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Imad BADI[1], Hassan BADI[2], Aziz KHAMJANE[1],
Karim EL MOUTAOUAKIL[2], Abdelkhalek BAHRI[1]

[1] ***Laboratory of Applied Sciences (LSA) National School of Applied Sciences (ENSAH), Adelmalek Esaadi University, Al Hoceima, Morocco***
[2] ***Laboratory of Engineering Sciences, Multidisciplinary faculty of Taza, Sidi Mohamed Ben Abdellah University, Fes, Morocco***

SUPERVISED IDENTIFICATION AND EQUALIZATION OF TRANSMISSION CHANNEL USING REPRODUCING KERNEL HILBERT SPACE

The subject matter of the article is to identify and equalize the parameters of telecommunication channels. The goal is to develop a new mathematical approach based on positive definite kernels on a Hilbert space. The tasks to be solved are: (a) to formulate a mathematical procedure based on a kernel; a kernel is a function that maps pairs of data points to a scalar value, and positive definite kernels are widely used in machine learning and signal processing applications; (b) to identify the channel parameters using the proposed method; and (c) to apply the Zero Forcing and MMSE equalizer to measure the performance of the proposed system. This article introduces a new method to address the problem of supervised identification of transmission channel parameters based on the positive definite kernel on Hilbert space, which implements Gaussian kernels. The input sequence, used as an input for a system or process, is assumed to be independent, have a zero mean, a non-Gaussian distribution, and be identically distributed. These assumptions are made to simplify the analysis and modeling. The proposed method for estimating the parameters of the channel impulse response yields promising results, indicating that the estimated parameters are close to the measured parameters of the model for various channels. The convergence of the estimated parameters toward the measured parameters of the model is particularly noticeable for BRAN A (indoor) and BRAN E (outdoor) channels. The method has been tested with different channel models, and the results remain consistent. Overall, the proposed method appears to be a reliable and effective approach for estimating channel impulse response parameters. The accuracy of the estimated parameters is particularly noteworthy considering the challenges inherent in modeling wireless channels, which can be influenced by various factors such as obstacles and interference. These findings have important implications for the design and optimization of wireless communication systems. Accurate estimates of channel impulse response parameters are essential for predicting and mitigating the effects of channel distortion and interference, and the proposed method represents a promising tool for achieving this goal. Further research and testing are needed to validate and refine the method and to explore its potential applications in different settings and scenarios. We evaluated the performance of the system using the estimated parameters obtained from the proposed method. Two equalizers, MMSE and ZF, were used, and the results show that MMSE outperforms ZF. Both equalizers produced highly satisfactory outcomes.

Keywords: *FIR channel; MC-CDMA; Equalization; identification; transmission channel; Reproducing Kernel Hilbert Space; BRAN; ZE equalizer; MMSE equalizer.*

1. Introduction

Wireless or cellular networks have undergone a great evolution, which requires operators to improve their offered services. Vocal conversations were a major challenge for researchers, especially due to the massive explosion of Internet traffic on fixed networks, which imposes an effective improvement in techniques based on mobile network technology. Several services have been introduced, such as 2G systems, including email, web access, and online bank transaction services. Then, the 3G architecture was developed to solve the defects of the 2G. The first is based on Direct Sequence Code Division Multiple Access (DSCDMA) [1]. However, the capabilities of the 3G system remain insufficient due to the needs expressed in permanence. The 4th generation that supports high-speed remains a solution adapted to the expectations of users [2]. The 4th generation, which supports broadband, comes as a solution adapted to the expectations of users' needs, but it is still insufficient to meet the needs and number of users, which is constantly increasing. For this reason, the generation 5G has taken place, which already promises speeds 10 times higher than those of 4G, with maximum speeds expected of 20 Gb/s. The latter is still in its infancy, and its deployment should still take a few years. Although 5G is still very young in

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the world market, telecommunications specialists are already preparing the ground for the network that will succeed it – we are talking about 6G. The migration of our approach to these last generations will be the subject of our future work.

The identification of channel parameters has become a major topic addressed mainly by researchers in the signal-processing field [3]. In this work, we will apply a new method based on a positive-definite kernels on Hilbert space using a Gaussian kernel to identify the channel parameters [4].

2. Related works.

In this article, we study the application of positive definite kernels on a Hilbert space for the identification and equalization of a transmission channel, so it is necessary to review existing studies and approaches already used in this context. The widespread use of the Internet has led to a massive increase in the data throughput of wireless communication systems, and many research studies have focused on the identification of channel parameters using different approaches such as order cumulants [2], which addressed the problem of blind identification and equalization using higher order cumulants for MC-CDMA systems. Another approach was introduced in [5]. This approach was based on neural networks, and this paper proposes a training scheme using the Cuckoo Search Algorithm (CSA) for Functional Link Artificial NN (FLANN) based channel equalizers. In [6], the authors have proposed an approach based on Recursive Least Square (RLS) and Least Mean Squares (LMS) algorithms for the supervised and blind identification of channel parameters. In [7 - 9], the authors have discussed the techniques that allows eliminating the effect of intersymbol interference (ISI) in multipath wireless channels, such as the MMA algorithm. Consequently, the design of a channel equalizer has become more and more demanding. For severe nonlinear distortions, the authors in [10] and [11] conclude that neural network (NN) based on nonlinear channel equalizers provide superior performance than the linear equalizers based on an adaptive filter. Unsupervised adaptive attenuation of inter-symbol interference in an additive impulse noise environment, modeled as a generalized Gaussian, has been discussed in [12].

This work introduces a new technique, based on very robust mathematical models, to solve the problem of identification and equalization of a transmission channel. Indeed, there are many other methods already used in other works. Our method proposes an additional alternative with encouraging and satisfactory parameters identified in terms of accuracy, which can be combined with others to obtain a perfect transmission. The technique proposed can be used across various domains, particularly in the efficient handling of spatial and temporal random signals in multi-channel radar systems deployed on mobile aerospace platforms [13]; this technique can also be used to address communication protocol issues in the context of the Internet of Things [14].

3. Objective

The main objective of this work is to apply algorithms based on RKHS properties to channel identification and equalization. In this respect, the paper presents an equalizer proceeding in two steps: identification of the channel parameters using RKHS methods and computation of the equalizer parameters given by ZF or MMSE criteria.

4. Mathematical formulations

A positive definite kernel on a set χ is a function K that associates two objects to a real number [15].

$$k: \chi \times \chi \rightarrow \mathbb{R}.$$

Symmetry and positivity are required. For any finite sequence, $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$ the matrix K of elements $\mathbf{K}(\mathbf{x_i}, \mathbf{x_j})$ (called matrix of Gram in the following sections) is positive definite if:

$$\forall \boldsymbol{\alpha} \in \mathbb{R}^{\mathbf{n}} : \boldsymbol{\alpha}^{\mathbf{T}} \mathbf{K} \boldsymbol{\alpha} \geq \mathbf{0}.$$

Examples of positive kernels defined for $\boldsymbol{\chi} = \mathbb{R}^{\mathbf{d}}$.
- Linear kernel $\mathbf{K}(\mathbf{x}, \mathbf{y}) = \mathbf{x}^{\mathbf{T}}\mathbf{y}$;
- Polynomial kernel $\mathbf{K}(\mathbf{x}, \mathbf{y}) = (\mathbf{1} + \mathbf{x}^{\mathbf{T}}\mathbf{y})^{\mathbf{d}}$;
- Gaussian kernel $\mathbf{K}(\mathbf{x}, \mathbf{y}) = \mathbf{exp}[-\frac{\|\mathbf{x}-\mathbf{y}\|^2}{2\sigma^2}]$.

Kernels can also be viewed from the functional analysis view point, since to each kernel K on χ is associated a Hilbert space H_k of real-valued functions on χ.

Definition 1:

A real-valued function $k : \boldsymbol{\chi} \times \boldsymbol{\chi} \rightarrow \mathbb{R}$ is a reproducing kernel of a Hilbert space $\mathbb{H}_{\mathbf{k}}$ of real-valued functions on χ if and only if:

– $\forall x \in \chi, k(x, .) \in \mathbb{H}_k$;

– $\forall x \in \chi, \forall f \in \mathbb{H}_k, < f, k(x, .) > = f(x)$.

We restrict ourselves to the case of a time invariant linear channel of communication, and we suppose a finite impulse response. It can be described as a convolution filter h(k) of the transmitted signal x(k) [2, 16].

We assume that we observe the output signal y(k) as a discrete-time signal, which is described by the following equations:

$$y(k) = h(k) * x(k) = \sum_{i=0}^{q} h(i)x(k-i), \quad (1)$$

and

$$s(k) = y(k) + n(k), \quad (2)$$

where $x(k)$ is the input sequence;

$h(k)$ is the channel parameters;

$y(k)$ represents the system output in the noiseless case and $s(k)$ is the observed system output corrupted by additive Gaussian noise $n(k)$;

$n(k)$ is an additive white Gaussian noise.

The following conditions are assumed to be satisfied:

– the model order q is supposed to be known;

– the input sequence $x(n)$ is independent and identically distributed (i.i.d) zero mean, the variance is $\sigma_x^2 \equiv 1$, and non-Gaussian;

– the system is causal and $h(0) = 1$;

– we assume that the measured noise $n(k)$ is zero mean;

– Gaussian, i.i.d, with unknown variance σ_n^2.

The problem addressed is to identify the channel parameters h using the positive definite kernels, so, if we set:

$$F(k) = h(k) * x(k). \quad (3)$$

Equation (2) becomes:

$$s(k) = F(k) + n(k). \quad (4)$$

Historically, kernels approaches were mentioned sixty years ago. The essential idea is to use the learning examples $\langle(x_i, y_i)\rangle_{i=1,m}$ to make parameter identification of the input space χ by measuring the outputs associated with inputs$(x_i)_{i=1,m}$.

Most often, the form of kernel functions generally used is of the form:

$$K(x, x') = g\left(\frac{d(x,x')}{\sigma}\right), \quad (5)$$

where $d(x, x')$ is a distance defined on a set χ ;

$g(.)$ is a decreasing function and σ is the covariance.

The use of this type of kernel function leads to estimate $\hat{y}$ as measured average of $(y_i)_{i=1,m}$ with a stronger measure to examples where the distance $d(x, x')$ is small, the notion of being determined by the small value of σ.

Using the inductive principle of minimization of empirical risk, the estimated parameters are obtained by minimizing the following function [17]:

$$R_H(F) = \frac{1}{n}\sum_{i=1}^{n}(y_i - F(x_i))^2, \quad (6)$$

where:

$$F(x_i) = \sum_{j=0}^{n} \alpha_j \, k(x_i, x_j) \,. \quad (7)$$

In our case, we try solving this non-linear regression problem in χ, we may be tempted to use this function of re-description ϕ of χ in a new space H, so that the problem becomes a linear regression problem in the new space. Our approach is to estimate the parameters of the vector F, which amounts to estimate the parameters α. Since it is supervised identification, the output of the channel is known, we can consider the simplified matrix form:

$$F = K.\alpha, \quad (8)$$

where K is the matrix of kernel and F is the output signal, this equation will be verified by experiments.

According to the principle of least squares, the vector of the coefficients α is obtained as follows:

$$\alpha = (K^T.K)^{-1}K^T.F\,. \quad (9)$$

Since F is defined by the parameters α and h is function of F, if we identify α we can find h by applying Laplace transform as follows:

$$L(F) = L(h).L(x), \quad (10)$$

$$L(h) = L(F)\,/L(x). \quad (11)$$

Then, we can apply the equalization of the defied parameters h by the ZF and MMSE equalizer.

5. The proposed approach

5.1. Definitions

The symmetric Gram matrix (or kernel matrix) contains all the information used by the kernel methods on the input data. The exclusive use of the information contained in this matrix on the training data results in the loss of part of the information on these data. For example, this matrix corresponds to rotationally invariant points in the input space χ.

Lets χ be a non-empty set and H a Hilbert space. We define a kernel k from: $\chi \times \chi \to R$ by the following function:

$$k(x, y) = \exp(-\|x - y\|^2). \quad (12)$$

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The vector x is the input signal, and y is the output signal.

In the following sections, we will apply the defined kernel in equation (12) to different channels.

5.2. Radio channel

There are many obstacles between a Mobile Station (MS) and a base station. Neighboring buildings cause a great loss of information during transmission between a fixed base and mobile station, especially near the MS. For this, the channel can be viewed as a time-varying linear filter of impulse response with $h(\tau)$ observed at the time t, which can be expressed as Safi et al [2, 18].

$$h(\tau) = \sum_{i=0}^{L-1} \psi_i\, \delta(\tau - \tau_i), \qquad (13)$$

where $\delta(n)$ is Dirac function, ψ_i is the magnitude of the target i, L = 18 is the number of target and τ_i is the time delay (from the origin) of target i.

We investigated two types of radio channels: (1) Mobile Channel Model (BRAN A) which is described in Table 1 and (2) Mobile Channel Model (BRAN E) which is considered the case of a channel in the interior of an office has been treated, it is imperative to handle the case in an outdoor environment (Table 2).

Table 1

Delays and magnitudes of the 18 trips BRAND A radio channel

Delays τ_i(ns)	magnitudes h_i(dB)	Delays τ_i(ns)	magni-tudes $\mathbf{h_i}$(dB)
0	0	90	-7.8
10	-0.9	110	-4.7
20	-1.7	140	-7.3
30	-2.6	170	-9.9
40	-3.5	200	-12.5
50	-4.3	240	-13.7
60	-5.2	290	-18
70	-6.1	340	-22.4
80	-6.9	390	-26.7

5.3. MC-CDMA System

In MC-CDMA systems, the codes are transmitted in the frequency domain on different subcarriers. Consequently, the subcarriers are sensitive to problems of synchronization, such as timing errors, the frequency offset of the carrier, and the amplitude and phase additive noise. In this section, we describe an equalization based on a new concept using positive definite kernels. It is a technology considered as candidates for wireless communication systems of the fourth generation [19].

Table 2

Delays and magnitudes of the 18 trips BRAND E radio channel

Delays τ_i(ns)	magnitudes h_i(dB)	Delays τ_i(ns)	magnitudes $\mathbf{h_i}$(dB)
0	-4.9	320	0.0
10	-5.1	560	-2.8
20	-5.2	710	-5.4
40	-0.8	400	-11.7
70	-1.3	880	-7.3
100	-1.9	1070	-10.6
140	-0.3	1280	-13.4
190	-1.2	1510	-17.4
240	-2.1	1760	-20.9

The MC-CDMA signal is derived from the concatenation of the operation spread spectrum by direct sequence modulation and multi-carrier [19, 20]. The MC-CDMA modulator spreads the data a_i for each user i in the frequency domain. The spreading is done by multiplying data ai by each element of ci code associated sprawl. Here, the MC-CDMA emulation system is given by:

$$x(t) = \frac{a_i}{\sqrt{N_p}} \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} \exp^{2jf_k t}, \qquad (14)$$

where $f_k = f_0 + \frac{1}{T_c}$, Nu the number of users and Np the number of subcarriers. The impulse response h of the channel can be written as in equation 15. This expression takes fixed paths when the transmitter and receiver are moving.

$$h(\tau, t) = \sum_{p=0}^{P-1} \beta_p\,(t) * \\ * \exp^{i\left(2\pi\nu_p t + \theta_p(t)\right)} \delta\left(\tau - \left(\tau_p\right)\right). \qquad (15)$$

The issued signal s and received signal r are linked by:

$$r(t) = (h * s)(t) + n(t) = \\ = \int_{-\infty}^{+\infty} \sum_{p=0}^{P-1} \beta_p \exp^{i\theta_p} \delta\left(\tau - \tau_p\right) * \\ * x(t-\tau)d\tau + n(t) = \qquad (16) \\ = \sum_{p=0}^{P-1} \beta_p \exp^{i\theta_p} x(t-\tau) + n(t).$$

To overcome disturbances introduced by the transmission channel, it is necessary to implement an equalization device. Two classes of detectors exist [21, 22], single-user detectors, and multi-user detectors. In the first case, only the sequence of the user is assumed to be known, and related interference to other users is then considered jammer. On the other hand, since the sequences for all users are known, multiple access interference is then considered a deterministic signal and not random. Therefore, where Nu users are active, the received signal can be expressed using the relationship in equation 15 of the impulse response h of the channel:

$$r(t) = \frac{1}{\sqrt{N_p}} \sum_{p=0}^{P-1} \sum_{j=0}^{N_u-1} \sum_{k=0}^{N_p-1} \mathrm{real}(\, \beta_p \exp^{i\theta_p} a_j * \\ * c_{k,j} \exp\text{“}\epsilon^{2i\pi\left(f_0+\frac{k}{T_c}\right)(t-\tau_p)}). \tag{17}$$

To facilitate the presentation of different detection techniques, we assume an adequate dimensionality of the system studied. Therefore, the duration of the guard interval Tg is sufficient, ensuring no interference between symbols and subcarriers. Furthermore, the duration of the MC-CDMA symbol is sufficient to consider the channel as invariant during this symbol, as well as the absence of frequency selectivity on each subcarrier. Then, the temporal and frequency interleaving will guarantee the independence of random processes affecting each subcarrier. Finally, and to facilitate the introduction of different detection techniques [23, 24], we will take Lc $=$ Np. We can represent our matrix system:

$$r = HCa + n, \tag{18}$$

where r is a vector containing the values received on each subcarrier:

$$r = \left[r_0 \dots r_{N_p-1} \right]. \tag{19}$$

The matrix H is the matrix of size parameters of the channel Np × Np. The hypotheses previously assumed on the proper sizing of the system allow us to consider this matrix as a diagonal matrix H:

$$H = \begin{pmatrix} h_0 & 0 & \cdots & 0 \\ 0 & h_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{N_p-1} \end{pmatrix}. \tag{20}$$

The matrix C represents the spreading codes. The spreading operation can therefore be represented as the multiplication of the matrix C by the vector a, constituted of the data of each user. We can therefore write:

$$C = \left[c_0, \cdots, c_{N_n-1} \right] = \\ = \begin{pmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,N_n-1} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,N_n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_p-1,0} & c_{N_p-1,1} & c_{N_p-1,2} & c_{N_p-1,N_n-1} \end{pmatrix}, \tag{21}$$

where $c_i = [c_{0,i}, c_{1,i}, \cdots, c_{N_p-1,i}]^T$;

$a = [a_0, a_1, \cdots, a_{N_p-1}]^T$.

The vector n presents the Np components of the noise affecting each subcarrier, it can be modeled as a Gaussian additive process: $n = [n_0, n_1, \cdots, n_{N_p-1}]^T$

In reception, the structure of the study detectors based on the use of an equalization stage followed by dispreading operations according to the user's sequence considered; see figure 1.

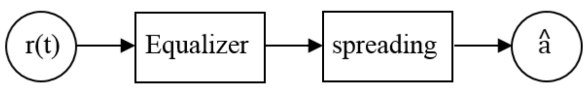

Figure 1. Principle of single-user detection

Single-user detectors consider only the active user signal; other users are treated as jammers. Single-user detectors typically use a linear equalization structure [3, 25], consisting of an equalizer into an outlet. Using the above matrix notation, it is possible to express G, a diagonal matrix composed of coefficients equalization G:

$$G = \begin{pmatrix} g_0 & 0 & \dots & 0 \\ 0 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{N_p-1} \end{pmatrix}. \tag{22}$$

After equalization and dispreading in the sequence of the user C_J considered, the estimated transmitted symbol $\hat{a}_i$ can be expressed as:

$$\hat{a}_i = c_i{}^{-T} G r = \\ = c_j{}^{-T} GHCa + c_j{}^{-T} n = \\ = \sum_{k=0}^{N_p-1} c_{i,k}{}^2 g_k h_k a_i + \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k}\, c_{q,k} g_k a_q + \\ + \sum_{k=0}^{N_p-1} c_{i,k}{}^2 g_k n_k. \tag{23}$$

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6. Equalization

6.1. The detector combination and orthogonally restoration COR (Zero Forcing (ZF))

This technique Orthogonally Restoring Combining (COR) cancels completely the dispersion introduced by the channel. The use factor applied to each subcarrier is given by:

$$g_k = \frac{1}{h_k} \text{avech}_k \neq 0. \tag{24}$$

Here, the expression of the estimate $\hat{a}_i$ becomes:

$$\hat{a}_i = \sum_{k=0}^{N_p-1} {c_{i,k}}^2 a_i + \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} a_q + \\ + \sum_{k=0}^{N_p-1} c_{i,k} \frac{1}{h_k} n_k. \tag{25}$$

The use of orthogonal spreading codes to the levels of the transmitter guaranties:

$$\sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} = 0 \quad \forall i \neq q. \tag{26}$$

This CESMM technical or MMSE [2, 18], for Minimum Mean Square Error, offers a compromise between minimizing the term multipath interference and maximizing the signal-to-noise ratio. It comes from the application of the Wiener filter [23, 24]. The calculation of the equalization coefficients must minimize the mean square error for each subcarrier between the transmitted signal and the equalized signal [7, 11]. This resolution leads to the expression of the coefficients g_k:

$$g_k = \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}}. \tag{27}$$

Where the channel normalization hypothesis in power: $E[|h_k|^2] = 1$. The coefficient γ_k is calculated from the estimated signal to noise ratio per subcarrier, inducing additional complexity. Here, the expression of the estimate $\hat{a}_i$ becomes:

$$\hat{a}_i = \sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{|h|_k^2}{|h|_k^2 + \frac{1}{\gamma_k}} a_i + \\ + \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} \frac{|h|_k^2}{|h|_k^2 + \frac{1}{\gamma_k}} a_q + \\ + \sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{|h|_k^*}{|h|_k^2 + \frac{1}{\gamma_k}} n_k. \tag{28}$$

It is also assumed that the spreading codes are orthogonal, so we can deduce that:

$$\sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} = 0 \; \forall i \neq q. \tag{29}$$

So, equation (28) becomes:

$$\hat{a}_i = \sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i + \\ + \sum_{k=0}^{N_p-1} {c_{i,k}}^2 \frac{|h_k|^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k. \tag{30}$$

7. Results and Discussion

In this section, we present the results of computer simulations for various SNR and assume that the input channel is driven by a non-Gaussian signal $x(k)$. Gaussian noise $N(k)$ corrupts the output channel $y(k)$. The equalization performance of MC-CDMA systems was evaluated using an algorithm previously studied. This assessment is made by calculating the Binary Error Rate (BER) for both ZF and MMSE equalizers, using the measured and estimated parameters of both channels.

7.1. BRAN A Identification using the Reproducing Kernal Hilbert Space algorithm (RKHS)

Figures 2 and 3 show that the estimated parameters of the channel impulse response approach the actual model parameters. These results were performed for SNR = 32 db and a number of samples N = 4096.

We represent the estimation of the BRAN A parameters using the RKHS algorithm, for an SNR = 16db the data length is 2048 and for 100 iterations, we observe an insignificant variation due to the noise on the estimation of channel parameters.

(28)

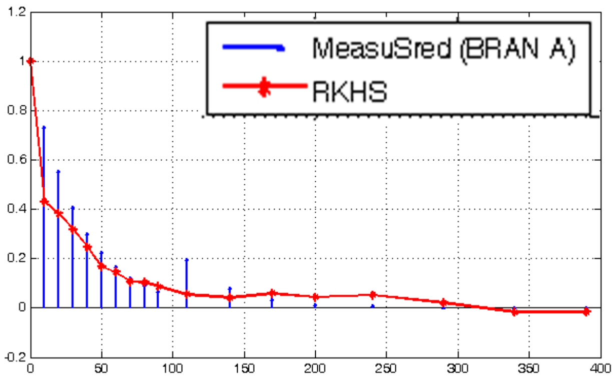

Figure 2. Parameter estimation of the impulse response of BRAN A channel using the RKHS algorithm for SNR=16 db

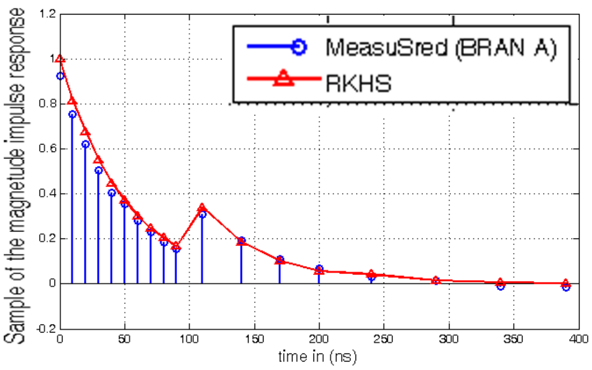

Figure 3. Parameter estimation of the impulse response of BRAN A channel using the RKHS algorithm for SNR=32 db

In our investigation of the BRAN E channel model, we used the reproducing kernel Hilbert space (RKHS) algorithm to estimate the impulse response. Figure 4 illustrates the results of this estimation at a signal-to-noise ratio (SNR) of 16 dB. It can be observed that the presence of noise has a slight impact on the accuracy of the estimated impulse response. However, when the SNR is increased to 32 dB, as shown in Figure 5, the effect of noise is significantly reduced, resulting in a more precise impulse response estimate. Overall, these results demonstrate the robustness of the RKHS algorithm in the presence of noise.

7.2. MC-CDMA system performance

To evaluate the performance (or performances?) of the MC-CDMA system using the Reproducing Kernel Hilbert Space methods, several channels are used. These performances are evaluated by the calculation of the Bit Error Rate (BER) for ZF and MMSE equalizers, using the measured and estimated BRAN A and BRAN E channel impulse responses. These studies were evaluated for different SNR values.

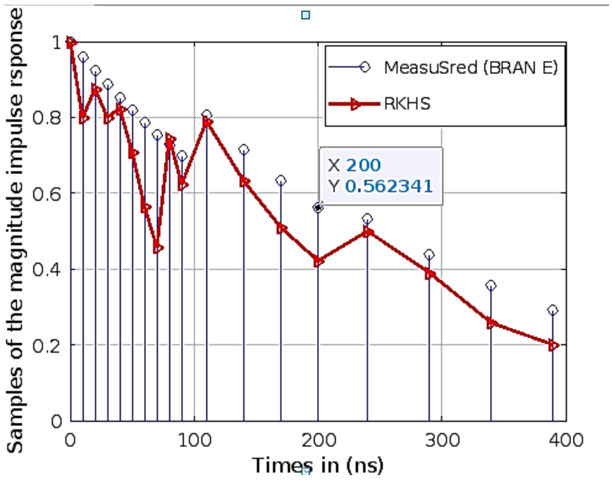

Figure 4. Parameter estimation of the impulse response of BRAN E channel using the RKHS algorithm for SNR=16 db

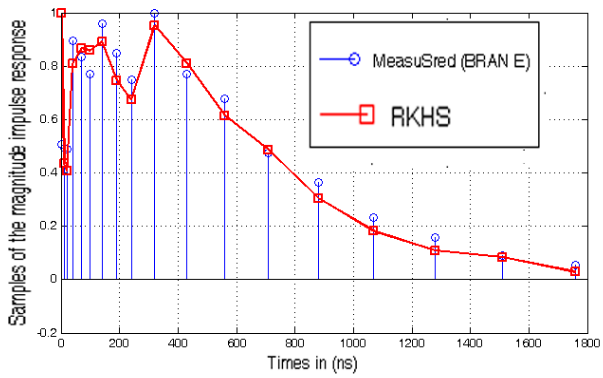

Figure 5. Parameter estimation of the impulse response of BRAN E channel using the RKHS algorithm for SNR=32 db

7.3. ZF and MMSE equalizers: the case of BRAN A channel

The results of our simulation, which can be seen in Figure 6, demonstrate the effectiveness of using the reproducing kernel Hilbert space method to estimate parameters for the BRANA channel. These estimated parameters were then used in conjunction with the measured parameters to implement an equalization process using the Zero-Forcing equalizer. The resulting equalization yielded satisfactory results.

Figures 6 and 7 show that for different SNR, the results obtained by RKHS follow the same form compared with those obtained using measured data. From the same figures, we conclude that the equalization results obtained using the MMSE equalizer are better than those obtained by the ZF equalizer, especially where the SNR > 20db, we have only a BER of 10^{-4}.

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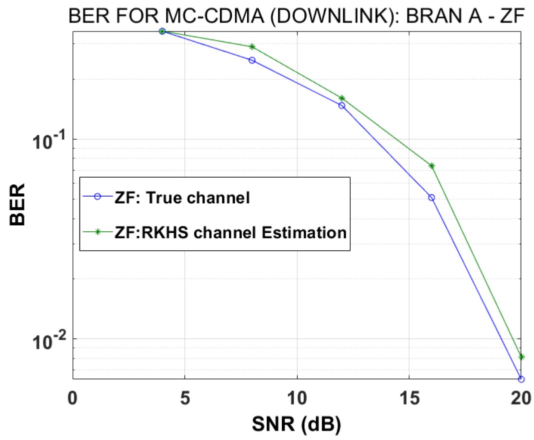

Figure 6. BER of the estimated and measured BRAN A channel, using the ZF equalizer

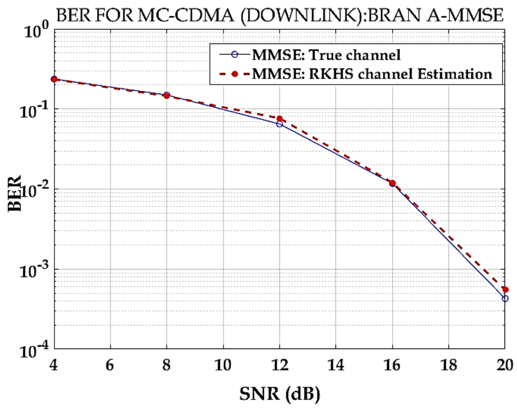

Figure 7 BER of the estimated and measured BRAN A channel, using the MMSE equalizer

7.4. ZE and MMSE equalizer: the case of BRAN E channel

In Figures 8 and 9, we present the results of our simulation of the bit error rate (BER) estimation using the measured and estimated impulse response of the BRAN E channel. The equalization process was carried out using both Zero-Forcing (ZF) and Minimum Mean Squared Error (MMSE) equalizers. The results demonstrate the effectiveness of these equalizers in improving the BER performance of the communication system. Overall, these results provide valuable insights into the behavior of the BRAN E channel and the performance of different equalization techniques.

7.5. Discussion

The Figures 6, 7, 8, and 9 present the bit error rate (BER) as a function of the number of carriers and the number of bits/symbol of each carrier (SNR). From the results obtained by the equalizers (ZF and MMSE), we can see that the bit error rate is low even if a large number of carriers are used, and if the number of bit/symbol (SNR) is increased, the performance will be weak.

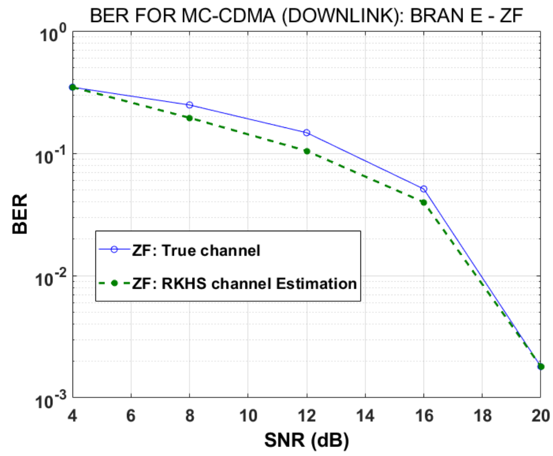

Figure 8. BER of the estimated and measured BRAN E channel, using the ZF equalizer

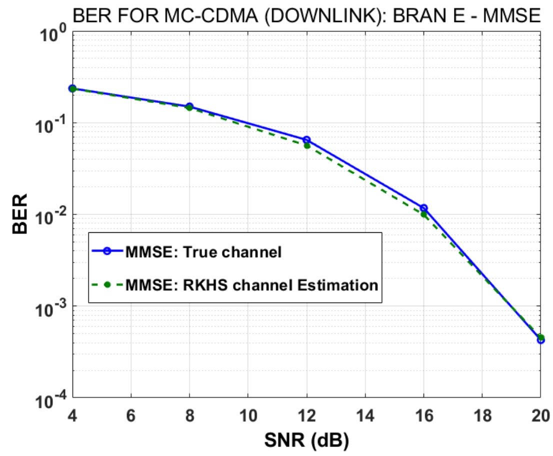

Figure 9. BER of the estimated and measured BRAN E channel, using the MMSE equalizer

We can conclude from the simulation results, that our method gives a good estimation of the channel compared to those obtained using the measured values of BRAN E and BRAN A channels, by both ZF and MMSE equalizers, except that the MMSE equalizer shows more precision compared to the ZF. Based on our analysis, it is clear that the signal-to-noise ratio (SNR) has a significant impact on the bit error rate (BER) of the communication system. When the SNR was above 20 dB, we observed that the BER was significantly reduced, with only one error occurring for every 10^4 bits received. This is in contrast to when the SNR was below 20 dB, where we encountered one error for every 10^3 bits received. These results underscore the importance of maintaining a high SNR to achieve reliable communication. Overall, our observations demonstrate a strong relationship between

SNR and BER, and highlight the importance of optimizing SNR values to improve the performance of the communication system.

8. Conclusions and perspectives

The development of kernel methods, especially for the identification and equalization of the channel parameters, marks the point of convergence of several essential concepts: passing the nonlinear, thanks to the astuteness kernels, a large family of linear algorithms relying only on scalar products in the space of inputs χ.

In this paper, the method of positive definite kernels has been applied for the parameter identification of a wireless transmission channel, for a single user of a finite impulse response signal, and for a stationary system and time-invariant. We applied this approach to different channels and it was observed, from the results of the simulation parameters in phase and amplitude, that they follow the shape of the measured response. The RKHS algorithm shows its efficiency in the impulse response channel (BRAN (A and E) normalized for the MC-CDMA system) identification with high precision. MMSE has demonstrated its effectiveness compared to ZF, knowing that both of them give very satisfactory results.

As a part of future research, a model that implements the blind identification and equalization of the channel based on RKHS will be very interesting and practical because we should reconstruct the signal without any information about the emitted signal.

Authors' contributions: Development of the mathematical tools, definition of the problematic – **Imad BADI**; Development of the algorithm and implementation of the method, revision of the document – **Hassan BADI**; analysis of the literature, setting and substantiation of the purpose and objectives of the study – **Aziz Khamjane**; Development of the algorithm and implementation of the method, research methodology and presentation of results, revision of the document – **Karim EL MOUTAOUAKIL** and **Abdelkhalek BAHRI**.

All the authors have read and agreed to the published version of the manuscript

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КОНТРОЛЬОВАНА ІДЕНТИФІКАЦІЯ ТА ЕКВАЛІЗАЦІЯ КАНАЛУ ПЕРЕДАЧІ З ВИКОРИСТАННЯМ ВІДТВОРЮЮЧОГО ЯДРА ГІЛЬБЕРТОВОГО ПРОСТОРУ

Імад Баді, Хасан Баді, Азіз Хамджане,
Карім Ель Мутауакіл, Абдельхалек Бахрі

Предметом статті є визначення та вирівнювання параметрів телекомунікаційних каналів. Метою є розроблення нового математичного підходу, заснованого на позитивно визначених ядрах у Гільбертовому просторі. Завдання, які вирішуються: по-перше, необхідно сформулювати математичну процедуру на основі ядер. Ядро – це функція, яка відображає пари точок даних у скалярне значення, а позитивно визначені ядра широко використовуються в програмах машинного навчання та обробки сигналів. Наступним завданням є ідентифікація параметрів каналу за допомогою запропонованого методу. Останнім завданням є застосування Zero Forcing і еквалайзера MMSE для вимірювання продуктивності запропонованої системи. У цій статті представлено новий метод для вирішення проблеми контрольованої ідентифікації параметрів каналу передачі на основі позитивно визначеного ядра в Гільбертовому просторі, який реалізує ядра Гауса. Припускається, що вхідна послідовність, яка використовується як вхідна інформація для системи або процесу, є незалежною, має нульове середнє, негаусів розподіл і однаково розподілена. Ці припущення зроблені для спрощення аналізу та моделювання. Запропонований метод оцінки параметрів імпульсної характеристики каналу дає багатообіцяючі результати, що свідчить про те, що оцінені параметри близькі до виміряних параметрів моделі для різних каналів. Збіжність оцінених параметрів до виміряних параметрів моделі особливо помітна для каналів BRAN A (внутрішній) і BRAN E (зовнішній). Метод перевірено на різних моделях каналів, і результати залишаються незмінними. Загалом, запропонований метод є надійним та ефективним підходом для оцінки параметрів імпульсної характеристики каналу. Точність оцінених параметрів заслуговує особливої уваги в світлі

проблем, пов’язаних із моделюванням бездротових каналів, на які можуть впливати різноманітні фактори, такі як перешкоди та втручання. Ці висновки мають важливе значення для проектування та оптимізації систем бездротового зв’язку. Точні оцінки параметрів імпульсної характеристики каналу є важливими для прогнозування та пом’якшення впливу спотворень каналу та перешкод, і запропонований метод є перспективним інструментом для досягнення цієї мети. Потрібні подальші дослідження та тестування щоб перевірити та вдосконалити метод, а також вивчити його потенційні застосування в різних умовах і сценаріях. Ми оцінили продуктивність системи за допомогою визначених параметрів, отриманих на підставі запропонованого методу. Було використано два еквалайзери, MMSE та ZF, і результати показують, що MMSE перевершує ZF. Обидва еквалайзери дали доволі задовільні результати.

Ключові слова: FIR канал; MC-CDMA; еквалізація; ідентифікація; канал передачі; Відтворення ядра Гільбертового простору; BRAN; ZE еквалайзер; MMSE еквалайзер.

Імад Баді – проф., Департамент математики та інформатики Університету Абдельмалека Есааді, Лабораторія прикладних наук Марокко, Ель-Хосейма.

Хасан Баді – PhD, Лабораторія інженерних наук, Мультидисциплінарний факультет Таза, Університет Сіді Мохамеда Бен Абделла, Фес, Марокко.

Азіз Хамджане – проф., Департамент математики та інформатики Університету Абдельмалека Есааді, Лабораторія прикладних наук Марокко, Ель-Хосейма.

Карім Ель Мутауакіл – проф., Лабораторія інженерних наук, Мультидисциплінарний факультет Тази, Університет Сіді Мохамеда Бен Абделлаха, Фес, Марокко.

Абдельхалек Бахрі – проф., Департамент математики та комп’ютерних наук Університету Абдельмалека Есааді, Лабораторія прикладних наук Марокко, Ель-Хосейма.

Imad Badi – Prof., National School of Applied Sciences Al-Hoceima, Laboratory of Applied Sciences Al-Hoceima, Abdelmalek Esaadi University, Tétouan, Morocco,
e-mail: ibadi@uae.ac.ma, ORCID: 0000-0002-2844-4629.

Hassan Badi – PhD, Laboratory of Engineering Sciences, Multidisciplinary faculty of Taza, Sidi Mohamed Ben Abdellah University, Fes, Morocco,
e-mail: hassan.badi@usmba.ac.ma, ORCID: 0000-0002-1568-9790.

Aziz Khamjane – Prof., National School of Applied Sciences Al-Hoceima, Laboratory of Applied Sciences Al-Hoceima, Abdelmalek Esaadi University, Tétouan, Morocco,
e-mail: akhamjane@uae.ac.ma, ORCID: 0000-0002-3508-8968.

Karim El Moutaouakil – Prof., Laboratory of Engineering Sciences, Multidisciplinary faculty of Taza, Sidi Mohamed Ben Abdellah University, Fes, Morocco,
e-mail: karim.elmoutaouakil@usmba.ac.ma, ORCID: 0000-0003-3922-5592.

Abdelkhalek Bahri – Prof., National School of Applied Sciences Al-Hoceima, Laboratory of Applied Sciences Al-Hoceima, Abdelmalek Esaadi University, Tétouan, Morocco,
e-mail: abahri@uae.ac.ma, ORCID: 0000-0002-8527-7281.