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TOPOLOGICAL OPTIMIZATION OF A SYMMETRICAL ADHESIVE JOINT. ISLAND MODEL OF GENETIC ALGORITHM

Modern additive technologies make it possible to create structures of variable thickness and of any shape. Thus, designers face problems of optimal design of a new type, and these are problems of topological optimization. Such problems are to determine the optimal form of the structure or the optimal distribution of material over the structure. As a rule, the criterion of optimality is the mass of the structure. However, the structure must retain its bearing capacity under a certain load. The symmetric two-shear adhesive joint of the main plate with two overlays of the same shape on both sides is the **object of study** in this article. The main **goal** of this study was to determine the optimal form of overlays with variable thicknesses under certain restrictions. The main restriction is the strength of the structure. Furthermore, additional restrictions are imposed on the minimum and maximum thickness of the overlay. Therefore, the solution to the problem is presented in the form of a set of the following tasks: building a mathematical model of the adhesive joint, building a numerical solution to the primal problem using the finite difference method, and building a genetic optimization algorithm. In the presented article, to improve the convergence of the genetic algorithm is proposed to use an island model that consists of several populations. The main feature of the proposed model of the genetic algorithm lies in the fact that on one of the “islands” mutations occur more frequently and with higher dispersion than on the other two “islands”. On the one hand, this decision ensures a high rate of evolutionary selection, and on the other hand, the stability of the results is achieved. Several modeling problems are solved in this article. The **main results** of this research include the following: nonlinear dependence of the overlay length on the applied load was determined; restrictions on the minimum thickness of the overlay, which cause the appearance of a certain “plateau” at the edge of the overlay, the thickness of which is equal to the minimum allowable were defined.

Keywords: constrained optimization; finite difference method; genetic algorithm.

1. Introduction

Adhesive lap joints are an integral part of modern composite structures. The widespread application of adhesive joints in composite structures is due to high manufacturability, tightness, lightweight and high aerodynamic efficiency. In addition, adhesive joints do not violate the composite structure in consequence adhesive joints allow to realize their high strength and mechanical properties in the structures. However, a well-known disadvantage of lap joints is the concentration of stresses in the adhesive layer at the edges of the gluing zone [Ошибка! Залка не определена., 78]. To reduce stress concentration and increase the strength of adhesive joints, the following design solutions are used: increasing the thickness of the adhesive layer at the edges of the joint [3], decreasing the thickness of the plates at the edge of the joint [4] (the sequence of the placement of the layers in the composite package is very important [5]), using of two or more types of adhesives [6], using of the transversal bonds in the adhesive joint [7], etc [9, 10]. Moreover, using the symmetrical double-lap adhesive joints makes it possible to eliminate the bending of the structure and therefore reduce the tear stresses in the adhesive layer [11, 12].

As a rule, mathematical models of three-layer rods or beams with thin pliable filler are used to describe the stress state of adhesive lap joints [1, 2]. The stress-strain state of the joint can be described in an analytical form in the case, if the elastic and geometric parameters of the layers are constant along the length of the joint. However, even in the trivial case, when the layer thickness varies linearly, such a problem doesn't have an analytical solution. Therefore, to find the stress state of adhesive joints with thicknesses variable along the length, such numerical methods as the finite difference method [7], the Ritz method, and the finite element method are used.

The problem of topological optimization of a structure is to find the optimal form of the structure [8]. It's a qualitatively more difficult problem compared to the problem of classical parametric optimization. The main reason for the problem is that the desired value will be the function of the distribution material into the structure, but is not a set of a small number of unknown parameters. As a rule, the topological optimization of the adhesive lap joint is to find the optimal length of the adhesive joint, as well as to determine the dependence of the change in the thickness of the joined plates on the length of the adhesive zone. One of the possible way to

solve this problem is the discretization of the desired function. In this case, the problem is to find the thickness of structural elements in a system of points [13]. The transition from continuous to discrete functions makes it possible to optimize the joint with a stepped thickness change [14, 15] (this solution is often used in joints of layers composites). If the desired function is implied to be continuous, in this case, it can be described by known values in the system of points using splines [16], of the Bezier functions [17, 18]. The search for a sufficiently large number of unknowns by gradient methods is rather difficult, that's why other methods, for example, genetic algorithms are used to solve the optimization problem. This approach implies finding the optimal parameters of the problem by solving a sequence of primal problems. The primal problem of finding the stress state of a structure at the defined parameters of the problem is usually solved in a two-dimensional problem setting by the finite element method [16 - 20]. Optimization can be also carried out for the thickness of the adhesive layer at the edge of the adhesive area [21], the size and shape of the squeezed-out excess glue at the edge of the joint [22], and the structure of the composite [23, 24].

A common disadvantage of using the finite element method for solving topological optimization problems is the relatively slow speed of the algorithm. This is due to the fact that optimization problems are formulated in a two-dimensional setting. In this case, the elements of the adhesive joint are considered as a continuum. As a result, at each iteration, it is necessary to build a new finite element mesh, while the adhesive layer is divided into sufficiently small elements.

The main goal of the research is to solve the problem of topological optimization of adhesive joints in a one-dimensional setting. The use of well-established mathematical models of joints [1], [25], which are used to describe the stress state of adhesive joints in an analytical form, makes it possible to reduce the dimension of the problem without a significant loss of accuracy and thereby increase the speed of calculations. In addition, the one-dimensional setting of the problem makes it possible to use a fairly simple and efficient finite difference method to calculate the stress state of the adhesive joint. This method is successfully used both for solving one-dimensional problems [7, 13] and for solving two-dimensional problems of joint mechanics [28].

In order to enhance the rate of convergence of the genetic algorithm, the improved island model of the genetic algorithm (Island Model GA) [26, 27] is used in this article. Island models can be classified according to several features, such as possible migration directions [26] and evolutionary selection conditions on islands. The model is called homogeneous [27, 28] if conditions are the same on all islands. The model is hetero-

geneous [29] if the conditions are different. In the version of the evolutionary algorithm proposed in this article, mutagenesis occurs more frequently and with higher dispersion on one of the three islands than on the other two islands. This combination has high variability on one "island" and stability on the other two. In addition, in combination with the regular migration of the best individuals between the "islands", it ensures a good rate of the evolutionary algorithm and the stability of the results achieved.

2. Problem Statement

Consider a structure which consists of two plates, moreover, these plates are connected to each other by symmetrical overlays, Fig. 1, a. This structure is symmetrical and does not experience bending under the tensile-compressive loading therefore it is often used in mechanical engineering. Since the structure is symmetrical, we can consider only its fourth part, Fig. 1, b. The transverse displacements of the middle layer of the main plate are equal to zero due to the symmetry of the structure. If we consider the deformation of this structure within the framework of the theory of rods, then, due to the symmetry of the structure, we can consider only the adhesive area and leave aside the deformation of the entire structure.

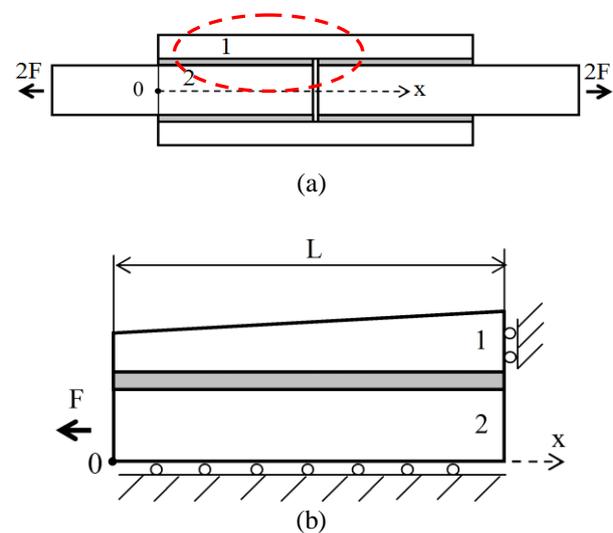


Fig. 1. Scheme of a structure

The structure is loaded with longitudinal forces. $2F$. The thickness of the overlay is variable along the length, as shown in Fig. 1, b. We have considered, that the thickness of the adhesive layer is constant along the length of the adhesive joint and uniform in all sections. The length of the adhesive area is L .

The differential element of the adhesive area and the acting force factors are shown in Fig. 2.

The equilibrium equations of the external (bearing) layers have the following form

$$\begin{aligned} \frac{dN_1}{dx} &= -\tau; & \frac{dN_2}{dx} &= \tau; & \frac{dQ_1}{dx} &= \sigma, \\ \frac{dM_1}{dx} - s_1(x)\tau - N_1 \frac{ds_1}{dx} + Q_1 &= 0, \end{aligned} \quad (1)$$

where N_1 , N_2 – longitudinal forces in the bearing layers; Q_1 – shear force in the overlay; M_1 – bending moment in overlay; τ and σ – shearing and normal stresses in the adhesive layer; s_1 – distance from the neutral axis of overlay to adhesive layer, in the case of a symmetrical overlay structure $s_1(x) = 0.5 \delta_1(x)$, where $\delta_1(x)$ – thickness of the overlay.

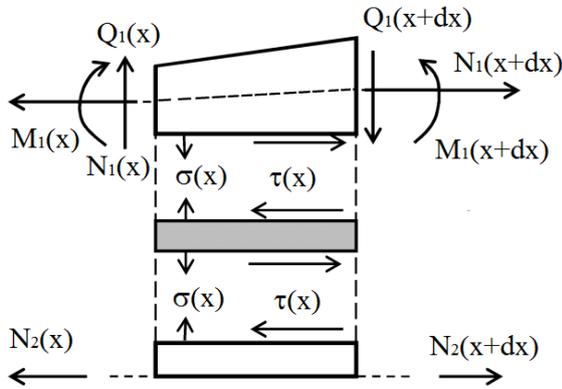


Fig. 2. Differential element of joining

The displacements of the bearing layers are described by the following equations

$$N_1 = B_1 \frac{dU_1}{dx}, \quad N_2 = B_2 \frac{dU_2}{dx}, \quad D_1 \frac{d^2 W_1}{dx^2} = M_1, \quad (2)$$

where U_1 and U_2 – longitudinal displacements of bearing layers; W_1 – transversal displacements of overlay; $B_1(x)$ and B_2 – tensile-compressive stiffness of the layers, in a case, if the layers are uniform in thickness, then $B_1(x) = \delta_1(x)E_1$, $B_2 = \delta_2 E_2$, where E_1 and E_2 – the modulus of elasticity of the corresponding layer; $D_1(x)$ – the bending stiffness of overlay, $D_1(x) = \delta_1^3(x)E_1/12$.

We have considered, that the stresses in the adhesive layer are uniformly distributed over the thickness and proportional to the difference in displacements of the inner sides of the bearing layers

$$\sigma = K \cdot w_1, \quad \tau = P \left(U_1 - U_2 + s_1(x) \frac{dW_1}{dx} \right), \quad (3)$$

where K , P – tensile-compressive and shear stiffness of the adhesive layer, which can be calculated, for example, as $K = E_0 \delta_0^{-1}$, $P = G_0 \delta_0^{-1}$, where, in turn δ_0 – thickness of the adhesive layer, E_0 and G_0 – modulus of elasticity and shear modulus of the adhesive, respectively.

The system of equations Eq. (1) - (3) is reduced to a system of three differential equations relative to the longitudinal displacements of both layers U_1 , U_2 and the transversal displacements for the overlay W_1

$$\frac{B_1}{P} \frac{d^2 U_1}{dx^2} + \frac{1}{P} \frac{dB_1}{dx} \frac{dU_1}{dx} - U_1 + U_2 - s_1 \frac{dW_1}{dx} = 0, \quad (4)$$

$$U_1 + \frac{B_2}{P} \frac{d^2 U_2}{dx^2} - U_2 + s_1 \frac{dW_1}{dx} = 0, \quad (5)$$

$$\begin{aligned} & \frac{D_1}{P} \frac{d^4 w_1}{dx^4} + \frac{2}{P} \frac{dD_1}{dx} \frac{d^3 w_1}{dx^3} + \\ & + \left(\frac{1}{P} \frac{d^2 D_1}{dx^2} - s_1^2 \right) \frac{d^2 w_1}{dx^2} - 2s_1 \frac{ds_1}{dx} \frac{dw_1}{dx} + \\ & + \frac{K}{P} w_1 - \frac{B_1}{P} \frac{ds_1}{dx} \frac{d^2 U_1}{dx^2} + \frac{ds_1}{dx} U_2 + \\ & + \left(-\frac{1}{P} \frac{ds_1}{dx} \frac{dB_1}{dx} - s_1 - \frac{B_1}{P} \frac{d^2 s_1}{dx^2} \right) \frac{dU_1}{dx} - \\ & - \frac{ds_1}{dx} U_1 + s_1 \frac{dU_2}{dx} = 0, \end{aligned} \quad (6)$$

The boundary conditions have the following form

$$\begin{aligned} N_2(0) &= F, \quad N_2(L) = 0, \quad N_1(0) = 0, \\ U_1(L) &= 0, \quad Q_1(0) = 0, \quad M_1(0) = 0, \\ Q_1(L) &= 0, \quad \left. \frac{dw_1}{dx} \right|_{x=L} = 0. \end{aligned} \quad (7)$$

The boundary conditions (7) can be written in displacements as well as system (4) - (6),

$$\begin{aligned} \left. \frac{dU_2}{dx} \right|_{x=0} &= \frac{F}{B_2}, \quad \left. \frac{dU_2}{dx} \right|_{x=L} = 0, \quad \left. \frac{dU_1}{dx} \right|_{x=0} = 0, \\ & s_1(0)P \left(u_1 - u_2 + s_1(0) \frac{dW_1}{dx} \Big|_{x=0} \right) - \\ & - \frac{d}{dx} \left(D_1(x) \frac{d^2 W_1}{dx^2} \right) \Big|_{x=0} + \frac{ds_1}{dx} B_1(0) \frac{dU_1}{dx} \Big|_{x=0} = 0; \end{aligned}$$

$$s_1(L)P \left(u_1 - u_2 + s_1(L) \frac{dW_1}{dx} \Big|_{x=L} \right) - \frac{d}{dx} \left(D_1(x) \frac{d^2 W_1}{dx^2} \right) \Big|_{x=L} + B_1(L) \frac{ds_1}{dx} \frac{dU_1}{dx} \Big|_{x=L} = 0;$$

$$U_1(L) = 0; \quad D_1(0) \frac{d^2 W_1}{dx^2} \Big|_{x=0} = 0, \quad \frac{dW_1}{dx} \Big|_{x=L} = 0.$$

The optimization problem was formulated as follows – it is necessary to find the length of the adhesive joint L and the dependence of the overlay's thickness on the longitudinal coordinate $\delta_1(x)$, which provides the extreme value of a certain optimality criterion. Such a criterion can be, for example, the mass of the overlay or the cost of the adhesive joint. The mass of the overlay, up to an arbitrary multiplier, is proportional to the cross-sectional area of the overlay. Thus, the optimality criterion can have the form

$$M = \int_0^L \delta_1(x) dx \rightarrow \min. \quad (8)$$

On the other hand, the cost of the structure can be an optimality criterion. The length of the adhesive joint can also be used as an optimality criterion. Since a significant contribution to the cost of the adhesive joint is made by the cost of the adhesive and the technological processes

$$L \rightarrow \min. \quad (9)$$

The desired values L and the function $\delta_1(x)$ must be such that the structural strength restrictions are fulfilled, as well as the geometric and technological restrictions are fulfilled.

There are various ways of the adhesive joint failure, such as rupture of the overlay, destruction of the adhesive layer, delamination and pulling away from the laminated composite, delamination of the adhesive layer from the bonded plate, etc. In this article, only one restriction is used. It's the condition of the strength of the adhesive layer. However, this is not principal for the running of the genetic optimization algorithm which is proposed in the article. The strength of the adhesive criterion and the adhesive adhesion to the joining surfaces depends on a lot of factors, such as the composition of the adhesive, the chemical composition and the quality of the joining surfaces processing, the temperature and curing time of the adhesive, etc. The strength criterion of the formed adhesive layer is determined as a result of physical experiments. Obviously, in this case,

there should be high adhesion of the adhesive to the joined surfaces, so that the destruction of the joint occurs along the adhesive, and not along the interface between the materials. It was shown in [25] that such a criterion is the criterion of maximum normal stresses.

$$\sigma_1(x) = \frac{|\sigma(x)|}{2} + \frac{\sqrt{\sigma^2(x) + 4\tau^2(x)}}{2} \leq \sigma_{\max}. \quad (10)$$

where $x \in [0; L]$; $\sigma_1(x)$ – the first principal stress; σ_{\max} – the ultimate strength of an adhesive.

It should also be noted, in this case, due to the symmetry of the structure, the normal stresses in the adhesive layer are assumed to be several times less than the shear stresses. Therefore, the influence of normal stresses was ignored in the adhesive joint strength evaluation. The criterion of strength was taken as the criterion of maximum shear stresses:

$$\tau(x) \leq \tau_{\max}. \quad (11)$$

where $x \in [0; L]$; τ_{\max} – the shear ultimate strength of an adhesive.

Criteria (10) and (11) assume that the adhesive has high adhesion to the joined surfaces. In addition, there isn't occur the tear-off failure of the adhesive from the surface along the interface between materials. Otherwise, it is also necessary to impose restrictions on the maximum normal stresses in the adhesive $\sigma(x)$.

Moreover, restrictions on the function from below and from above are imposed. In the first case, the thickness of overlay should not be less than some certain value

$$\delta_1(x) \geq \delta_{\min}. \quad (12)$$

where δ_{\min} – some technologically minimum possible overlay thickness, which can be equal, for example, to the thickness of one monolayer of the laminated composite.

On the other hand, in some cases, a restriction is imposed on the maximum thickness of the overlay

$$\delta_1(x) \leq \delta_{\max}. \quad (13)$$

Such a restriction is dictated, for example, by some considerations such as aerodynamic efficiency, quality control capabilities, manufacturing technique etc.

In addition to the above restrictions (10) - (13), restrictions on the maximum stresses in the overlay itself can also be introduced. This is due to the fact that the

destruction of the connection can occur in the form of a rupture of the lining. Using such additional restrictions does not introduce principal changes in the algorithm for solving the problem and, therefore, is not covered in this article.

3. Numerical solution of the primal problem

The primal problem is to determine the stress state of an adhesive joint at a defined length L and known function $\delta_1(x)$. For the numerical solution of a system of ordinary differential equations with variable coefficients Eq. (4)-(6) different methods are applied. The direct finite difference method is the simplest and fastest in the case of software implementation. This method has shown high efficiency in calculating the stress state of adhesive joints this method [7, 30].

If there is a function $\delta_1(x)$, therefore, the following functions $s_1(x)$, $B_1(x)$ and $D_1(x)$ are also known. The adhesive zone $x \in [0; L]$ is divided into a system of nodal points with numbers from zero to N_{fem} . The partition interval is $h = L / N_{\text{fem}}$. Points with numbers 0 and N_{fem} are boundary points ($x = 0$ and $x = L$ and respectively). The coordinates of a point with a number i is calculated using the formula $x_i = h \cdot i$. Let us also introduce points to the left and right of the region, which lie outside the region $x \in [0; L]$ and have numbers -1 and $N_{\text{fem}} + 1$, respectively. The displacements of the bearing layers at the points $U_1(x_i) = u_i^{(1)}$, $U_2(x_i) = u_i^{(2)}$ and $W_1(x_i) = w_i^{(1)}$ are denoted.

Thus, the solution of the system of differential equations Eq. (4)-(6) is reduced to finding displacements $u_i^{(1)}$, $u_i^{(2)}$ and $w_i^{(1)}$. The boundary conditions (7) allow us to introduce displacements at external nodes outside the adhesive zone as unknowns. However, the function $\delta_1(x)$ and related functions $s_1(x)$, $B_1(x)$ and $D_1(x)$ are defined only in the adhesive zone $x \in [0; L]$. That is only the values $\delta_0^{(1)}, \dots, \delta_{N_{\text{fem}}}^{(1)}$, $B_0^{(1)}, \dots, B_{N_{\text{fem}}}^{(1)}$, $D_0^{(1)}, \dots, D_{N_{\text{fem}}}^{(1)}$ are defined. Therefore, the extreme points x_0 and $x_{N_{\text{fem}}}$ derivatives of displacements in Eq. (4)-(6) and in the boundary conditions (7) are written in the difference form according to the symmetric finite-difference pattern.

However, to write the derivatives of the $s_1(x)$, $B_1(x)$ and $D_1(x)$ in the difference form at the boundary points, one-sided templates should be used. We write in difference form the differential equations Eq. (4)-(6) for the points $0, 1, \dots, N_{\text{fem}}$, as well as the boundary conditions (7), we obtain a system of linear equations for the unknowns $u_{-1}^{(1)}, u_0^{(1)}, u_1^{(1)}, \dots, u_{N_{\text{fem}}+1}^{(1)}$, $u_{-1}^{(2)}, \dots, u_{N_{\text{fem}}+1}^{(2)}$, and $w_{-2}^{(1)}, w_{-1}^{(1)}, w_0^{(1)}, \dots, w_{N_{\text{fem}}+2}^{(1)}$, which consist of $3N_{\text{fem}} + 11$ equations. Having solved the system of linear equations, we find the displacements of the bearing layers at the nodal points. This makes it possible to determine stresses in the adhesive layer (3) (i.e. a set of stress values and at the nodal points), the longitudinal forces in the bearing layers, as well as all other force factors in the joining elements.

4. Solution of the optimization problem. Island genetic algorithm

As mentioned above, the solution of the problem of topological optimization of an adhesive joint in an analytical form is associated with significant difficulties. Therefore, to solve this problem, a genetic optimization algorithm is proposed. The length of the adhesive joint L and the thickness of the overlay at the nodal points $\delta_i^{(1)}$ are accepted as the desired variables. It means to determine the optimal values, which provide, for example, a minimum mass of the overlay (8) when the strength restrictions (10) are fulfilled. However, in contrast to the problem of finding the optimal distribution of the material along the beam [13], if the thickness values $\delta_i^{(1)}$ at neighboring points differ significantly (this may occur due to crossing or mutations during the execution of the genetic algorithm), then the stresses in the adhesive layer (3), calculated using the finite difference method will have implausible peaks. This suggests that the mathematical model is losing its adequacy. Therefore, it is proposed to determine the optimal dependence $\delta_i^{(1)}$ among functions that a priori have smoothness. This should be from the intuitive considerations that, most likely, the desired function $\delta_1(x)$ is smooth, and hasn't discontinuities, corner points, and peaks. In articles, devoted to topological optimization, as a rule, Bezier functions or splines are used [16]. In this article, a function $\delta_1(x)$ is proposed in the form of a Fourier series expansion in cosines on the interval $\xi \in [0; 1]$ and further scaling along the horizontal axis to

the segment $x \in [0; L]$.

$$\delta_i^{(1)}(\xi) = \frac{a_0}{2} + \sum_{n=1}^M a_n \cos \pi n \xi, \quad (14)$$

If we divide the interval $\xi \in [0; 1]$, as well as the interval $x \in [0; L]$, into $N+1$ points ξ_i numbered from 0 to N , then the thickness of the overlay at the nodal points can be calculated as follows:

$$\delta_i^{(1)} = \frac{a_0}{2} + \sum_{n=1}^M a_n \cos \pi n \xi_n, \quad (15)$$

where M - the number of terms of the Fourier series used.

The description of the geometric shape of the overlay in the form of a Fourier series (15) makes it quite easy to calculate the mass of the overlay (8)

$$M = \int_0^L \delta_1(x) dx = \frac{a_0 L}{2}.$$

To implement a genetic algorithm, it is necessary to create a fitness-function that would make it possible to rank various sets of desired parameters L and a_0, a_1, \dots, a_M by quality (each set in the terminology of genetic algorithms is called an individual).

If the mass of the structure (8) is used as an optimality criterion, and restrictions on the maximum first principal stresses in the adhesive layer (10) and the minimum overlay thickness (12) are added, then the fitness-function will have the following form:

$$\Phi = 0,5 a_0 L + \left\{ \begin{array}{l} Z_1 \left(\frac{\max_i \left(\left| \sigma_i^{(1)} \right| \right)}{\sigma_{\max}} - 1 \right)^2, \max_i \left(\sigma_i^{(1)} \right) \geq \sigma_{\max} \\ 0, \max_i \left(\sigma_i^{(1)} \right) \leq \sigma_{\max} \end{array} \right. + \left\{ \begin{array}{l} Z_2 \left(\frac{\delta_{\min}}{\min_i \left(\delta_i^{(1)} \right)} - 1 \right)^2, \min_i \left(\delta_i^{(1)} \right) < \delta_{\min} \\ 0, \min_i \left(\delta_i^{(1)} \right) \geq \delta_{\min} \end{array} \right. \quad (15)$$

where Z_1, Z_2 - some large numbers that determine the size of the penalty for leaving the solution out of the

allowable area; $\sigma_i^{(1)}$ - the first principal stresses in the adhesive layer at nodal points, which are calculated according to (10); $\max_i \left(\left| \sigma_i^{(1)} \right| \right)$ - maximum principal stresses; $\min_i \left(\delta_i^{(1)} \right)$ - respectively, the minimum value of the overlay thickness.

Thus, if the solution (i.e., the set of values L and a_0, a_1, \dots, a_M) is valid, then the value of the fitness-function is equal to the cross-sectional area of the overlay $0.5 a_0 L$. However, if at least in one node the stresses in the adhesive layer exceed the allowable values, or (and) the thickness of the overlay at least in one node is less than the permissible value, then penalty components are added to the specified area, the value of which is the higher, the higher the violation of the corresponding restriction.

Genetic algorithms have some disadvantages, the most significant of which is the complexity of customization. It is necessary to strike a balance between variability and stability. At high variability, convergence is violated and even good values of the desired parameters found are at risk of being lost as a result of mutations. An approximate solution, at low variability is found quickly, after that the convergence slows down and the population degenerates. In the future, the value of the objective function will change little even at the large number of iterations. One of the possible ways out of this contradiction is to use the island model of the evolutionary algorithm. In this case, the total population is divided into several isolated subpopulations (islands). On each of the islands, the evolutionary process occurs independently and in parallel with other islands. At regular intervals, the best individuals migrate randomly from island to island. In this article, a model with three islands is proposed. On the one of them the probability and dispersion of mutations is higher than on the other two. This combination of two relatively stable islands with one island, where the mutation rate is higher, allows you to combine the speed of finding good solutions with the stability and preservation of the best solutions in the general population.

The flowchart of evolutionary selection on the one island is shown in fig. 3.

1) Creation of the initial population of vectors $\vec{h}^{(j)}$, where $j=1, \dots, N_g$, (N_g - number of individuals in a population). Each vector $\vec{h}^{(j)}$ (individual) which contains components $L^{(j)}$ and $a_0^{(j)}, a_1^{(j)}, \dots, a_M^{(j)}$. According to the sets of parameters, we calculate the corresponding values $\Phi_j = \Phi(\vec{h}^{(j)})$ using the formula (15).

To achieve this, it is necessary to find the thickness of the overlay at the nodal points from the values of the coefficients $a_0^{(j)}, a_1^{(j)}, \dots, a_M^{(j)}$ and $L^{(j)}$, then calculate the discretization step $h^{(j)} = L^{(j)}N^{-1}$ and solve the primal problem of finding displacements in the carrier layers. By which to calculate the stress in the adhesive layer.

2) *Selection.* We rank the vectors that are present in the population $\vec{h}^{(j)}$ according to the corresponding values of the fitness-function Φ_j and select from the population $2k$ (where $2k < N_g$) elements $\vec{h}^{(j)}$. In this case, the probability of getting into the sample depends on the number in the ranked list, or on the values of Φ_j . The sample should include mainly the best individuals $\vec{h}^{(j)}$ of the population, which have lower values of the fitness function.

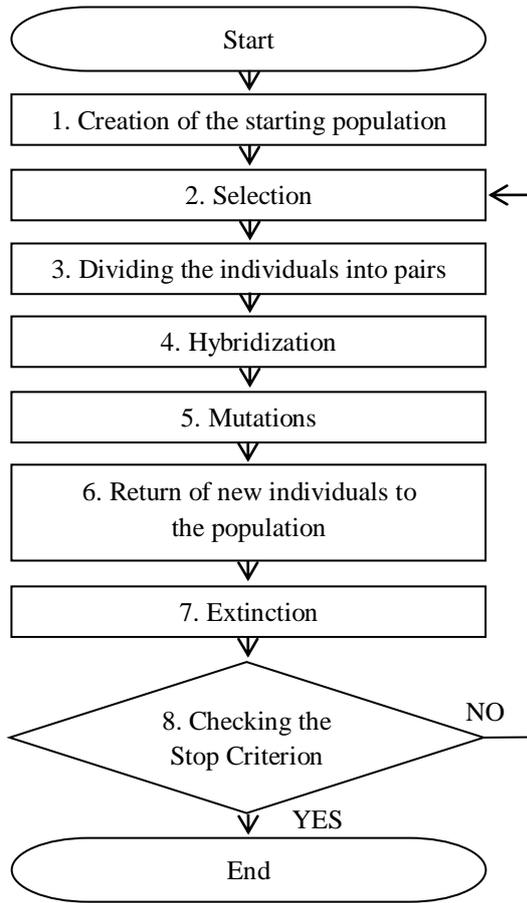


Fig. 3. Flowchart of a simple genetic algorithm (single population)

3) *Division of the parents into pairs.* We divide $2k$ the selected individuals into pairs and get k pairs of “parents”. We can divide individuals into pairs accord-

ing to different strategies, according to similarity, or vice versa according to differences in vectors $\vec{h}^{(j)}$. A measure of the similarity-difference of individuals $\vec{h}^{(j)}$ and $\vec{h}^{(i)}$ is, for example, the scalar product of vectors, which is related to the product of the norms of both vectors

$$\cos \theta_{i,j} = \frac{(\vec{h}^{(j)}, \vec{h}^{(i)})}{|\vec{h}^{(j)}| \cdot |\vec{h}^{(i)}|},$$

Another criterion of similarity-difference of individuals $\vec{h}^{(j)}$ and $\vec{h}^{(i)}$ is the root-mean-square deviation of functions (14) in the range $\xi \in [0;1]$

$$\Delta_{i,j}^2 = \int_0^1 \left(\frac{a_0^{(j)}}{2} + \sum_{n=1}^M a_n^{(j)} \cos \pi n \xi \right) \left(\frac{a_0^{(i)}}{2} + \sum_{n=1}^M a_n^{(i)} \cos \pi n \xi \right) d\xi.$$

However, such a criterion does not take into account the difference in the length of individuals. For each pair (i, j) , the values of the selected criterion are calculated. After that, k pairs are selected from this set. After that, pairs are selected from this set. In this work, the principle of outbreeding is applied, according to which pairs are selected randomly. However, the probability of being selected from a pair is higher, the more individuals differ $\vec{h}^{(j)}$ and $\vec{h}^{(i)}$. This is a certain guarantee against the degeneration of the population.

It should also be noted that the algorithm proposed in the article assumes the occurrence of pairs in which one of the “parents” is common. That is, the best individuals will create descendant of several different partners at the same stage of reproduction.

In the simplest case, individuals are randomly paired.

4) *Hybridization.* For each new individual, parameters $L^{(\cdot)}$ and $a_0^{(\cdot)}, a_1^{(\cdot)}, \dots, a_M^{(\cdot)}$ are randomly selected from both parent individuals. The result of this operation is a population k of new individuals, that is, “descendants”.

5) *Mutations.* In the version of the algorithm presented by the authors, mutations occur only in some of the “descendants” and only with a small fraction of the vector components $\vec{h}^{(j)}$. In this case, a mutation is a change in the values of the vector components by a slight deviation. The magnitude of random deviation is described, for example, by a Gaussian distribution with

zero mathematical expectation. In this case, the distribution dispersion depends on the absolute value of the coefficient a_n . In this way, the Fourier coefficients that are large in absolute value mutate with a larger dispersion, and the smaller ones mutate with a smaller one. If the coefficient a_n is zero, then the standard deviation for mutations has a certain non-zero value σ_0 .

6) *Return of new individuals to the population.* New individuals are returned to the main population, which increases from N_g to $N_g + k$ individuals after changes are made to the gene code.

7) *Extinction.* After the return of new individuals to the population, all individuals are again ranked according to the values of the fitness function Φ_j and k individuals are removed from the population. There are also several ways to remove excess individuals. In the simplest case, individuals k with the worst values of the fitness function are removed. In a more complex case, individuals are selected randomly. Moreover, the worst values of the fitness-function increase the probability of removing an individual from the population. Both approaches are used in the program proposed by the authors. The selection of approach is random for each reproducing-extinction cycle.

8) *Checking the Stop Criterion.* If the Stop Criterion (for example, the number of reproducing cycles is equal K) is not reached, then return to step 2.

But since several populations are considered in the island model of the evolutionary algorithm, the above algorithm is applied to each of them separately. Thus, in each of the subpopulations (islands), evolutionary selection occurs in parallel with the others.

The flowchart of the island model of the evolutionary algorithm is shown below in Fig. 4.

Three islands are considered in the proposed version of the island model of the genetic algorithm. On one of the islands, mutations occur with a higher probability and higher dispersion than on the other two. After K cycles of generation change, two islands are randomly selected and the best individuals (migrants) are exchanged. This provides an influx of new genetic information into the population. The number of migrants must be significantly less than the total number of individuals on a given island. The Stop Criterion can be, for example, the execution of a given number of migrations.

After stopping the algorithm, it is necessary to select the optimal solution from the entire population of individuals. Since the parameters of one, even the best individual, are the result of random mutations and hybridizations, they can differ slightly from each other with different implementations of the algorithm. The average value of the parameters in the population is more resistant to random deviations. Therefore, as a

solution to the optimization problem, it is proposed to take the sample average of the parameters of the best individuals of all (or one of the three) populations. To calculate the sample, mean, you can use, for example, half of the individuals in the population. This approach allows leveling random deviations of parameter values from their optimal values resulting from mutations.

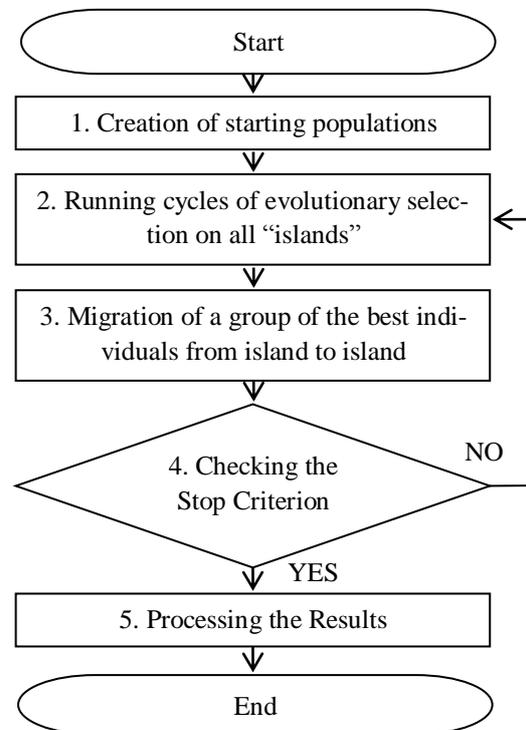


Fig. 4. The flowchart of the island model of the evolutionary algorithm

5. Numerical implementation and results

Consider the results of solving the algorithm for topological optimization of an adhesive joint proposed in this article using a specific example. Consider an adhesive joint, with the following parameters: $E_1 = 100$ GPa, $E_2 = 70$ GPa, $\delta_2 = 3$ mm, $\delta_0 = 0.1$ mm, $E_0 = 2.274$ GPa, $G_0 = 0.54$ GPa, $\sigma_{\max} = 30$ MPa, $\delta_{\min} = 0.1$ mm. The adhesive joint is loaded with longitudinal force: a) $F_1 = 150$ kN/m; b) $F_2 = 300$ kN/m.

The initial population is formed as follows: the length of adhesion joint is randomly assigned by the Gaussian distribution with the mathematical expectation $m_L = 20$ mm and the root-mean-square deviation $\sigma_L = 4$ mm. The Fourier coefficients are calculated based on the assumed linear dependence of the overlay thickness, which starts from some random variable

$\delta_1(0)$ with the mathematical expectation $m_\delta = 1$ mm and dispersion $\sigma_\delta = 0.1$ mm at $x = 0$ and $\delta_1(L) = 3$ mm. The number of terms of the Fourier series is assigned $M = 30$. Calculation of the stress state of the adhesive joint is performed by dividing the area into $N_{fem} = 100$ nodal points. The number of individuals in the population of each island is $N_g = 120$. Of these, $2k = 40$ individuals are selected for hybridization at each iteration.

On two islands, the probability of a length mutation is set equal to 0.2. The length of the adhesion region during mutation changes by a random value, which has a Gaussian distribution with zero mathematical expectation and the root-mean-square deviation of 0.2 mm. Fourier coefficients mutate with a probability also equal to 0.2. During mutations, they change by a random value, this value has a Gaussian distribution with the root-mean-square deviation $\sigma_a = 2 \cdot 10^{-8}$ and if the corresponding Fourier coefficient is equal zero, and the coefficient of variation $c_v = 0,02$. On one of the three islands, these parameters will be doubled. This will ensure greater diversity among the subpopulation of a particular island.

The number of cycles of hybridization and reproduction in the interval between migrations will be set equal to $K = 200$. That is for every $K = 200$ we randomly select two islands, and these islands exchange $m = 10$ the best individuals (migration). There are $N_m = 20$ such cycles of migrations. Consequently, the total number of reproduction cycles is 4000. When processing the results, to smooth out random deviations, an island with the smallest truncated mean value of the objective function was selected. Then, the average values of the parameters of the best half of the individuals of this population were calculated.

5.1. The first calculation case

As a result, in the first calculation case ($F_1 = 150$ kN/m) the optimal value of the joint length $L_1 = 12.09$ mm was obtained. The diagram of the change in the thickness of the overlay along the length of the adhesive joint is shown in Fig. 5. The straight line on the diagram is the thickness of the main plate and this line is shown for scale.

The diagrams of the corresponding stresses in the adhesive layer are shown in Fig. 6.

As we can see, the ratio of the first principal stresses to the maximum allowable normal stresses at the right edge of the joint is equal to one. A curious feature of the obtained optimal shape of the overlay is the

presence of a section of constant thickness on the left edge $\delta_1 = \delta_{min}$.

As a result of solving the topological optimization problem, we see that it makes sense to have a small section of constant thickness at the thin end of the overlay. In contrast to the known design solutions, where the thickness of the lining increases monotonously. In addition, the shape of the overlay at the right edge also has its own characteristics that require analysis. The presence of an inflection point for the function $\delta_1(x)$ in the vicinity of the right edge is most likely dictated by the fact that when solving the optimization problem, the strength criterion for the maximum principal stresses (10) was chosen, the diagram of which $\sigma_1(x)$, Fig. 6 has a kink in this region.

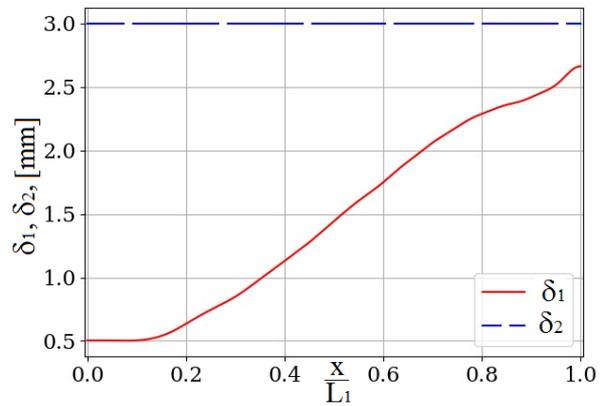


Fig. 5. The thickness of overlay and main plate

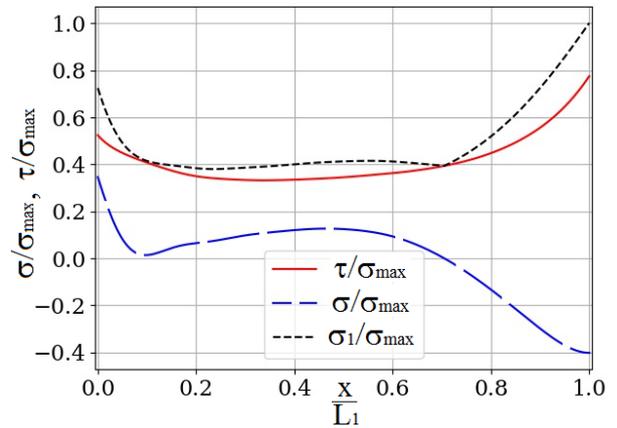


Fig. 6. Stresses in the adhesive layer

To estimate the rate of convergence of the algorithm, consider the sample mean values of some population parameters on each of the islands at the stages before migrations. The parameters were averaged over 80% of individuals from each of the populations.

Diagrams of changes in the sample means of the objective function as a result of optimization are shown in Fig. 7.

Different marker diameters correspond to different islands. The island with the highest mutation rate corresponds to the smallest marker. The average values of the objective function of the starting populations are not shown on the graph, since they differ many times from subsequent values.

The change in the average values of the adhesive joint length during the optimization process is shown in Fig. 8.

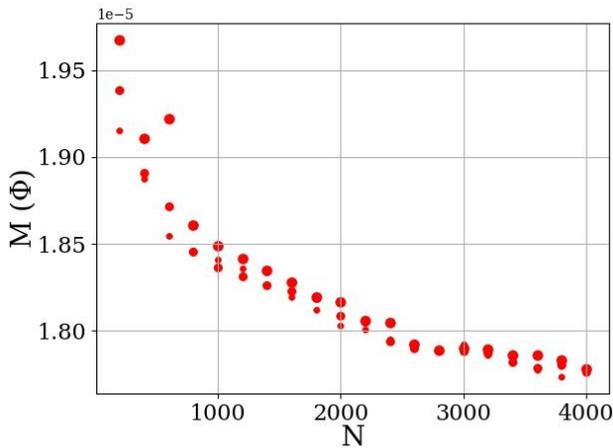


Fig. 7. Average objective function values

The values of the objective function and the length of the starting population are not shown in Fig. 7 and Fig. 8. The diagrams start with $N = 200$ cycles of the algorithm. This is due to the fact that the optimization at the initial stage is quite quickly, and then the rate of the parameters change decreases. That is, at the initial stage, the average value of the adhesive joint length in each of the populations was 20 mm, but in 200 reproduction-hybridization cycles, it is already less than 13 mm. However, after 4000 cycles of the algorithm is $L_1 = 12,09$ mm.

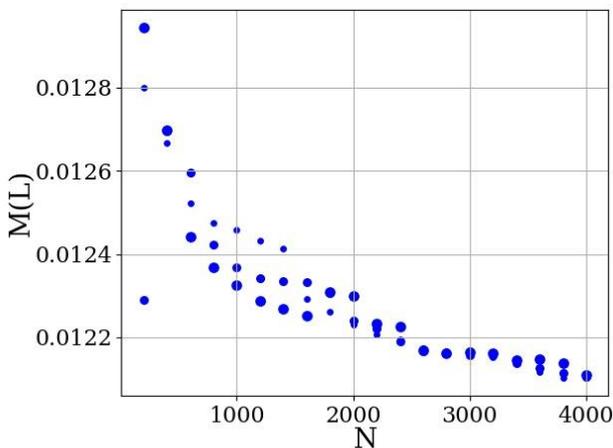


Fig. 8. Average objective function values

The change in the first three coefficients of the Fourier series during the optimization algorithm is shown in Fig. 9 “a” – “c”.

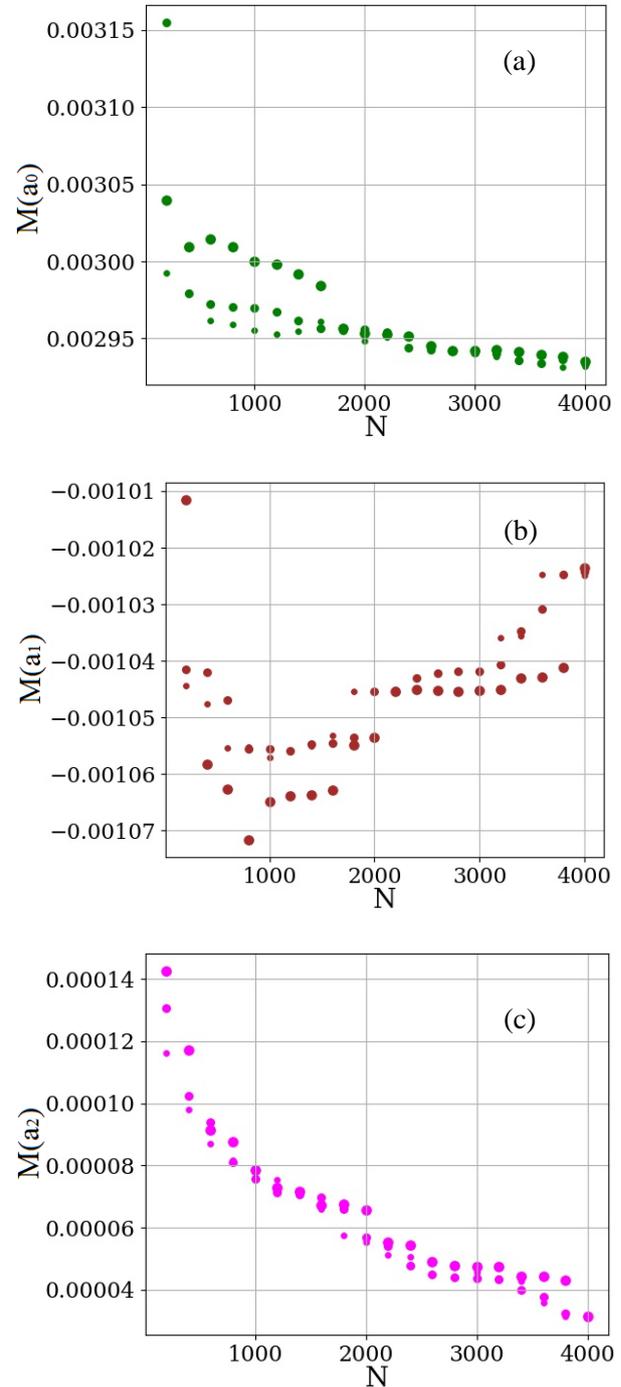


Fig. 9. The Fourier coefficients:
 a_0 – «a», a_1 – «b», a_2 – «c»

5.2. The second calculation case

As a result, in the second calculation case ($F_1 = 300$ kN/m), the optimal value of the joint length $L_2 = 92,48$ mm was obtained. Thus, doubling the load in comparison with the first calculation case leads to the

fact that the length of the overlay increases by almost 8 times.

The diagram of the change in the thickness of the overlay along the length of the adhesive joint is shown in Fig. 10. The straight line on the diagram is the thickness of the main plate.

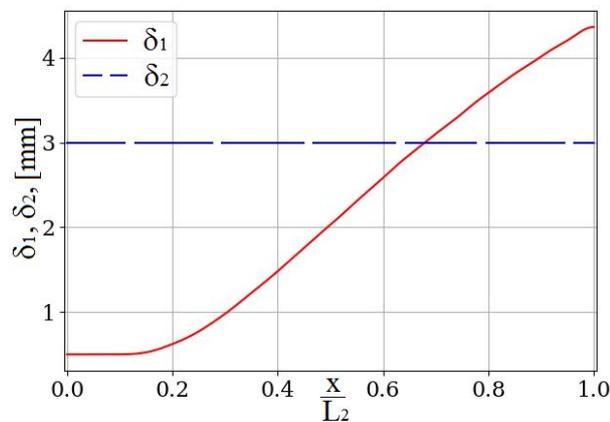


Fig. 10. The thickness of overlay and main plate

The diagrams of the corresponding stresses in the adhesive layer are shown in Fig. 11.

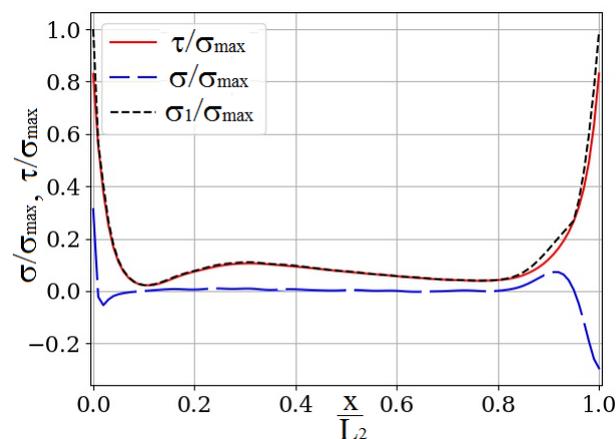


Fig. 11. Stresses in the adhesive layer

In this case, at increasing the load, it can be seen, that at both ends of the joint, the first principal stresses are equal to the maximum allowable.

6. Conclusion

A mathematical model of a double-lap adhesive joint with a variable thickness of overlay along the length is proposed in the article. Moreover, an improved genetic algorithm for optimization of the length joint and the cross-sectional shape of the overlay is represented. The proposed genetic algorithm refers to island models of genetic algorithms. These algorithms imple-

ment the idea of parallel evolutionary processes with the migration of the best individuals.

The stress state of the joint is described using the classical Goland-Reissner model [1]. The objective function of the optimization problem is the cross-sectional area of the overlay (i.e., in fact, it's the mass of the joint), and the desired parameters are the length of the joint and the Fourier series coefficients that describe the cross-sectional shape of the overlay. There are restrictions on the desired parameters in the form of the strength conditions of the adhesive layer and the minimum allowable thickness of the overlay. The solution to the primal problem is to determine the stress state of the adhesive joint for certain parameters and checking the strength condition of the adhesive layer is carried out using the finite difference method. The proposed approach is based on the classical one-dimensional models of the stress state of the structure and the genetic optimization algorithm and it showed high efficiency and speed. The use of classical one-dimensional models of the stress state of the joint made it possible to combine a fairly accurate description of the stress state of the structure with the speed of numerical calculation. The latter is the most important for solving optimization problems using genetic algorithms.

The proposed algorithm is highly flexible and can be generalized to other optimality criteria, strength criteria, and constraints.

As a result of solving a number of problems and analyzing the results, it was found that:

1. The dependence of the length and shape of the overlay on the transmitted load is non-linear.

2. There is a restriction in the optimization problem on the minimum allowable thickness of the overlay leads to the fact that the found optimal shape contains a horizontal area of the minimum allowable thickness at the unloaded edge of the overlay. This design solution, as far as the authors of the work know have not been previously proposed by anyone.

3. It is impossible to achieve a uniform distribution of stresses in the joint under the given conditions of the problem. The key limitation seems to be the constant thickness of the main plate along the length of the joint. Therefore, the load capacity of the adhesive joint is limited. The task of designing the joint has no solutions when the load F exceeds a certain value, which depends on the ultimate tensile strength of the adhesive, the elastic moduli of the joint components, etc.

4. The number of iterations as well as the calculation time was reduced due to the fact that the proposed island model of the evolutionary algorithm was used in the article. Calculations have shown that in order to achieve similar results, which are achieved in this case in 4000 iterations while the classical model of the genetic algorithm requires about 20000 iterations. A model

with 300 individuals was used for calculations and 100 individuals at each iteration were selected for hybridization. All other parameters of a task remained unchanged.

7. Future research directions

The proposed approach can be developed and generalized in the following directions:

1. The use of finite-difference templates of increased accuracy in solving the primal problem, this will make it possible to carry out calculations with high accuracy with a smaller number of nodal points, which in turn will increase the speed of calculations.

2. The number of restrictions in the problem can be increased. In addition to restrictions on the overlay thickness and the strength of the adhesive layer, some restrictions, such as amount of deflection, the strength of the overlay can be added to the optimization problem.

3. Topological optimization of joints, in which pliable adhesive is used at the edges of the adhesive area, and more rigid adhesive is used in the depth of the adhesive area [31, 32].

4. The proposed island genetic optimization algorithm can be applied for solving problems of topological optimization of joints of coaxial cylindrical pipes [33], joints with circular symmetry [12, 34] and for solving problems of optimization of joints in a two-dimensional formulation [30, 35, 36].

5. The proposed method can be applied to solve the problems of optimizing structures with honeycomb core [37].

6. The island model of the genetic algorithm can be further developed and complicated. For example, different objective functions on each of the islands [38], as well as combinations of genetic algorithms with other modern optimization methods [8, 39, 40] can be used.

7. Optimization of joints can be carried out taking into account thermal and technological stresses in the structure [41, 42].

Contribution of the authors: theoretical studies on the creation of a mathematical model of the adhesive joint, the finite-difference solution of the problem and the construction of the genetic algorithm – **Sergiy Kurennov**; numerous studies and parameters of the genetic algorithm, creation of an objective function and a system of restrictions – **Kostiantyn Barakhov**; analysis of literary sources, analysis of modelling results – **Olexiy Vambol**.

All authors have read and agreed with the published version of the manuscript.

References (GOST 7.1:2006)

1. *Analytical models of adhesively bonded joints. Part I: Literature survey [Text]* / L. F. M. da Silva, P. J. C. das Neves, R. D. Adams, J. K. Spelt // *Int. Journal Adhes. & Adhesiv.* – 2009. – Vol. 29, No. 3. – P. 319–330. DOI: 10.1016/j.ijadhadh.2008.06.005.

2. *Shishesaz, M. Effects of joint geometry and material on stress distribution, strength and failure of bonded composite joints: an overview [Text]* / M. Shishesaz, M. Hosseini // *The Journal of Adhesion.* – 2018. – Vol. 96, No. 12. – P. 1053–1121. DOI: 10.1080/00218464.2018.1554483.

3. *Barakhov, K. P. Influence of Joint Edge Shape on Stress Distribution in Adhesive Film [Text]* / K. P. Barakhov, I. M. Taranenko // In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering – 2021. ICTM 2021. Lecture Notes in Networks and Systems.* – 2022. – Vol. 367. – P. 123–132. Springer, Cham. DOI: 10.1007/978-3-030-94259-5_12.

4. *Haghani, R. Effect of Laminate Tapering on Strain Distribution in Adhesive Joints: Experimental Investigation [Text]* / R. Haghani, M. Al-Emrani, R. Kliger // *Journal of Reinforced Plastics and Composites.* – 2009. – Vol. 29, No. 7. – P. 972–985. DOI: 10.1177/0731684408102698.

5. *Effect of the geometric shapes of repair patches on bonding strength [Text]* / H. Lee, S. Seon, S. Park, R. Walallawita, K. Lee // *The Journal of Adhesion.* – 2021. – Vol. 97, No. 3. – P. 207–224. DOI: 10.1080/00218464.2019.1649660.

6. *A comprehensive experimental study on bi-adhesive single lap joints using DIC technique [Text]* / F. Ramezani, M. R. Ayatollahi, A. Akhavan-Safar, L. F. M. da Silva // *International Journal of Adhesion and Adhesives.* – 2020. – Vol. 102. – Article No. 102674. DOI: 10.1016/j.ijadhadh.2020.102674.

7. *Karpov, Ya. S. Jointing of high-loaded composite structural components. Part 2. Modeling of stress-strain state [Text]* / Ya. S. Karpov // *Strength of Materials.* – 2006. – Vol. 38, No. 5. – P. 481–491 DOI: 10.1007/s11223-006-0067-9.

8. *Hu, Z. A hybrid multilevel method for simultaneous optimization design of topology and discrete fiber orientation [Text]* / Z. Hu, O. Vambol, S. Sun // *Composite Structures.* – 2021. – Vol. 266. – Article No. 113791. DOI: 10.1016/j.compstruct.2021.113791.

9. *Review on techniques to improve the strength of adhesive joints with composite adherends [Text]* / X. Shang, E. A. S. Marques, J. J. M. Machado, R. J. C. Carbas, D. Jiang, L. F. M. da Silva // *Composites Part B: Engineering.* – 2019. – Vol. 177. – Article No. 107363. DOI: 10.1016/j.compositesb.2019.107363.

10. Kupski, J. *Design of adhesively bonded lap joints with laminated CFRP adherends: Review, challenges and new opportunities for aerospace structures* [Text] / J. Kupski, S. Teixeira de Freitas // *Composite Structures*. – 2021. – Vol. 268. – Article No. 113923. DOI: 10.1016/j.compstruct.2021.113923.
11. Amidi, S. *An analytical model for interfacial stresses in double-lap bonded joints* [Text] / S. Amidi, J. Wang // *The J. Adhesion*. – 2019. – Vol. 95, No. 11. – P. 1031–1055. DOI: 10.1080/00218464.2018.1464917.
12. Kurennov, S. *Stress-Strain State of a Double Lap Joint of Circular Form. Axisymmetric Model* [Text] / S. Kurennov, N. Smetankina // In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) *Integrated Computer Technologies in Mechanical Engineering - 2021. ICTM 2021. Lecture Notes in Networks and Systems*. – 2022. – Vol. 367. – P. 36–46. Springer, Cham. DOI: 10.1007/978-3-030-94259-5_4.
13. *A genetic algorithm of optimal design of beam at restricted sagging* [Text] / S. Kurennov, K. Barakhov, I. Taranenko, V. Stepanenko // *Radioelectronic and Computer Systems*. – 2022. No. 1(101). – P. 83–91. DOI: 10.32620/reks.2022.1.06.
14. Ichikawa, K. *A three-dimensional finite-element stress analysis and strength evaluation of stepped-lap adhesive joints subjected to static tensile loadings* [Text] / K. Ichikawa, Y. Shin, T. Sawa // *International Journal of Adhesion and Adhesives*. – 2008. – Vol. 28, No. 8. – P. 464–470. DOI: 10.1016/j.ijadhadh.2008.04.011.
15. Veisytabar, M. *Stress analysis of adhesively-bonded single stepped-lap joints based on three-parameter fractional viscoelastic foundation model* [Text] / Mehdi Veisytabar, Arash Reza, Younes Shekari // *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*. – 2022. – Vol. 236, No. 5. – P. 933–949. DOI: 10.1177/14644207211062497.
16. Groth, H. L. *Shape optimization of bonded joints* [Text] / H. L. Groth, P. Nordlund // *International Journal of Adhesion and Adhesives*. – 1991. – Vol. 11, No. 4. – P. 204–212. DOI: 10.1016/0143-7496(91)90002-y.
17. Ayaz Ümütlü, H. C. *Airfoil shape optimization using Bézier curve and genetic algorithm* [Text] / H. C. Ayaz Ümütlü, Z. Kiral // *Aviation*. – 2022. – Vol. 26, No. 1. – P. 32–40. DOI: 10.3846/aviation.2022.16471.
18. Zou, P. *Optimization of submerged floating tunnel cross section based on parametric Bézier curves and hybrid backpropagation - genetic algorithm* [Text] / P. Zou, J. Bricker, W. Uijtewaal // *Marine Structures*. – 2020. – Vol. 74. – Article No. 102807. DOI: 10.1016/j.marstruc.2020.102807.
19. Kaye, R. *Through-thickness shape optimisation of typical double lap-joints including effects of differential thermal contraction during curing* [Text] / R. Kaye, M. Heller // *International Journal of Adhesion and Adhesives*. – 2005. – Vol. 25, No. 3. – P. 227–238. DOI: 10.1016/j.ijadhadh.2004.07.006.
20. *Topology optimisation of adhesive joints using non-parametric methods* [Text] / H. Ejaz, A. Mubashar, I. A. Ashcroft, E. Uddin, M. Khan // *International Journal of Adhesion and Adhesives*. – 2018. – Vol. 81. – P. 1–10. DOI: 10.1016/j.ijadhadh.2017.11.003.
21. *Shape optimisation of adhesive fillets* [Text] / A. R. Rispler, L. Tong, P. G. Steven, M. R. Wisnom // *International Journal of Adhesion and Adhesives*. – 2000. – Vol. 20, No. 3. – P. 221–231. DOI: 10.1016/s0143-7496(99)00047-0.
22. Belingardi, G. *On the Optimization of Single Lap Metal/Plastics Adhesive Joints* [Text] / G. Belingardi, L. Goglio, A. Tarditi // *Key Engineering Materials*. – 2001. – Vols. 221–222. – P. 161–172. DOI: 10.4028/www.scientific.net/kem.221-222.161.
23. Symonov, V. S. *Optimization of a Panelled Smooth Composite Shell with a Closed Cross-Sectional Contour by Using a Genetic Algorithm* [Text] / V. S. Symonov, I. S. Karpov, J. Juračka, // *Mechanics of Composite Materials*. – 2013. – Vol. 49, No. 5. – P. 563–570. DOI: 10.1007/s11029-013-9372-0.
24. Hassan Vand, M. *Optimization of adhesive single-lap joints under bending moment* [Text] / M. Hassan Vand, H. Abbaszadeh, M. Shishesaz // *The Journal of Adhesion*. – 2022. – Vol. 98, No. 11. – P. 1687–1712. DOI: 10.1080/00218464.2021.1932485.
25. Kurennov, S. S. *Refined Mathematical Model of the Stress State of Adhesive Lap Joint: Experimental Determination of the Adhesive Layer Strength Criterion* [Text] / S. S. Kurennov // *Strength Mater.* – 2020. – Vol. 52, Iss. 5. – P. 779–789. DOI: 10.1007/s11223-020-00231-5.
26. Palomo-Romero, J. M. *An island model genetic algorithm for unequal area facility layout problems* [Text] / J. M. Palomo-Romero, L. Salas-Morera, L. García-Hernández // *Expert Systems with Applications*. – 2017. – Vol. 68, Iss. C. – P. 151–162. DOI: 10.1016/j.eswa.2016.10.004.
27. Niu, Y. *Structural Optimization Design of a Typical Adhesive Bonded Honeycomb-Core Sandwich T-joint in Side Bending Using Multi-Island Genetic Algorithm* [Text] / Y. Niu, X. Xu, S. Guo // *Applied Composite Materials*. – 2021. – Vol. 23, No. 8. – P. 3932–3965. DOI: 10.1007/s10443-021-09882-2.
28. *Quality-Oriented Study on Mapping Island Model Genetic Algorithm onto CUDA GPU* [Text] / X. Sun, P. Chou, C.-C. Wu, L.-R. Chen // *Symmetry*. – 2019. – Vol. 11, Iss. 3. – Article No. 318. DOI: 10.3390/sym11030318.

29. Lucas Angelo da Silveira. *Reconfigurable Heterogeneous Parallel Island Models [Electronic resource]* / Lucas Angelo da Silveira, Thaynara Arielly de Lima, Mauricio Ayala-Rincón // *arXivLabs*. – 2022. – URL: <https://doi.org/10.48550/arXiv.2205.02916>. – 12.06.2022.
30. Rapp, P. *Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part II: Displacement formulation for orthotropic adherends [Text]* / P. Rapp // *Archives of Civil and Mechanical Engineering*. – 2015. – Vol. 15, No. 2. – P. 603–619. DOI: 10.1016/j.acme.2014.06.004.
31. Kumar, S. *Behaviour of Bi-adhesive Joints [Text]* / S. Kumar, P. C. Pandey // *Journal of Adhesion Science and Technology*. – 2010. – Vol. 24, No. 7. – P. 1251–1281. DOI: 10.1163/016942409x12561252291982.
32. Öz, Ö. *On the von Mises elastic stress evaluations in the bi-adhesive single-lap joint: a numerical and analytical study [Text]* / Ö. Öz, H. Özer // *Journal of Adhesion Science and Technology*. – 2014. – Vol. 28, No. 21. – P. 2133–2153. DOI: 10.1080/01694243.2014.948110.
33. Santi G. M. *Explicit analytical shape function calculation for macro-element in glue connection [Text]* / G. M. Santi, D. Francia, F. Cesari // *Mechanics of Advanced Materials and Structures*. – 2021. – P. 1–9. DOI: 10.1080/15376494.2021.1992049.
34. Barakhov, K. *One-Dimensional Axisymmetric Model of the Stress State of the Adhesive Joint [Text]* / K. Barakhov, D. Dvoretzka, O. Poliakov // In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) *Integrated Computer Technologies in Mechanical Engineering – 2020. ICTM 2020. Lecture Notes in Networks and Systems*. – 2021. – Vol. 188. – P. 310 – 319. Springer, Cham. DOI: 10.1007/978-3-030-66717-7_26.
35. Kurennov, S. S. *Two-Dimensional Stressed State of an Adhesive Joint. Nonclassical Problem [Text]* / S. S. Kurennov, O. G. Polyakov, K. P. Barakhov // *Journal of Mathematical Sciences*. – 2021. – Vol. 254, No. 1. – P. 156–163. DOI: 10.1007/s10958-021-05295-5.
36. *Stress State of Two Glued Coaxial Tubes Under Nonuniform Axial Load [Text]* / S. Kurennov, K. Barakhov, D. Dvoretzka, O. Poliakov // In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) *Integrated Computer Technologies in Mechanical Engineering – 2020. ICTM 2020. Lecture Notes in Networks and Systems*. – 2021. – Vol. 188. – P. 389–400. Springer, Cham. DOI: 10.1007/978-3-030-66717-7_33.
37. Kondratiev, A. *Optimal Design of Composite Shelled Sandwich Structures with a Honeycomb Filler [Text]* / A. Kondratiev, O. Potapov, A. Tsaritsynskiy, T. Nabokina // In: Ivanov, V., Trojanowska, J., Pavlenko, I., Zajac, J., Peraković, D. (eds) *Advances in Design, Simulation and Manufacturing IV. DSMIE 2021. Lecture Notes in Mechanical Engineering*. – 2021. – P. 546–555. Springer, Cham. DOI: 10.1007/978-3-030-77719-7_54
38. *Heterogeneous Parallel Island Models [Text]* / L. A. Da Silveira, J. L. Soncco-Alvarez, T. A. De Lima, M. Ayala-Rincon // *2021 IEEE Symposium Series on Computational Intelligence (SSCI)*. – 2021. – P. 1–8. DOI: 10.1109/SSCI50451.2021.9659938.
39. Zhang, Y. *Handling multi-objective optimization problems with a multi-swarm cooperative particle swarm optimizer [Text]* / Y. Zhang, D. Gong, Z. Ding // *Expert Systems with Applications*. – 2011. – Vol. 38. – P. 13933–13941. DOI: 10.1016/j.eswa.2011.04.200.
40. Sheikhi Azqandi, M. *A Novel Hybrid Genetic Modified Colliding Bodies Optimization for Designing of Composite Laminates [Text]* / M. Sheikhi Azqandi // *Mechanics of Advanced Composite Structures*. – 2021. – Vol. 8. – P. 203–212. DOI: 10.22075/mac.2020.20281.1254
41. *On the fracture toughness of metal-composite adhesive joints with bending-extension coupling and residual thermal stresses effect [Text]* / P. Tsokanas, T. Loutas, G. Kotsinis, V. Kostopoulos, W. M. van der Brink, F. Martin de la Escalera // *Composites Part B: Engineering*. – 2019. – Article No. 107694. DOI: 10.1016/j.compositesb.2019.107694.
42. Apalak, M. K. *On non-linear thermal stresses in an adhesively bonded single lap joint [Text]* / M. K. Apalak, R. Gunes // *Computers & Structures*. – 2002. – Vol. 80, Iss. 1. – P. 85–98. DOI: 10.1016/s0045-7949(01)00139-0.

References (BSI)

1. da Silva, L. F. M., das Neves, P. J. C., Adams, R. D., Speltio J. K. Analytical models of adhesively bonded joints. Part I: Literature survey. *International Journal of Adhesion and Adhesives*, 2009, vol. 29, iss. 3, pp. 319–330. DOI: 10.1016/j.ijadhadh.2008.0005.
2. Shishesaz, M., Hosseini, M. Effects of joint geometry and material on stress distribution, strength and failure of bonded composite joints: an overview. *The Journal of Adhesion*, 2018, vol. 96, iss. 12, pp. 1053–1121. DOI: 10.1080/00218464.2018.1554483.
3. Barakhov, K. P., Taranenko, I. M. Influence of Joint Edge Shape on Stress Distribution in Adhesive Film. In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) *Integrated Computer Technologies in Mechanical Engineering - 2021. ICTM 2021. Lecture Notes in Networks and Systems*, 2022, vol. 367, pp. 123–132. Springer, Cham. DOI: 10.1007/978-3-030-94259-5_12.
4. Haghani, R., Al-Emrani, M., Kliger, R. Effect of Laminate Tapering on Strain Distribution in Adhesive Joints: Experimental Investigation. *Journal of Rein-*

- forced Plastics and Composites*, 2009, vol. 29, iss. 7, pp. 972–985. DOI: 10.1177/0731684408102698.
5. Lee, H., Seon, S., Park, S., Walallawita, R., & Lee, K. Effect of the geometric shapes of repair patches on bonding strength. *The Journal of Adhesion*, 2021, vol. 97, iss. 3, pp. 207–224. DOI: 10.1080/00218464.2019.1649660.
6. Ramezani, F., Ayatollahi, M. R., Akhavan-Safar, A., & da Silva, L. F. M. A comprehensive experimental study on bi-adhesive single lap joints using DIC technique. *International Journal of Adhesion and Adhesives*, 2020, vol. 102, article no. 102674. DOI: 10.1016/j.ijadhadh.2020.102674.
7. Karpov, Ya. S. Jointing of high-loaded composite structural components. Part 2. Modeling of stress-strain state. *Strength of Materials*, 2006, vol. 38, iss. 5, pp. 481–491. DOI: 10.1007/s11223-006-0067-9.
8. Hu, Z., Vambol, O., Sun, S. A hybrid multi-level method for simultaneous optimization design of topology and discrete fiber orientation. *Composite Structures*, 2021, vol. 266, article no. 113791. DOI: 10.1016/j.compstruct.2021.113791.
9. Shang, X., Marques, E. A. S., Machado, J. J. M., Carbas, R. J. C., Jiang, D., da Silva, L. F. M. Review on techniques to improve the strength of adhesive joints with composite adherends. *Composites Part B: Engineering*, 2019, vol. 177, article no. 107363. DOI: 10.1016/j.compositesb.2019.107363
10. Kupski, J., & Teixeira de Freitas, S. Design of adhesively bonded lap joints with laminated CFRP adherends: Review, challenges and new opportunities for aerospace structures. *Composite Structures*, 2021, vol. 268, article no. 113923. DOI: 10.1016/j.compstruct.2021.113923.
11. Amidi, S., Wang, J. An analytical model for interfacial stresses in double-lap bonded joints. *The Journal of Adhesion*, 2019, vol. 95, iss. 11, pp. 1031–1055. DOI: 10.1080/00218464.2018.1464917.
12. Kurennov, S., Smetankina, N. Stress-Strain State of a Double Lap Joint of Circular Form. Axisymmetric Model. In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2021. ICTM 2021. Lecture Notes in Networks and Systems*, 2022, vol. 367, pp. 36–46. Springer, Cham. DOI: 10.1007/978-3-030-94259-5_4.
13. Kurennov, S., Barakhov, K., Taranenko, I., Stepanenko, V. A genetic algorithm of optimal design of beam at restricted sagging. *Radioelectronic and Computer Systems*, 2022, no. 1(101), pp. 83–91. DOI: 10.32620/reks.2022.1.06.
14. Ichikawa, K., Shin, Y., Sawa, T. A three-dimensional finite-element stress analysis and strength evaluation of stepped-lap adhesive joints subjected to static tensile loadings. *International Journal of Adhesion and Adhesives*, 2018, vol. 28, iss. 8, pp. 464–470. DOI: 10.1016/j.ijadhadh.2008.04.011.
15. Veisytabar, Mehdi., Reza, Arash., Shekari, Younes. Stress analysis of adhesively-bonded single stepped-lap joints based on three-parameter fractional viscoelastic foundation model. *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, 2022, vol. 236, no. 5, pp. 933–949. DOI: 10.1177/14644207211062497.
16. Groth, H. L., Nordlund, P. Shape optimization of bonded joints. *International Journal of Adhesion and Adhesives*, 1991, vol. 11, iss. 4, pp. 204–212. DOI: 10.1016/0143-7496(91)90002-y.
17. Ayaz Ümütlü, H. C., Kiral, Z. Airfoil shape optimization using Bézier curve and genetic algorithm. *Aviation*, 2022, vol. 26, iss. 1, pp. 32–40. DOI: 10.3846/aviation.2022.16471.
18. Zou, P., Bricker, J., Uijtewaal, W. Optimization of submerged floating tunnel cross section based on parametric Bézier curves and hybrid backpropagation - genetic algorithm. *Marine Structures*, 2020, vol. 74, article no. 102807. DOI: 10.1016/j.marstruc.2020.102807.
19. Kaye, R., Heller, M. Through-thickness shape optimisation of typical double lap-joints including effects of differential thermal contraction during curing. *International Journal of Adhesion and Adhesives*, 2005, vol. 25, iss 3, pp. 227–238. DOI: 10.1016/j.ijadhadh.2004.07.006.
20. Ejaz, H., Mubashar, A., Ashcroft, I. A., Uddin, E., Khan, M. Topology optimisation of adhesive joints using non-parametric methods. *International Journal of Adhesion and Adhesives*, 2018, vol. 81, pp. 1–10. DOI: 10.1016/j.ijadhadh.2017.11.003.
21. Rispler, A. R., Tong, L., P. Steven, G., Wisnom, M. R. Shape optimisation of adhesive fillets. *International Journal of Adhesion and Adhesives*, 2000, vol. 20, iss. 3, pp. 221–231. DOI: 10.1016/s0143-7496(99)00047-0.
22. Belingardi, G., Goglio, L., Tarditi, A. On the Optimization of Single Lap Metal/Plastics Adhesive Joints. *Key Engineering Materials*, 2001, vols. 221–222, pp. 161–172. DOI: 10.4028/www.scientific.net/kem.221-222.161.
23. Symonov, V. S., Karpov, I. S., Juračka, J. Optimization of a Panelled Smooth Composite Shell with a Closed Cross-Sectional Contour by Using a Genetic Algorithm. *Mechanics of Composite Materials*, 2013, vol. 49, iss. 5, pp. 563–570. DOI: 10.1007/s11029-013-9372-0.
24. Hassan Vand, M., Abbaszadeh, H., Shishesaz, M. Optimization of adhesive single-lap joints under bending moment. *The Journal of Adhesion*, 2022, vol. 98, iss. 11, pp. 1687–1712. DOI: 10.1080/00218464.2021.1932485.

25. Kurennov, S. S. Refined Mathematical Model of the Stress State of Adhesive Lap Joint: Experimental Determination of the Adhesive Layer Strength Criterion. *Strength Mater*, 2020, vol. 52, iss. 5, pp. 779–789. DOI: 10.1007/s11223-020-00231-5.
26. Palomo-Romero, J. M., Salas-Morera, L., & García-Hernández, L. An island model genetic algorithm for unequal area facility layout problems. *Expert Systems with Applications*, 2017, vol. 68, iss. C, pp. 151–162. DOI: 10.1016/j.eswa.2016.10.004.
27. Niu, Y., Xu, X., Guo, S. Structural Optimization Design of a Typical Adhesive Bonded Honeycomb-Core Sandwich T-joint in Side Bending Using Multi-Island Genetic Algorithm. *Applied Composite Materials*, 2021, vol. 23, iss. 8, pp. 3932–3965. DOI: 10.1007/s10443-021-09882-2.
28. Sun, X., Chou, P., Wu, C.-C., Chen, L.-R. Quality-Oriented Study on Mapping Island Model Genetic Algorithm onto CUDA GPU. *Symmetry*, 2019, vol. 11, iss. 3, article no. 318. DOI: 10.3390/sym11030318.
29. Lucas Angelo da Silveira, Thaynara Arielly de Lima, Mauricio Ayala-Rincón. Reconfigurable Heterogeneous Parallel Island Models. *ArXivLabs*, 2022. Available at: <https://doi.org/10.48550/arXiv.2205.02916> (accessed 12.06.2022).
30. Rapp, P. Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part II: Displacement formulation for orthotropic adherends. *Archives of Civil and Mechanical Engineering*, 2015, vol. 15, iss. 2, pp. 603–619. DOI: 10.1016/j.acme.2014.06.004.
31. Kumar, S., Pandey, P. C. Behaviour of Bi-adhesive Joints. *Journal of Adhesion Science and Technology*, 2010, vol. 24, iss. 7, pp. 1251–1281. DOI: 10.1163/016942409x12561252291982.
32. Öz, Ö., Özer, H. On the von Mises elastic stress evaluations in the bi-adhesive single-lap joint: a numerical and analytical study. *Journal of Adhesion Science and Technology*, 2014, vol. 28, iss. 21, pp. 2133–2153. DOI: 10.1080/01694243.2014.948110.
33. Santi, G. M., Francia, D., Cesari, F. Explicit analytical shape function calculation for macro-element in glue connection. *Mechanics of Advanced Materials and Structures*, 2021, pp. 1-9. DOI: 10.1080/15376494.2021.1992049.
34. Barakhov, K., Dvoretzka, D., Poliakov, O. One-Dimensional Axisymmetric Model of the Stress State of the Adhesive Joint. In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. ICTM 2020. Lecture Notes in Networks and Systems*, 2021, vol. 188, pp. 310-319. Springer, Cham. DOI: 10.1007/978-3-030-66717-7_26.
35. Kurennov, S. S., Polyakov, O. G., Barakhov, K. P. Two-Dimensional Stressed State of an Adhesive Joint. Nonclassical Problem. *Journal of Mathematical Sciences*, 2021, vol. 254, iss. 1, pp. 156–163. DOI: 10.1007/s10958-021-05295-5.
36. Kurennov, S., Barakhov, K., Dvoretzka, D., Poliakov, O. Stress State of Two Glued Coaxial Tubes Under Nonuniform Axial Load. In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering – 2020. ICTM 2020. Lecture Notes in Networks and Systems*, 2021, vol. 188, pp. 389-400. Springer, Cham. DOI: 10.1007/978-3-030-66717-7_33.
37. Kondratiev, A., Potapov, O., Tsaritsynskiy, A., Nabokina, T. Optimal Design of Composite Shelled Sandwich Structures with a Honeycomb Filler. In: *Ivanov, V., Trojanowska, J., Pavlenko, I., Zajac, J., Peraković, D. (eds) Advances in Design, Simulation and Manufacturing IV. DSMIE 2021. Lecture Notes in Mechanical Engineering*, 2021, pp. 546-555. Springer, Cham. DOI: 10.1007/978-3-030-77719-7_54.
38. Da Silveira, L. A., Soncco-Alvarez, J. L., De Lima, T. A., and Ayala-Rincon, M., Heterogeneous Parallel Island Models. *2021 IEEE Symposium Series on Computational Intelligence (SSCI)*, 2021, pp. 1-8, DOI: 10.1109/SSCI50451.2021.9659938.
39. Zhang, Y., Gong, D., Ding, Z. Handling multi-objective optimization problems with a multi-swarm cooperative particle swarm optimizer. *Expert Systems with Applications*, 2011, vol. 38, pp. 13933-13941. DOI: 10.1016/j.eswa.2011.04.200.
40. Sheikhi Azqandi, M. A Novel Hybrid Genetic Modified Colliding Bodies Optimization for Designing of Composite Laminates. *Mechanics of Advanced Composite Structures*, 2021, vol. 8, pp. 203-212. DOI: 10.22075/mac.2020.20281.1254.
41. Tsokanas, P., Loutas, T., Kotsinis, G., Kostopoulos, V., van der Brink, W. M., & Martin de la Escalera, F. On the fracture toughness of metal-composite adhesive joints with bending-extension coupling and residual thermal stresses effect. *Composites Part B: Engineering*, 2019, article no. 107694. DOI: 10.1016/j.compositesb.2019.107694.
42. Apalak, M. K., Gunes, R. On non-linear thermal stresses in an adhesively bonded single lap joint. *Computers & Structures*, 2002, vol. 80, iss. 1, pp. 85–98. DOI: 10.1016/s0045-7949(01)00139-0.

**ТОПОЛОГІЧНА ОПТИМІЗАЦІЯ СИМЕТРИЧНОГО АДГЕЗІЙНОГО З'ЄДНАННЯ.
ОСТРІВНА МОДЕЛЬ ГЕНЕТИЧНОГО АЛГОРИТМУ**

С. С. Курєннов, К. П. Барахов, О. О. Вамболь

Сучасні адитивні технології дозволяють створювати конструкції змінної товщини і практично будь-якої форми. Це ставить перед конструкторами задачі оптимального проектування нового типу – задачі топологічної оптимізації, які полягають у знаходженні оптимальної форми конструкції або оптимального розподілу матеріалу по конструкції. Критерієм оптимальності є, як правило, маса конструкції. При цьому конструкція повинна зберігати несну здатність під дією прикладених до неї навантажень. **Предметом** вивчення у цій статті є симетричне двозрізне клейове з'єднання основної пластини з двома накладками однакової форми, з обох її сторін. **Метою** цієї статті є знаходження оптимальної форми прикладених накладок, які можуть мати змінну товщину за наявності ряду обмежень. Основним обмеженням є міцність конструкції. Крім того, на мінімальну та максимальну товщину накладки можуть бути накладено додаткові обмеження. Отже, розв'язок поставленої задачі може бути подано у вигляді сукупності наступних завдань: побудова математичної моделі розглянутого з'єднання, побудова чисельного розв'язку прямої задачі за допомогою методу скінченних різниць, побудова генетичного алгоритму оптимізації. Для поліпшення збіжності генетичного алгоритму в представленій роботі запропоновано використовувати острівну модель, що складається з декількох популяцій. Відмінність запропонованої моделі генетичного алгоритму у тому, що на одному з «островів» мутації відбуваються частіше і з більшою дисперсією, ніж на двох інших «островах». Таке рішення забезпечує як швидкість еволюційного відбору, так і стабільність досягнутих результатів. У роботі розв'язано кілька модельних задач. До **основних результатів роботи** можна віднести наступне: виявлено нелінійну залежність довжини накладки від прикладеного навантаження; наявність обмеження на мінімальну товщину накладки зумовлює появу деякого «майданчика» на краю накладки, товщина якої дорівнює мінімально допустимій.

Ключові слова: умовна оптимізація; метод скінченних різниць; генетичний алгоритм.

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