sqrt{\hat{p}_n(1-\hat{p}_n)}}{\sqrt{n}}$ does not exceed $\frac{c}{2\sqrt{n}}$. If we need to guarantee that $|\hat{p}_n - p|<\varepsilon$ with the confidence level δ, the number of experiments n should be such that $\frac{c}{2\sqrt{n}}<\varepsilon$, or $n>\left(\frac{c}{2\varepsilon}\right)^2$. For $\delta=0.95$, $\varepsilon=0.03$ the corresponding value n is approximately equal to $\left(\frac{1.96}{2\times 0.03}\right)^2 \approx 1068$.

3. Case study: results and discussion

We consider the situation when a heterogeneous group of UAVs, consisting of false and real drones, is used to reconnoiter and destroy an important enemy object. Due to the statistical nature of the model, the ranges for the probabilities of detecting real\false UAVS are taken wide enough to serve illustrative purposes (such as detecting significant qualitative changes in the behavior of a swarm with rather small changes in the aforementioned probabilities) and to reflect practical needs.

We assume that to guarantee the destruction of the enemy object, at least 3 (or 5) real UAVs (with a probability of at least 0.95) must reach the target. Given the importance of the object, the enemy has taken measures to protect it from air strikes. The analysis of the tactical and technical characteristics of modern enemy air defense systems and the variety of ways to perform UAV tasks led to the following assumptions on the parameters of the model: the interval of values of the statistical probability of detecting a UAV is 0.6...0.9 for a real UAV and 0.6...0.9 for a false UAV, and the probability of defeating a UAV belongs to the interval 0.6...0.9 for a real UAV and to 0.6...0.9 for a false one (the step for modelling purposes is 0.1 in all cases). According to intelligence, the enemy can defeat UAVs at the rate of one missile per UAV.

No more than 20 real UAVs and no more than 50 false UAVs were available. The enemy is in possession of at least 2 missiles.

Note that if the number of missiles is such that $n_1+n_2 \le d$, he probability of hitting a real UAV hitting a target no longer depends on the number of false UAVs and is completely determined by the values of (p_1, q_1).

Under these circumstances, several issues must be addressed:

1. How many false drones should be included in the group so that 3 (5) real drones reach the target.
2. If no more than 10 drones are available to be used as decoys, what is the size of the UAV group to ensure that 3 (5) real drones reach the target.
3. If the maximum acceptable losses are 3 (5) real UAVs out of 6 (10) real UAVs, what number and type of false UAVs should be included in the heterogeneous group.
4. What is the predicted loss level of real UAVs from the group if there are no more than 10 false drones.

The results of the corresponding modeling are presented in Figures 5-12.

Regarding question 1. Figure 5 shows the minimum required number of false UAVs that must be included in a heterogeneous group of false and real UAVs to reach the target by 3 real UAVs. The absence of a point on a graph means that the target cannot be hit given the corresponding constraints on the numbers of UAVs (real and false) and other parameters.

Figure 6 shows similar information for the case of 5 UAVs. Note that in both cases, there are signs of qualitatively different limit regimes: for example, for 20 UAVs in Figure 6, increasing the probability of UAV detection from 0.6 to 0.7 leads to a sharp increase in the number of required UAVs, while increasing this probability further does not lead to such changes.

Figures 7-8 indicate the minimum number of missiles that renders impossible a succesful attack while using no more than 30 false UAVs for all combinations of acceptable probabilities (for the event of a hit by 3 and 5 UAVs, respectively). It is assumed that the enemy cannot use more than 45 missiles.

Regarding question 2. Figures 9-10 show the minimum number of false UAVs required to achieve a

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

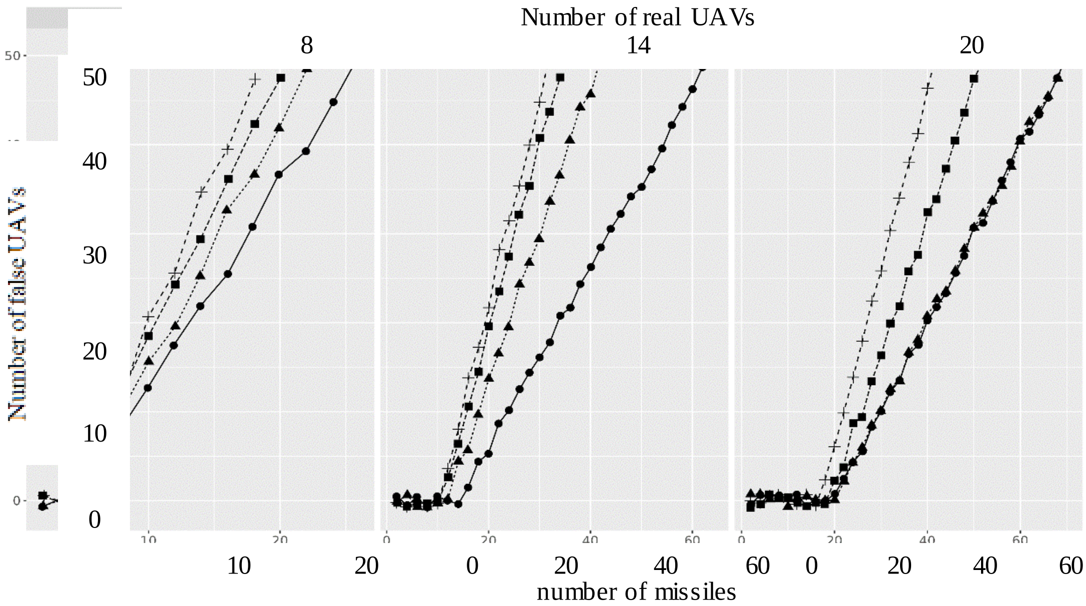

Figure 5. The minimum number of false UAV that guarantees a hit by 3 real UAVs probability to detect a real UAV: • – 0.6; ‣ – 0.7; ▪ – 0.8; + – 0.9

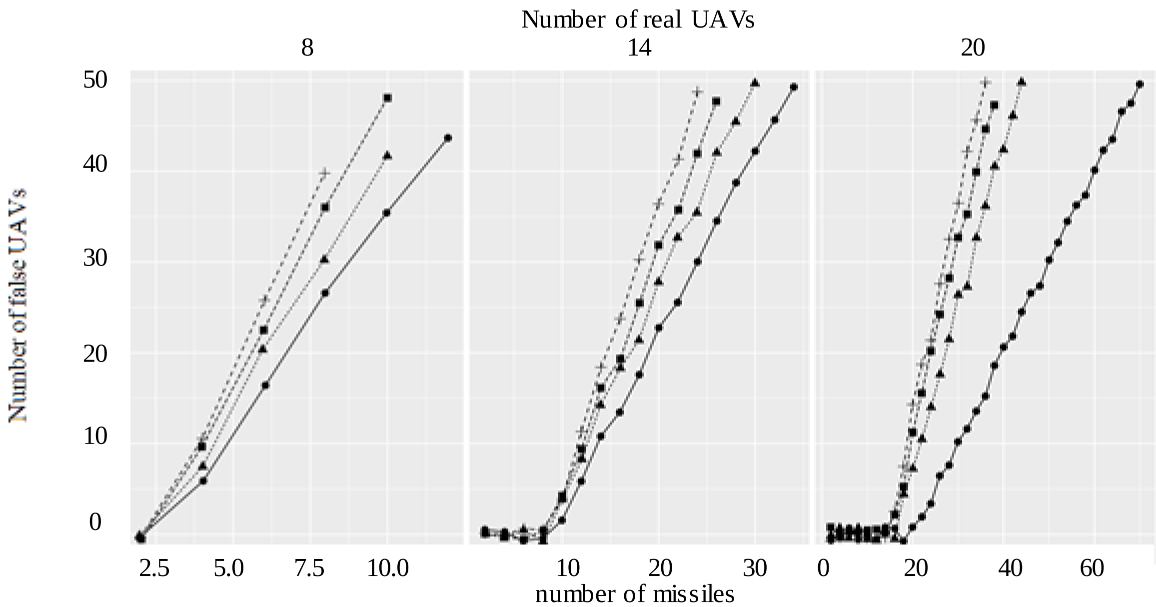

Figure 6. The minimum number of false UAV that guarantees a hit by 5 real UAVs probability to detect a real UAV: • – 0.6; ‣ – 0.7; ▪ – 0.8; + – 0.9

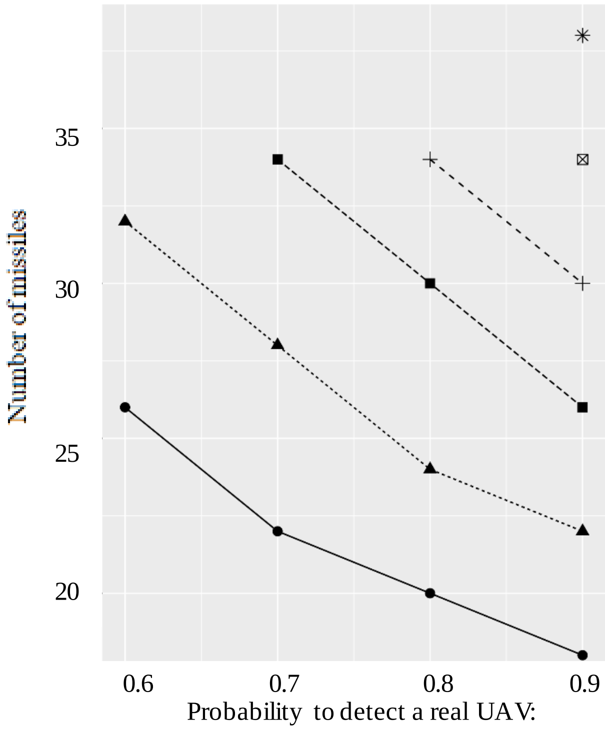

Figure 7. The minimum number of missiles at which target cannot be hit by 3 UAVs

number of real UAVs: • – 6; ‣ – 7; ▪ – 8; + – 9; □ – 10; * – 11

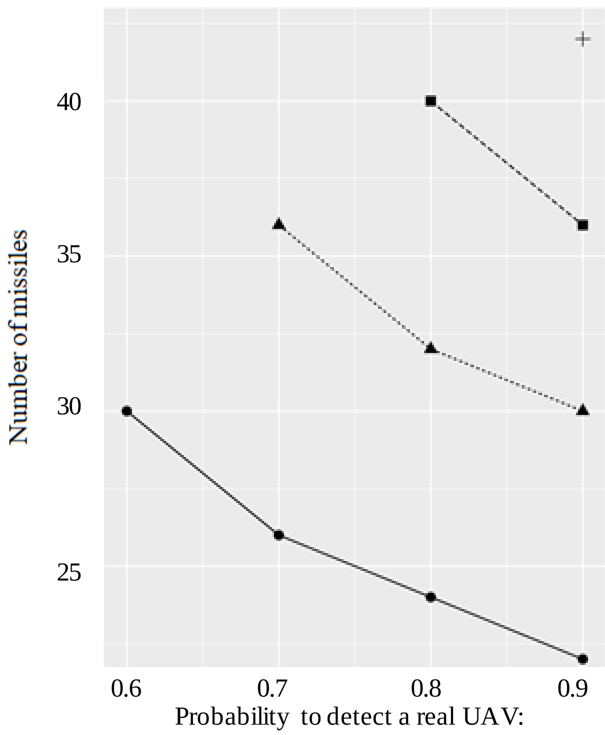

Figure 8. The minimum number of missiles at which target cannot be hit by 5 UAVs

number of real UAVs: • – 10; ‣ – 12; ▪ – 14; + – 16

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ISSN 2663-2012 (online)

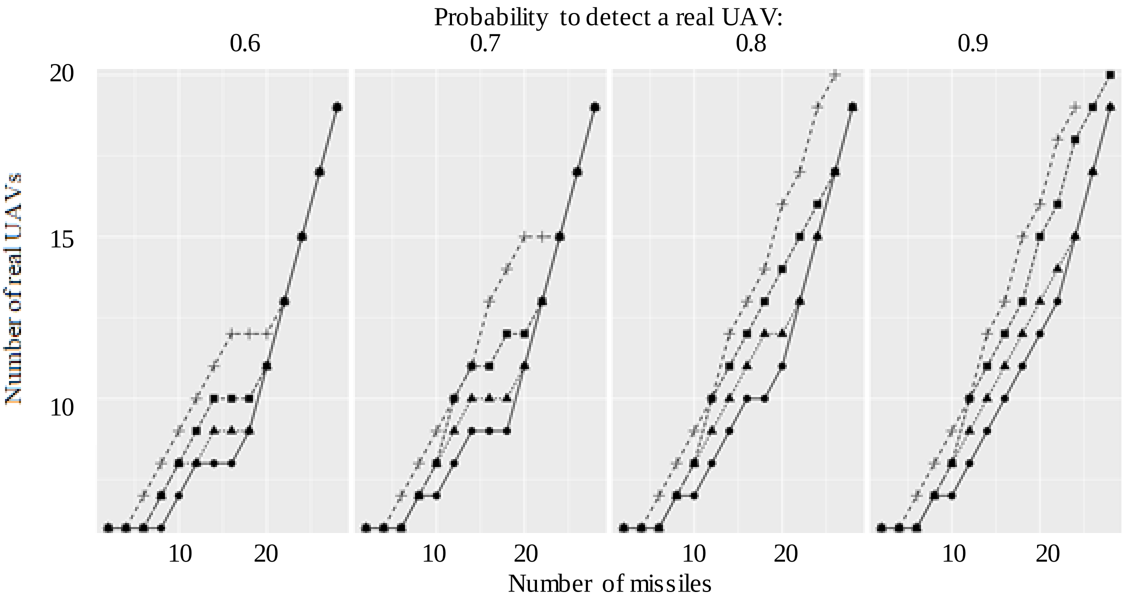

Figure 9. The minimum number of false UAVs to defeat the target by 3 real UAVs
(with equal probabilities of detecting different classes of UAVs)
probability to hit a real UAV: • – 0.6; ‣ – 0.7; ▪ – 0.8; + – 0.9

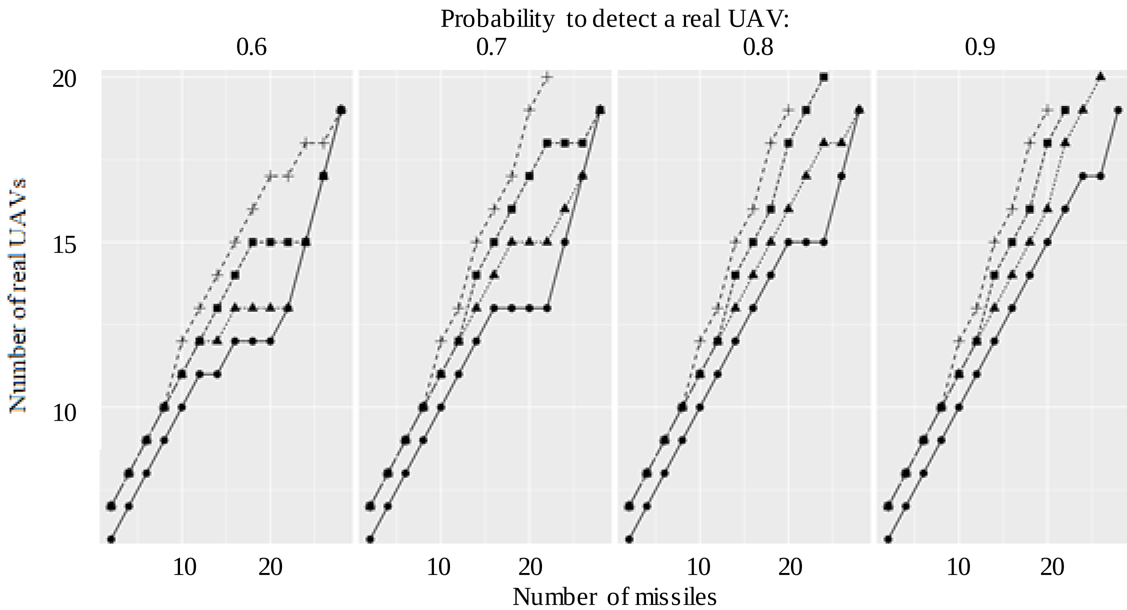

Figure 10. The minimum number of false UAVs to defeat the target by 5 real UAVs
(with equal probabilities of detecting different classes of UAVs)
probability to hit a real UAV: • – 0.6; ‣ – 0.7; ▪ – 0.8; + – 0.9

hit by 3 (5) real UAVs, respectively, assuming that the probabilities of detecting false and real UAVs are equal. Note that it is not always possible to achieve the desired result, as in the case of Question 1.

Regarding question 3. First, assume that the probability of hitting a real UAV is 0.6, and the probability of detecting a false UAV is unknown. Losses are considered acceptable if, with probability not lower than 0.95, no more than 3 (5) real UAVs out of 6 (10) launched real UAVs are lost. Figure 11 shows the minimum number of false UAVs for which losses were acceptable for all possible values of p_2 (the probability of detection for false UAVs).

Assuming that the probability of hitting a real UAV is q_1=0,9 , the minimum number of false UAVs with p_2=0,6 is shown in Figure 12. At the same time, given 20 real UAVs and 12 missiles, it is impossible to achieve acceptable losses without increasing p_2 . In particular, it is necessary to achieve $p_2=p_1$ in this case (with at least 50 false UAVs). It is worth noting the high requirements for the number of false UAVs. For example, if only 20 false UAVs are available and 20 real UAVs are launched, it is impossible to achieve an acceptable level of losses if the enemy uses at least 12 missiles, and if 8 missiles are used, condition $p_2=p_1+0.1$ must be met, and so on.

Regarding question 4. To illustrate the numerical calculations, the average loss rate is plotted against the ratio n_2/d (the number of false UAVs launched divided by the number of missiles available) in Figure 13. The probabilities of detecting false and real UAVs were taken as equal.

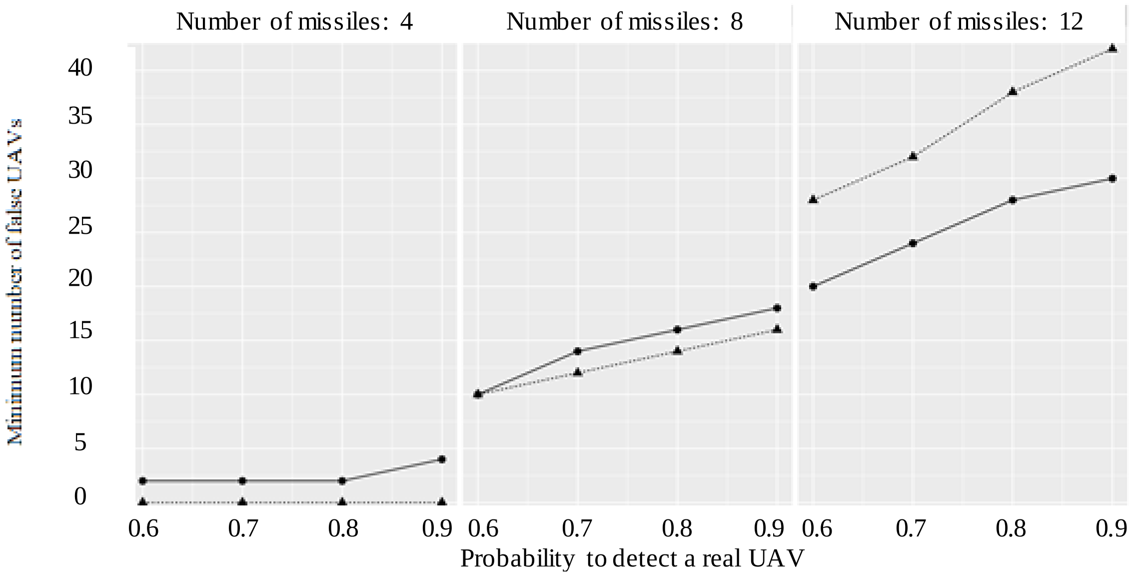

Figure 11. The minimum number of false UAVs that guarantees an acceptable level of losses with a probability of defeating a real UAV 0.6
number of real UAVs: • – 6; ‣ – 20

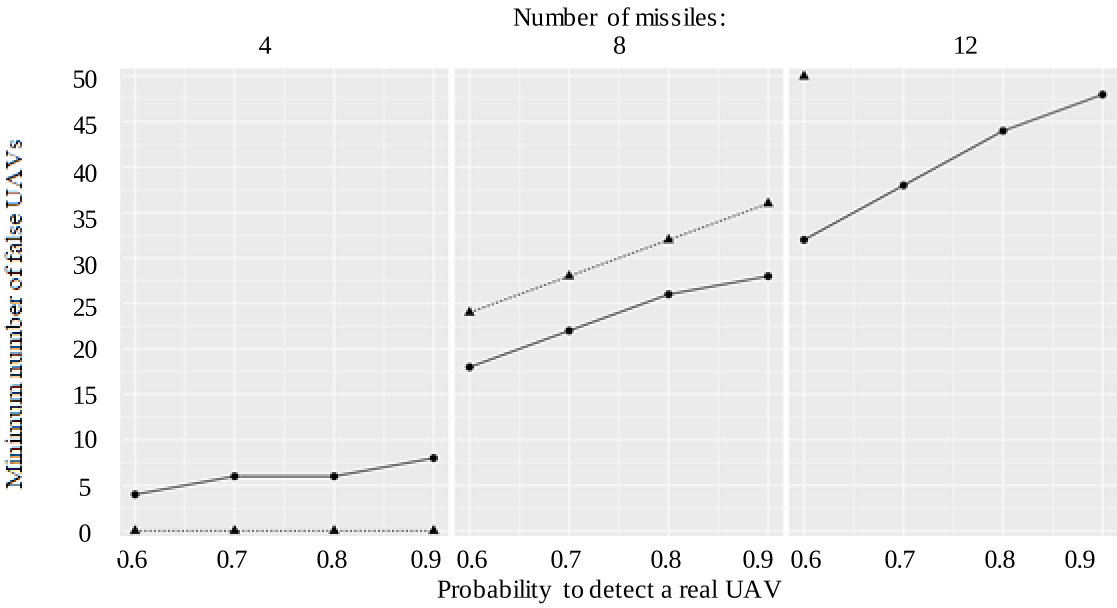

Figure 12. The minimum number of false UAVs that guarantees an acceptable level of losses with the probability of defeating a real UAV 0.9 and the probability of detecting a false UAV 0.6
number of real UAVs: • – 6; ‣ – 20

ISSN 1814-4225 (print)
ISSN 2663-2012 (online)

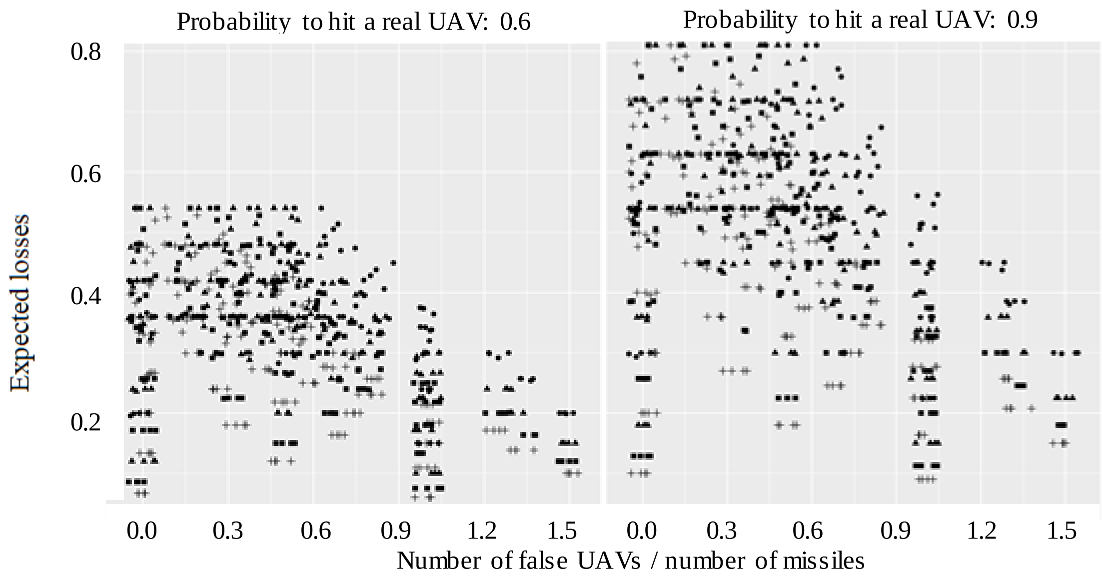

Figure 13. Average proportion of losses in the presence of no more than 10 false UAVs (with equal probabilities of detecting different classes of UAVs)
number of real UAVs: • – 6; ‣ – 10; ▪ – 14; + – 18

4. Conclusions

Thus, this article presents a methodology, an algorithm, and program code for finding the probability of defeating an enemy using a group of false and real UAVs. The calculations were performed analytically and using the Monte Carlo method. These calculations can be used to determine the optimal numbers of false and real UAVs (drones) in a heterogeneous group of UAVs to achieve reliable performance in combat missions to destroy important enemy targets covered by air defense systems. In addition, based on this algorithm and program code, a practical problem was solved, and relevant recommendations were provided. The process of determining the optimal number of false and true UAVs (drones) in a heterogeneous group of UAVs is carried out in several stages. In the first stage, it is determined whether an object (real or false UAV) was detected by an enemy reconnaissance system during a combat mission. In the second stage, the number of missiles launched at an UAV is determined. The third step is to determine the number of real UAVs that were attacked but not shot down. The next step is to determine the number of survivors among real UAVs, depending on the number of false UAVs included in the UAV group. The proposed model predicts the required number of UAVs of different types to achieve success with a predetermined probability. The proposed model is multifunctional because, on the one hand, it can be used to find the required number of false drones in a heterogeneous group of UAVs to ensure reliable performance during combat missions and, on the other hand, to determine the predicted losses of real and false UAVs under different combat use conditions. In addition, this methodology considers the possibility of using different types of UAVs in the same group.

The example input data is derived from a possible case of combat operations, and the obtained results indicate the possibility of using the presented mathematical model and program code in practice to make appropriate military decisions.

The availability of this model and program code will allow, based on the initial parameters, an assessment of the possible results of the combat use of heterogeneous groups of UAVs (drones) and substantiation of recommendations on their possible composition and the conditions of the operational and tactical situations being considered.

Thus, the existence of this mathematical model, program code and adequate practical results indicate achievement of the research goal.

5. Directions for further research

Further research in this direction may include finding ways to ensure the stability of a heterogeneous group of UAVs during their tasks (including the case of large swarms) and expanding and improving the mathematical model.

Contributions of the authors: introduction, motivation, analysis of previous studies – **Serhii Bazilo**, problem statement, formalization of the process of using a heterogeneous group of UAVs, analysis of research results – **Volodymyr Prymirenko**, analysis of existing mathematical models of UAV swarms, identification of problematic issues and shortcomings in existing models, development of a mathematical model of UAVs group use – **Andrey Pilipenko**, writing the program code of the mathematical model, modeling the use of a UAVs group and description of modeling results, plotting and description of the results – **Mykola Vovchanskyi**, objectives and methodology, formulation of the input data, conditions and tasks of the practical example for testing the mathematical model – **Andrii Demianiuk**, state of the art, analysis and interpretation of the modeling results, creation of the graphs and figures, conclusions, prospects for further research – **Roman Shevtsov**.

Conflict of Interest

The authors declare that they have no conflict of interest concerning this research, whether financial, personal, authorship, or otherwise, that could affect the research and its results presented in this paper.

Financing

This study was conducted without financial support.

Data Availability

Data will be made available upon reasonable request.

Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence technologies in their work.

Acknowledgments

A. Pilipenko was supported by the project Mathematical modeling of complex dynamical systems and processes caused by the state security (Ukraine, Reg. №0123U100853).

All the authors have read and agreed to the published version of this manuscript.

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Received 06.06.2024, Accepted 20.08.2024

ФОРМУВАННЯ НЕОДНОРІДНОЇ ГРУПИ БПЛА З ДОЦІЛЬНОЮ КІЛЬКІСТЮ ХИБНИХ І ДІЙСНИХ ДРОНІВ

В. М. Приміренко, А. В. Дем'янюк, Р.В. Шевцов, С.М. Базіло, А. Ю. Пилипенко, М. В. Вовчанський

Предметом дослідження є процес спільного застосування хибних та дійсних БПЛА у складі неоднорідної групи БПЛА для виконання завдань військового призначення. **Метою** статті є визначення доцільної кількості хибних та дійсних БПЛА у складі неоднорідної групи БПЛА для забезпечення прольоту до цілі визначеної кількості дійсних БПЛА з метою ураження цілі. Вирішувалися такі **завдання**. Формалізовано процес спільного застосування хибних дронів у складі групи БПЛА для ураження цілей із заданим ступенем їх ураження. Розроблено математичну модель визначення складу хибних та дійсних БПЛА у складі неоднорідної групи БПЛА, застосування якої дасть змогу виконати завдання з ураження цілі противника з потрібним ступенем ураження. Розроблено програмний код, який значно спрощує математичні розрахунки представленої математичної моделі та дозволяє використовувати його у процесі прийняття відповідного військового рішення. Запропоновано алгоритм для знаходження числа дійсних і хибних БПЛА у складі неоднорідної групи БПЛА. **Методи.** Математична модель базується на основі комбінаторних методів теорії ймовірностей. Програмування обчислення аналітичних формул та комп'ютерне моделювання методу Монте-Карло здійснено на основі комп'ютерної мови R. Отримані **результати**. Представлено алгоритм, який є багатофункціональним, оскільки, з одного боку, його застосування дає змогу визначити оптимальну кількість хибних БПЛА у складі неоднорідної групи БПЛА для забезпечення прольоту до цілі потрібної кількості дійсних БПЛА, а з іншого боку – визначити прогнозований рівень втрат дійсних БПЛА із неоднорідної групи БПЛА при застосуванні певної кількості хибних дронів. При цьому, результати практичного прикладу, отримані шляхом аналітичного розв'язку, збігаються з результатами, одержані методом комп'ютерного моделювання. **Висновки.** Наявність математичної моделі, алгоритму та програмного коду дозволяє прогнозувати можливі результати бойового застосування неоднорідних груп БПЛА на основі вихідних параметрів та обґрунтовувати рекомендації щодо їх можливого складу.

Ключові слова: безпілотний літальний апарат; дрон; приманка; неоднорідна група.

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