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Review of Research and Validation of New Mathematical Methods for Modeling the Aeroelasticity of a Wing with Control Surfaces Free Play

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The phenomenon of instability in flight due to the interaction of aerodynamic, elastic and inertial forces. could prove disastrous. Therefore, due importance is given to the aeroelastic analysis of the aircraft at the design stage. Free play, sometimes called play, occurs as a result of wear on parts such as lugs or moving mounts, often due to the aging of the aircraft. Excessive free play is detrimental to the flight safety of the aircraft. This can lead to catastrophic aeroelastic instabilities such as limit cycle oscillation (LCO), a phenomenon associated with sustained vibration of a fixed amplitude. LCO can accelerate the accumulation of structural fatigue damage and compromise controllability during flight. Aircraft control systems play a critical role in ensuring stability, maneuverability and flight safety. Over the past few decades, nonlinear aeroelasticity has received increasing attention. Advanced computational capabilities have stimulated renewed interest among researchers in re-examining the control surface free play problem with the goal of refining analytical models for greater accuracy. Both modern theoretical and experimental analyzes of free play have opened up new possibilities for the study of aeroelasticity. Despite the apparent understanding of the fundamental problem and the availability of numerous modeling approaches, freewheeling research continues to generate varied predictions regarding aeroelastic dynamic behavior and associated properties. Free play, defined as the sag or movement of mechanical connections between control inputs and aircraft control surfaces, is a ubiquitous aspect of control system design. While a little play may be tolerable, excessive play can lead to non-linearities and uncertainties, potentially compromising flight performance and safety. The aim of the article is to review existing methods of numerical research, analyze new mathematical methods for modeling aeroelastic vibrations and formulate the problem of developing safety criteria for transport category aircraft in terms of preventing elastic instability in the presence of free play in the control mechanism.

Key words: free play; control surface; flutter; limit cycle oscillation; degree-of-freedom; nonlinear aeroelasticity

Introduction

Aerodynamic forces act on the aircraft structure which, being flexible, deforms. The interaction of the aerodynamic forces with the flexible structure is termed aeroelasticity. Fig. 1 shows primary and secondary flight controls of aircraft. Wing flutter is probably the most commonly known and studied of all the dynamic aeroelastic phenomena. Flutter occurs as a result of interactions between aerodynamics, stiffness, and inertial forces on a structure. In an aircraft, as the speed of the wind increases, there may be a point at which the structural damping is insufficient to damp out the motions which are increasing due to aerodynamic energy being added to the structure. This vibration can cause structural failure and therefore considering flutter characteristics is an essential part of designing an aircraft. In this paper research is focused mainly on the wing flutter with aileron and horizontal stabilizer with elevator under free play conditions.

If the wing twists and bends in a certain manner the unsteady aerodynamic loads start feeding the elastic motion of the wing causing the amplitudes to grow, eventually leading to structural failure or Limit Cycle Oscillation (LCO) [1].

Free play within aircraft control surfaces, all-moving wings, and external stores represents a focused structural nonlinearity characterized by diminished or absent stiffness within specific ranges of motion of the actuating structures. Free play, occasionally known as backlash, arises from deteriorated components such as worn hinges or insecure attachments, frequently stemming from the aging of an aircraft. Excessive free play is detrimental to the safety of aircraft flight cycle oscillations LCO, a phenomenon involving sustained fixed amplitude vibration. The occurrence of LCO can expedite the accrual of structural fatigue damage and jeopardize controllability during flight. LCO can accelerate the accumulation of structural fatigue damage and compromise controllability during flight. One can observe that vertical and horizontal tail plane control surfaces (Elevator and Rudder) represent the major cause of airframe vibrations (72%). These vibrations may have multiple consequences on the aircraft life cycle from the design to operations. Indeed, spurious oscillations of the control surfaces impact both the structural airframe design and the flight control system design. Undetected oscillations may lead to several problems including local structural load augmentation, flight handling qualities deterioration, actuator operational life reduction, cockpit and cabin comfort deterioration as well as maintenance cost augmentation.

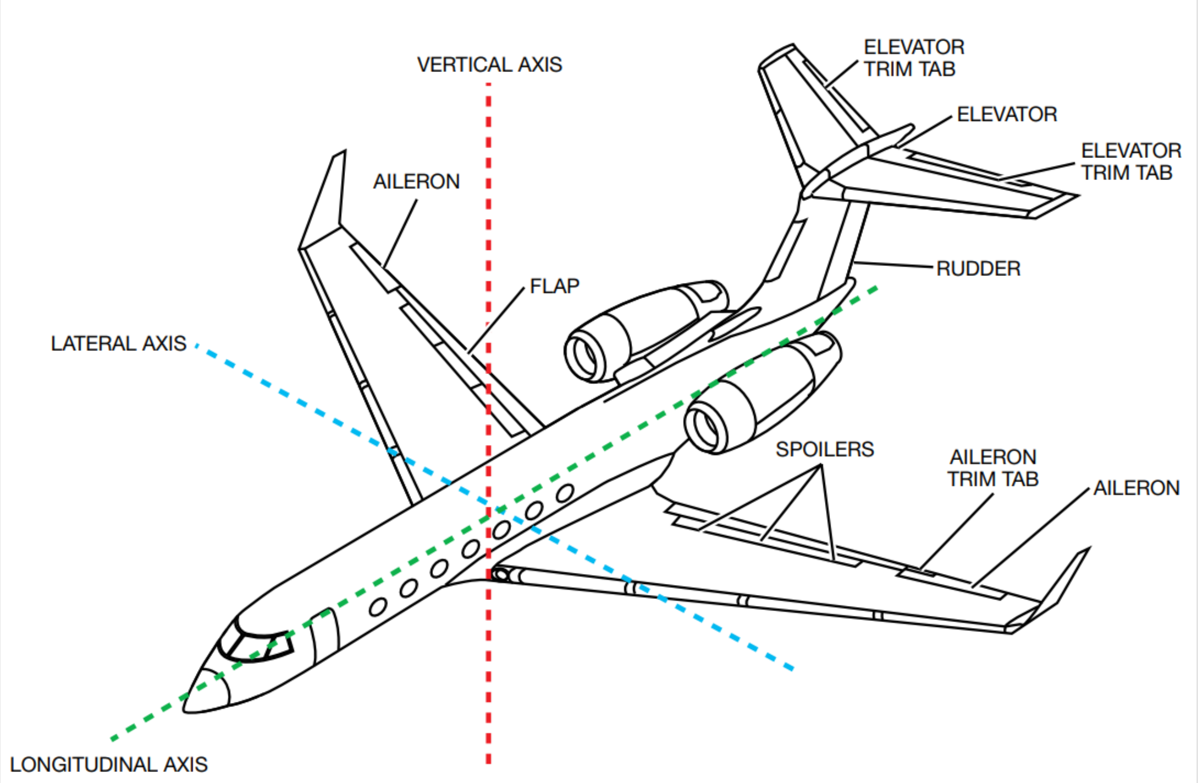


Fig. 1. Primary and secondary flight controls

Furthermore, it has been well-documented that such instabilities can occur below the predicted linear flutter velocities, rendering potentially hazardous flight scenarios such as premature flutter. The understanding and identification of free play and its associated aeroelastic effects are further complicated by the simultaneous presence of other structural nonlinearities, such as friction and damping within moving parts [2–4]. Since free play is a complex, time-dependent phenomenon, there is difficulty in both identifying when free play is experienced during flight and characterizing the nonlinear dynamical behavior associated with free play, which can yield wide-ranging predictions from stable to chaotic response. Therefore, the free play problem is of both practical concern as well as theoretical interest.

The Federal Aviation Administration (FAA) has established guidelines that define allowable free play quantities, limitations, and considerations that would meet regulatory compliance to freedom from flutter in their Advisory Circular (AC) 23.629-1B (normal, utility, and acrobatic category airplanes) [5] and 25.629-1B (transport category airplanes) [6]. The recommendations in these guidelines, however, need updating. For example, AC 23.629-1B [5], which governs current design of small airplanes, gives numerical limitations on free play angle based on data from experiments done in the 1950s, directly referencing [7]. The costs and dangers of free play are further exacerbated with aging aircraft. In a 2006 Airworthiness Directive issued by the FAA [8], Boeing Model 737 airplanes had to undergo repetitive lubrication and more frequent maintenance checks to ensure that free play-induced vibration be avoided for the aging fleet.

Instances of corrosion have been reported in the mass-balance weights of the elevators in the Embraer-505 fleet. This corrosion may cause a loss of mass or detachment of the mass-balance weights, leading to an unbalanced elevator. In certain flight conditions, this imbalance could result in a loss of aircraft control. Addressing free play as part of the issue with mass and balance weights attachment for control surfaces is mandatory according to a service bulletin for Embraer-505 aircraft [9].

Within the last two decades, many similar Airworthiness Directives associated with free play have been issued more frequently for different aircraft fleets [10–11]. These proposed rule changes not only highlight both the logistical and financial challenges of free play management, but they also echo the urgency of addressing free play as a serious matter with potential for excessive airframe vibrations, divergent flutter, and subsequent loss of safe flight.

A majority of the literature on the topic of free play initiate their studies by expanding upon fundamental aeroelastic theories. Dynamical equations of motion are developed for an assumed cross-sectional structural model of a wing or control surface that is subjected to selected aerodynamic loads. Then the free play nonlinearity is introduced into the structural model and various types of engineering analyses are performed to map regions of aeroelastic instability or flutter, and identify parameters of influence and their sensitivities to aeroelastic response. The accuracy of these models is usually assessed by comparing numerical results to data from experimental studies involving similar structural and aerodynamic setups. While many research studies aim to better understand free play related instabilities and their implications for aircraft reliability, some researchers choose to exploit the instabilities, such as using wing-based piezoelectric systems to harvest energy [12]. Others wish to explore novel means of active control and other suppression techniques to prevent the instabilities resulting from nonlinear aeroservoelastic systems [13,14].

The goals of this paper are to: 1) provide a cohesive understanding of the numerous modeling approaches that exist and the continued development of newer ideas, 2) evaluate connection to the maintenance process, 3) discuss the important results pertaining to each subtopic of the research and 4) identify major gaps in the research and set tasks for further research.

The free play research presented is broadly categorized as either theoretical or experimental. Within the theoretical research, four major subtopics exist: free play models, structural models, aerodynamic models, and means of analysis. There is also a growing subgroup that explores control theory applications to the free play problem.

1. Free play models

Free play is more complicated than the idealized form described further. The physical manner by which it is present is rooted in looseness and excess tolerances in bolts, bearing, hinges, and actuators of the control surface mechanism [15,16]. Isolating the specific locations at which the free play originates can be a nontrivial task. Danowsky et al. [17] provides a material, depicting a typical electro-hydrostatic actuator of a control surface. In the diagram, multiple possible locations are shown where free play can be likely to manifest. Due to the complexities of many control surface mechanisms, free play can both originate from multiple locations (i.e., hinge line and actuator bearings) and also be simultaneously present with several other structural nonlinearities, such as friction and damping within moving parts, resulting in unique mappings on the moment-deflection graph. Another type of control surface controller of Embraer 505 is presented in Fig. 2. The aileron hydraulic servo actuators are mounted on the fixed trailing edge of each wing. Each aileron actuator is a moving body type actuator that receives hydraulic pressure from two sources (tandem actuator) and incorporates mechanical feedback. When the actuator extends or retracts, motion is transmitted through the aileron output bellcrank (sloppy link) [25].

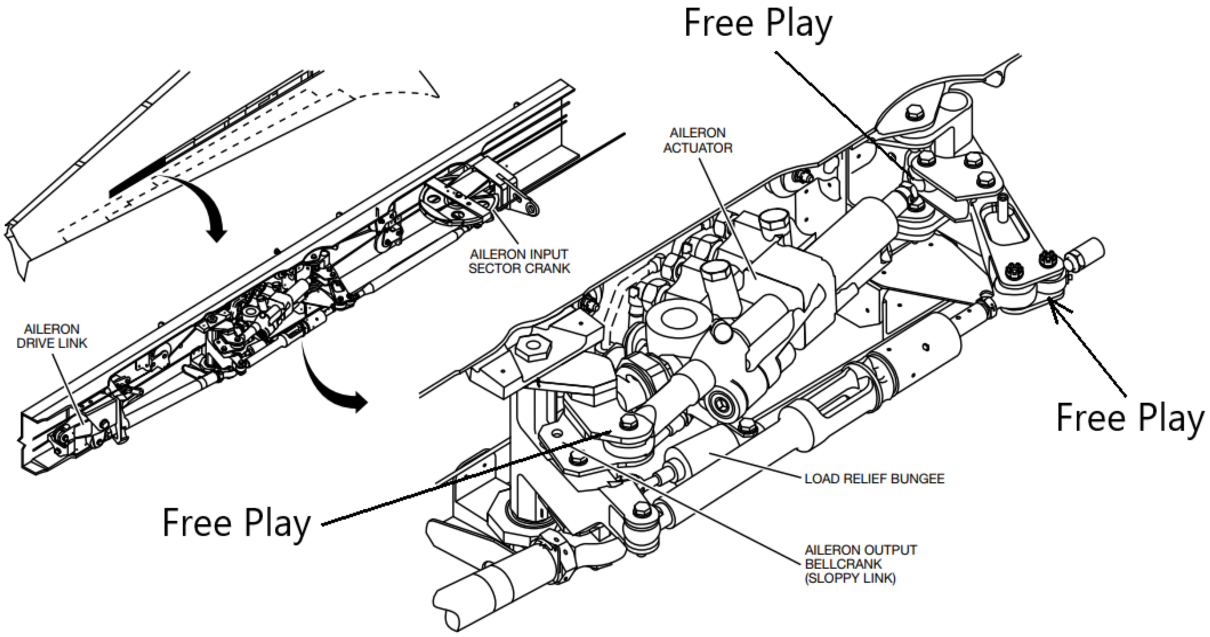


Fig. 2. Electro-hydro mechanical aileron actuator

Fig. 3. Represents elevator actuator assembly, located in the tail compartment and cradled in the power boost linkages between the input sectors and the output cranks are two moving body type hydraulic servoactuators that provide mechanical feedback. Each elevator has its own hydraulic servoactuator.

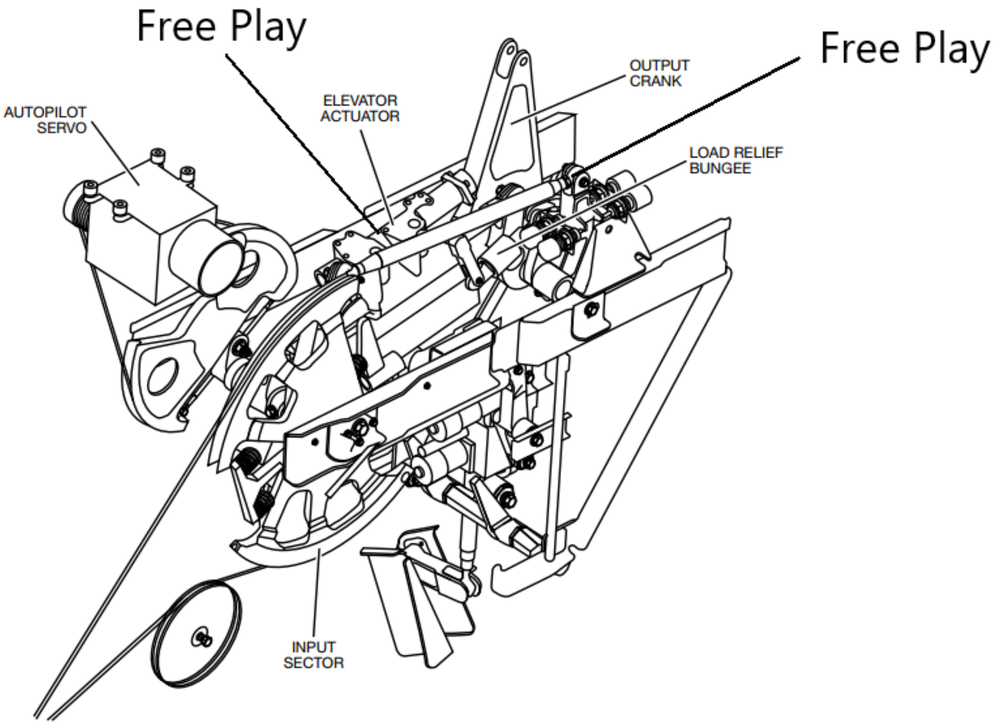


Fig. 3. Elevator Actuator Assembly

The mathematical modeling of the free play nonlinearity is the first major subtopic in the theoretical research of free play. In the simplest sense, the engineering idealization of free play involves first assuming that the system's structural stiffness obeys linear Hooke's law for some range of a control surface displacement. Then, the nonlinearity is introduced as a flat spot or a gap in the structural stiffness versus deflection graph, as shown in Fig. 4, splitting the graph into three piecewise linear subdomains. The graph shows the rotation angle of a control surface flap, denoted in the x-axis as "Flap displacement, β", and the corresponding hinge moment necessary to rotate control surface, denoted in the y-axis as "Torque". The free play region is seen in the center where there is a flat spot in the graph about ± δ, which is the variable for the amount of rotational free play.

MIL-A-8870C [18] outlines the maximum allowable free play within various control surfaces and movable wing structures through the service life of military aircraft in order to prevent the aircraft from potentially encountering aeroelastic instability within its flight envelope. These free play limits are also recommended by the FAA. However, some major concerns associated with the free play guidance still exist, such as:

- there may exist conditions where the onset of aeroelastic instability is not dependent on a specific free play amount, such as a situation where a control surface, with free play and no structural damping, is in an unloaded state (zero hinge moment) [19],
- meeting the free play limit requirements is often challenging or too constraining for certain aircraft configuration or control surfaces (i.e., all-moving horizontal tail on the F-16) [20], and
- efforts to comply with strict free play requirements can result in expensive and frequent maintenance procedures, which may require laborious repetitive actions such as re-lubrication or replacement of major parts [9,21].

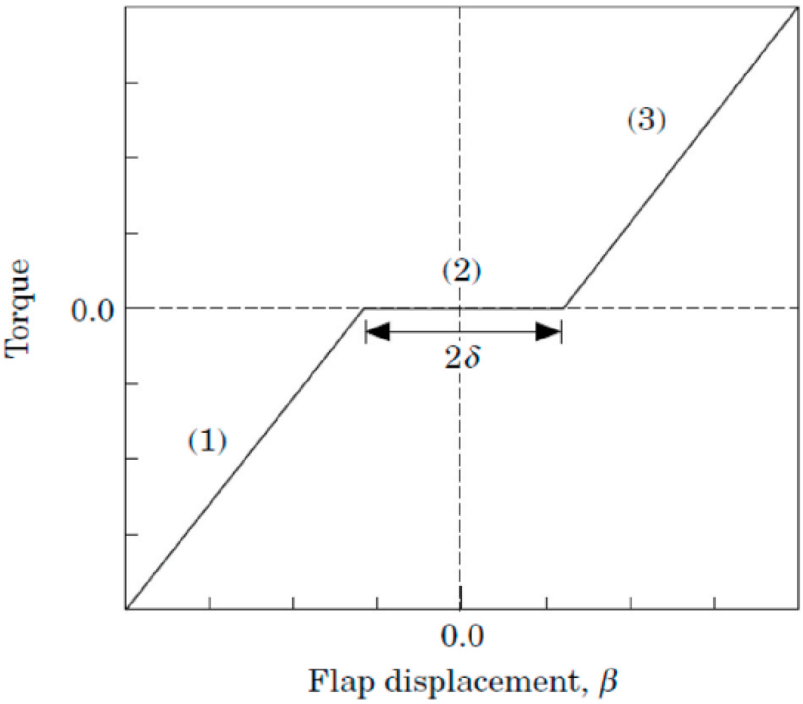


Fig. 4. Typical airfoil section with free play nonlinearity.

To combat these issues, several common actions are taken during the design, testing, and maintenance stages of an aircraft's life to avoid undesired aeroelastic penalties resulting from excessive free play. All three items mentioned above are normally addressed simultaneously during the design and testing of an aircraft. It is understood that free play will naturally exist and accumulate due to the wearing and loosening of parts as an operating aircraft ages. Classical aeroelastic theory demonstrates that the presence of free play reduces the effective global stiffness of an aeroelastic system, thereby potentially causing reductions in flight flutter speed. It is also known that the existence of free play can be one of the precursors to the aeroelastic instability of LCO, and the degree of free play can proportionally affect the size of the LCO [22]. Therefore, the first set of actions are: 1) employ regular inspection of free play to keep values within regulated design limits and 2) track, monitor, and control any vibration that can be associated with the presence of excessive free play. At the same time, it is not uncommon to discover that certain control surfaces of operating aircraft have difficulty meeting the free play limit requirements and can have free play significantly greater than the prescribed limits in MIL-A-8870C [18], as sometimes found in the horizontal tail control surfaces for fighter jets like the M346 [21] and the F-16 aircraft [23]. Despite some aircraft having free play well beyond their specified military limits, none of the aircraft ever reported experiencing LCO during their flights. Such findings lead to the second set of actions that push for the relaxation of free play limits to reduce manufacturing and maintenance cost, or to tailor a high-performance aircraft for specific aeroelastic characteristics. For example, the F-22 was designed to allow transient, small-amplitude LCO in certain control surfaces to attenuate undesired vibration at the pilot seat. This required establishing non-standard, design-specific free play limits that exceeded the traditional military limits. To achieve their design exception, exhaustive efforts were made during test planning to uncover flight conditions and aircraft maneuvers that yielded zero or near zero hinge moment in control surfaces with maximum designed free play. Then, successful controlled flight flutter tests were demonstrated at those conditions to justify the relaxation of the free play limits. To summarize, the leading industry approaches to tackling free play related aeroelastic instabilities, such as LCO, are: to [21,16]

1) ensure compliance with free play limits based on regulatory guidance by performing regular free play measurements and subsequent actions during maintenance.

2) track, monitor, and actively control vibration associated with free play, and/or.

3) justify relaxation of free play limits through exhaustive flight flutter testing during the design phase of an aircraft.

Approach #1 is required for all aircraft and is usually the only approach taken to addressing free play in most commercial aviation. It the most well-established method, especially for operating aircraft whose economics often cannot permit redesign of parts or introduction of new methods to monitor free play via Approach #2.

The industry standard for assessing free play during maintenance inspections involves static testing while the aircraft is grounded. This testing procedure entails applying a load to a control surface and measuring the resulting linear or angular deflection. A commonly used measurement device for this purpose is the Rotation Variable Differential Transformer (RVDT) [24]. Kiiskila et al. note that free play testing can be labor-intensive, time-consuming, and may necessitate complex assembly for configuring test mechanisms, particularly for larger aircraft. Fig. 5 illustrates an example of one such test [25].

If excessive free play is found during maintenance, part replacement or re-lubrication of components is necessary. However, residual free play after such maintenance actions is common and worsens as an aircraft ages [16]. Consequently, older aircraft frequently experience a trend towards reduced intervals between maintenance checks and increased frequency of maintenance tasks. This leads to elevated long-term maintenance expenses and a diminishing effectiveness of preventive measures.

Additionally, significant ongoing research efforts are directed towards exploring diverse methods for identifying and diagnosing free play through Approach #2. However, documentation regarding the widespread implementation of various solutions remains limited. Tools for actively monitoring free play have been developed, as evidenced by a 2011 patent from The Boeing Company [26]. This patent outlines a dynamic measurement system integrated with a flight control computer, capable of detecting specific low and high-frequency vibrations corresponding to control surface motion both within and outside free play zones. Similarly, Urbano [16] detailed another monitoring tool in which a statistical software solution was created to detect free play-related vibrations using data from flight control computers onboard Airbus aircraft. Additional methods are discussed in the authoritative review on active flutter suppression technology by Livne [27]. Despite these advancements, overcoming technology gaps in this area remains a significant challenge.

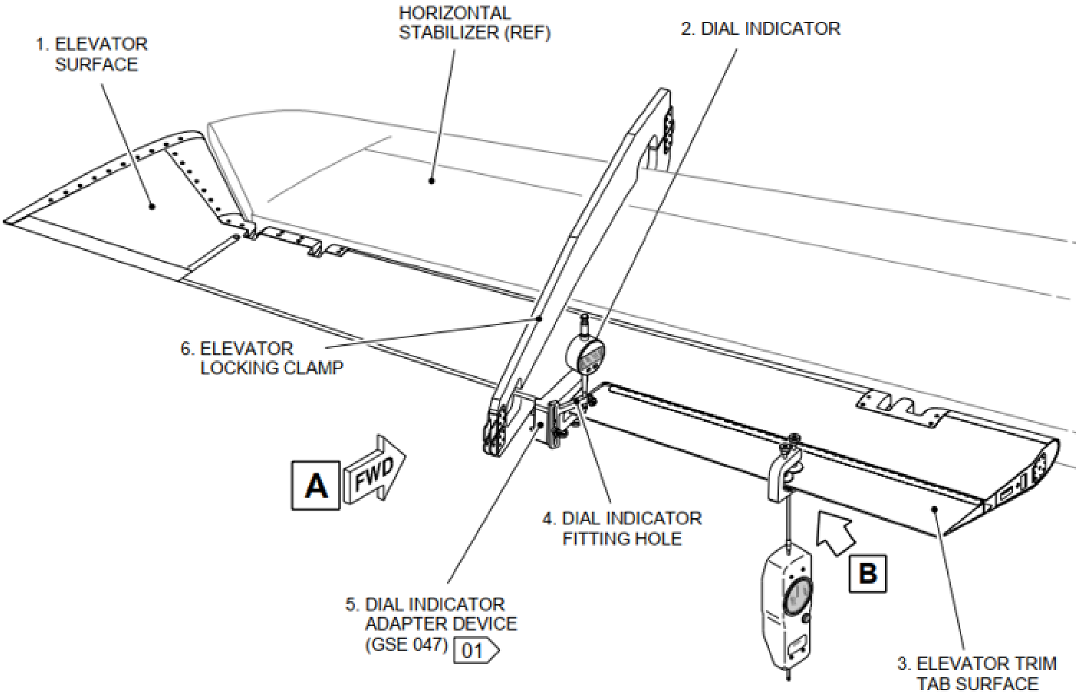
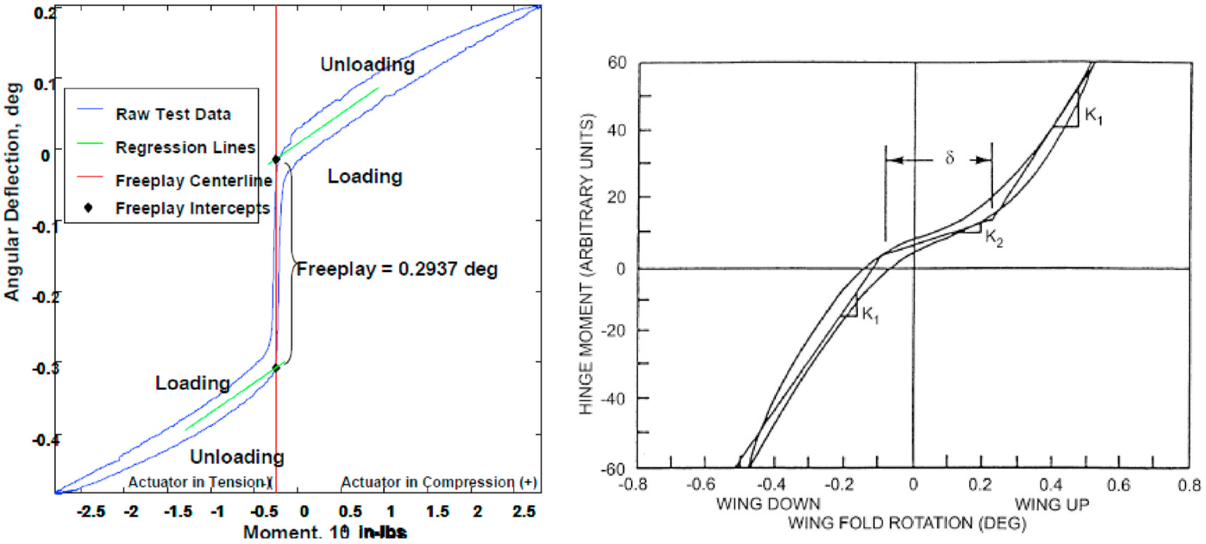


Fig. 5. Embraer-505 Elevator Autotab Backlash - Functional Check

Theoretical modeling of free play originates from the process of fitting mathematical functions to control surface stiffness. These functions aim to accurately represent typical moment-deflection curves derived from static free play tests, similar to those illustrated in Fig. 6. That displays moment-deflection curves obtained from free play tests on different components. In image (a), the curves depict the deflection of a F-22 flaperon, with deflection mapped on the y-axis. Image (b) shows moment-deflection curves from a free play test conducted on a wing-fold hinge of a CF-18 wing, with deflection mapped on the x-axis.



(a) (b)

Fig. 6. Moment-deflection curves : (a) F-22 flaperon, and (b) wing-fold hinge CF-18

These graphs reveal several distinct behaviors that deviate from the idealized model of free play depicted in Fig. 2. Firstly, both sets of test data exhibit hysteresis phenomena. Secondly, the free play observed in Fig. 6b is not centered around the zero rotation angle, likely due to non-uniform wear effects on the control surface mechanism. Thirdly, Fig. 6b may not display a flat spot nonlinearity. Finally, the free play curves appear to transition smoothly from inside to outside the free play region and display slight spring hardening curvatures in the stiffness beyond the free play region. These nuances in the moment-displacement graphs from free play testing data may stem from various underlying phenomena within the control surface, such as friction and damping. However, these effects could also largely arise from nonlinear structural stiffness, and thus could be incorporated in some form within the mathematical modeling of free play.

Theoretical studies often begin with an idealized, piecewise model of free play, especially when there's limited prior knowledge or data regarding free play nonlinearity at the outset of an aeroelastic investigation. This simplified model is commonly favored in initial design phases or exploratory analyses. The mo ment equation for the idealized free play model is [28]:

$$M(\alpha)=\begin{cases}K(\alpha+\delta) & \text{for } \alpha\geq\delta\\ 0 & \text{for } -\delta\leq\alpha\leq\delta,\\ K(\alpha-\delta) & \text{for } \alpha\geq\delta\end{cases} \tag{1}$$

where K represents the rotational stiffness coefficient outside of the free play region,

α denotes the pitching variable or rotational angle of deflection,

δ signifies the amount of free play angle referenced from the neutral position of the control surface (zero-centered). It's important to note that the moment equation for free play can be formulated with respect to any variable representing control surface displacement. Hence, Equation (1) serves as a representative model of Fig. 4, with a change in variable from α to β. Within the literature on nonlinear aeroelasticity, equation (1) is commonly adopted as the standard definition for zero-centered free play. This piecewise model of free play comprises three linear subdomains, resulting in a non-smooth function. In Fig. 4, subdomains (1) and (3) correspond to regions where stiffness is nominal, while subdomain (2) represents the region where zero-stiffness free play exists. When such piecewise linear functions are integrated into a set of governing equations of motion, analytical derivation of solutions becomes impractical. Specialized numerical methods are often necessary to accurately compute time-histories and conduct stability analyses.

Alternatively, the challenges associated with non-smooth free play models can be circumvented by transforming equation (1) into a smooth function through approximation techniques. One commonly used method is the rational polynomial approach, where the resulting moment equation takes the form of the ratio of two polynomials [28]:

$$M(\alpha)=\frac{a_0+a_1\alpha+a_2\alpha^2+a_3\alpha^3}{b_0+b_1\alpha+b_2\alpha^2}, \tag{2}$$

where a_i (i = 0, 1, 2, 3) and b_i (i = 0, 1, 2) are constants that allow for the selection of the best fit curve. Usually, these constants are obtained from experimental data. Another approach to approximating equation (1) is by using a tangent hyperbolic function, represented as [28]

$$M(\alpha)=\frac{\left[1-\tanh\varepsilon(\alpha+\delta)\right](\alpha+\delta)}{2}+\frac{\left[1-\tanh\varepsilon(\alpha-\delta)\right](\alpha-\delta)}{2}, \tag{3}$$

where ε is the smoothness variable. As $\varepsilon \rightarrow \infty$, equation (3) converges to the piecewise linear free play in equation (1). Asjes et al. [29] represented the hyperbolic model differently, but required similar limiting conditions to match the hyperbolic function with the piecewise function. A third good alternative is using equivalent linearized stiffness methods to produce an approximation of the piecewise model.

The discussion up to now has only considered free play as a flat spot nonlinearity in the moment-deflection graph centered about zero displacement of the control surface. As mentioned previously, variations in the shape and location of a free play model are expected in reality and may be important to consider for specific aeroelastic studies. In many cases, these variations can occur both progressively and frequently throughout an aircraft's life. Fig. 7a shows one such model where free play is both nonzero-centered and characterized by reduced stiffness. The nonzero-centered free play is typically the result of an aerodynamic preload being applied to a control surface. This scenario occurs commonly during normal flight due to changes in flight conditions (i.e., Mach number variation) and flight maneuvers (i.e., angle-of-attack variation). The preload causes the control surface's hinge-moment equilibrium position to shift into the nominal stiffness region outside of the location of free play [30]. Thus, the free play and the control surface equilibrium no longer coincide. In Fig. 7a, the preload amount is quantified as the distance M_0 between the control surface's equilibrium position and the nearest boundary edge of the free play region. Additionally, in Fig. 7a the free play region has a nonzero slope value of M. The bilinear free play moment function associated with Fig. 7a is obtained in [31]. Image (a) shows a bilinear free play model with a slight, sharp reduction in stiffness in the free play region before switching back to its nominal stiffness values outside. Image (b) shows a gradually changing stiffness that is nearly flat at the center

2. Aeroelastic Models

Aeroelastic modeling constitutes the second significant subtopic within free play research. These models mathematically capture the interactions between the dynamics of an aircraft structure and the aerodynamic forces it encounters.

The structural aspect of aeroelastic modeling typically involves approximations of an aircraft wing in its simplest structural form as a long, thin, and symmetric plate with uniformly distributed mass and structural properties. These models also assume that the plate has uniform transverse and rotational stiffness along its span. In many cases, the only boundary condition imposed on this plate is that one edge is fixed to a wall, representing the attachment of a wing's root to the fuselage body of an aircraft. Because of the uniformity and symmetry of such a plate, a comprehensive understanding of its dynamic behavior can be achieved by examining just a single cross-section, referred to as a typical wing section [32, 33]. This simplification reduces the 3D aeroelastic problem to a 2D problem through structural approximations. For instance, Fig. 8 illustrates a typical wing section without additional control surfaces like flaps and trim tabs. In scenarios where the wing possesses all-moving capabilities and can rotate entirely about its root, the wing itself can function as a control surface. Typical wing section is shown here with the two possible degrees of freedom in pitch α and plunge h. Modified versions of these airfoils can be used to model different model different aircraft control mechanisms and parts.

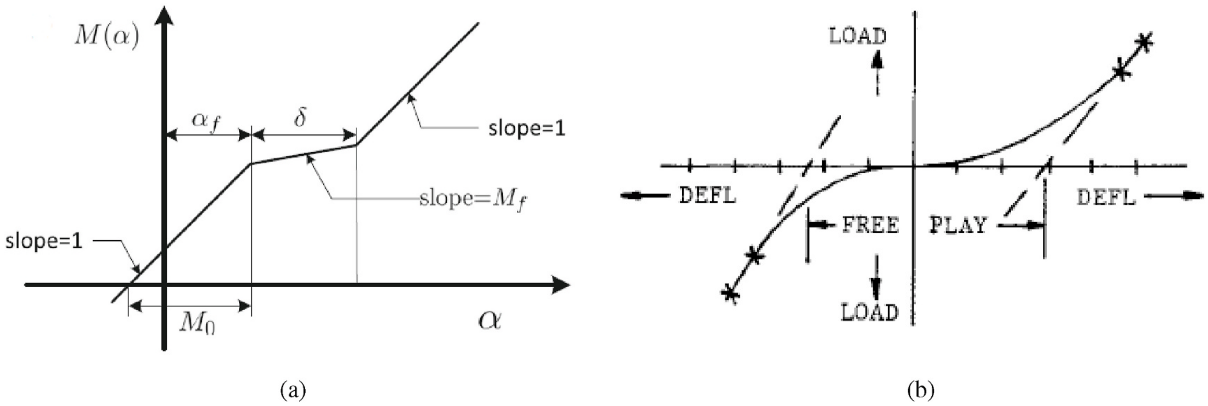


Fig. 7. (a) Bilinear free play model; (b) Gradually changing stiffness

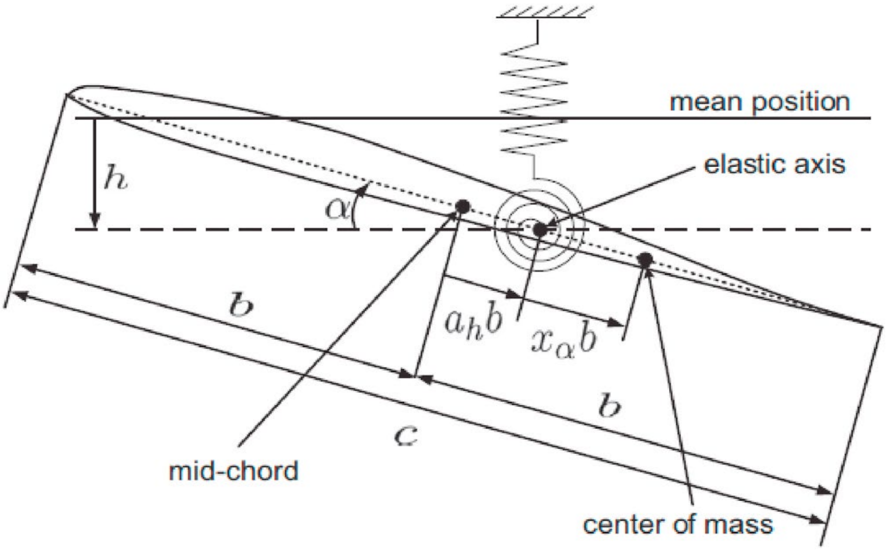
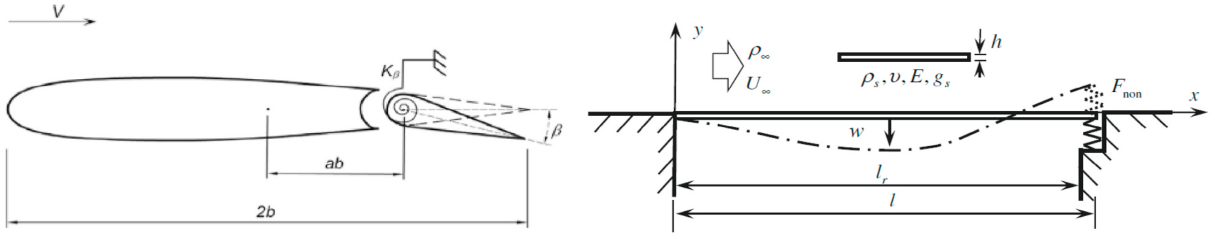


Fig. 8. Typical aeroelastic wing section

In Fig. 8, two types of airfoil motion are possible: rotational (or pitching), denoted by α, and bending (or plunging), denoted by h. The wing's bending and torsional stiffness properties are associated with each degree of freedom. Both pitching and plunging displacements are referenced from the wing's elastic axis, located chordwise at a distance $a_h b$ relative to the airfoil's midchord. Additionally, information regarding the chordwise location of the wing's center of mass is provided in the figure.

Typical wing section models serve as potent analytical tools, leveraging the renowned thin airfoil theory of aerodynamics. Initial assumptions involve considering incompressible, inviscid airflow around an airfoil of negligible thickness (and infinite span). To simplify further, it is assumed that the airfoil is rigid, with no airflow penetration, and limited to small amplitude oscillations, thereby maintaining the assumption of attached airflow on the airfoil's trailing edge. Expanding existing aeroelastic models to include additional degrees of freedom, more precise aerodynamic considerations, well-defined constraints, and suitable initial and boundary conditions can enhance the accuracy of aircraft flight descriptions, better capturing its true nature.

The most basic aeroelastic models utilized in free play investigations are those where free play's physical impact generates isolating effects primarily around a single degree of freedom in a structure's response. It is classified this type of research as "single degree of freedom" within the scope of free play studies. Typically, such investigations explore phenomena associated with specific, localized structural aspects of an aeroelastic system. This subsection provides two examples. The corresponding structural models can range from simple one-degree-of-freedom models, as depicted in Fig. 9a, to more intricate continuous, infinite-degree-of-freedom structures illustrated in Fig. 9b. Figure illustrates two structural models: Image (a) depicts a structural model for control surface buzz, featuring a fixed wing with a rotating control surface experiencing pitching motion about the hinge line of the control surface airfoil; Image (b) illustrates a continuous, infinite degree-of-freedom structural model of a plate with free play in the right boundary, representing loose aircraft skin. The free play affects only the single plunging degree of freedom along the longitudinal direction of the plate model.



(a) (b)

Fig. 9. (a) Structural model of control surface buzz; (b) Structural model of a plate with free play

In the first example, corresponding to Fig. 9a, He et al. [34] investigated the phenomenon of control surface buzz for a control surface with and without free play. Control surface buzz is a single degree-of-freedom flutter instability that occurs in control surfaces during transonic flight due to the presence of shock waves on the wing or the control surface itself. This phenomenon primarily affects the control surface flap rotation degree-of-freedom β on a wing, where the motion relative to the flap is insignificant. Hence, the structural model employed is as shown in Fig. 9a. The governing equation of motion is [34]:

$$I_\beta \ddot{\beta} + M(\beta) = 2\rho V^2 b^2 c_m \tag{4}$$

where I_β is the control surface moment of inertia;

ρ denotes the air density;

V stands for the airspeed;

b is the reference half-chord length of the airfoil;

c_m represents the aerodynamic pitching moment coefficient about the hinge line. In equation (4), (β) represents the system's hinge-moment function, where the defined free play moment equation. In nonlinear aeroelasticity, it's common practice to convert the equation of motion into a dimensionless form. This transformation makes it easier to identify crucial parameter groups that have the most significant influence on the model's aeroelastic response. Additionally, non-dimensional analytical results facilitate correct scaling and similitude, essential for conducting accurate wind tunnel experiments and flight tests. Transforming equation (4) into a non-dimensional form results in:

$$\frac{\ddot{\beta}}{\omega_\beta^2} + \frac{M(\beta)}{K_\beta} = \frac{2U^2}{\pi\mu r_\beta^2} c_m \quad (5)$$

where ω_β is the frequency of oscillation for the control surface given as the ratio of K_β/I_β,

K_β is the rotational stiffness coefficient control surface,

μ is the mass ratio defined by $\mu = m/\pi\rho b^2$,

r_β is the radius of gyration,

U is the reduced (non-dimensional) velocity given by $U = V/b\omega_\beta$.

It's worth noting that the aeroelastic model described in equation (5) assumes no structural damping at the hinge line where the control surface flap attaches to the wing. True structural damping is often omitted across much of the literature due to the complexity it introduces. Structural damping within most aircraft systems is a nonlinear and intricate phenomenon [2]. If included, it would become challenging to isolate specific dynamical behavior attributable to free play nonlinearities alone. Consequently, a majority of free play research does not incorporate structural damping. The trade-off is a reduction in the accuracy of the actual aeroelastic model and more conservative predictions of aeroelastic instability speeds. Additionally, aerodynamic damping tends to have a more significant impact on system response than structural damping [16, 35]. Thus, the assumption of no structural damping is generally deemed acceptable for flutter analysis.

3. Means of Analysis

In the realm of aeroelasticity, a significant portion of the literature leans towards deterministic methods. However, nonlinear deterministic calculations pose challenges, as few purely analytical methods can address all states of dynamic instabilities. Nonetheless, numerous computational advancements have been integrated into aeroelasticity, catering to both deterministic and non-deterministic approaches.

The literature on free play shows two predominant analysis approaches: time-domain methods and frequency-domain methods. The time-domain approach requires numerically integrating the aeroelastic system in the state space for different airspeeds and parameters of interest. For each time history produced, additional data is numerically extracted from the system's steady state oscillatory response to determine and characterize the structural behavior over a long time period. This process is repeated for many velocity and parameter increments until a detailed understanding is gained of how the system states evolve. The time-domain approach can be tedious and computationally intensive. On the other hand, the frequency-domain methods can be quicker because they only involve looking for solutions that are periodic and dominated by a few number of harmonics. Using a finite number of Fourier series components, various semi-analytical approaches can be developed to approximate such solutions with good accuracy. These procedures are often referred to as the describing functions technique or the harmonic balance approach [36]. The strength of the harmonic balance approach is that it can be successfully scaled to high-dimensional models with many degrees of freedom, as done by refs. [37, 36, 38].

Four sources of uncertainties in formulating the free play problem include stochastic variations in material properties, structural dimensions, boundary conditions, and external excitations [39, 40, 41].

In free play research, studies typically begin by examining the behavior of the aeroelastic system with linear structural stiffness. Subsequent nonlinear dynamic analyses are then carried out on equivalent aeroelastic systems, now including free play, to derive their results. These results are subsequently compared with those of the baseline linear system. A significant consensus across the literature is that the presence of free play can trigger premature flutter instability, leading to LCO.

Flutter occurs due to the presence of negative damping coefficient terms associated with the structural modes of the aeroelastic system. In nonlinear systems, LCO manifest as post-stability phenomena following flutter. Hence, the occurrence of LCO is influenced by various factors, including inertial coupling, global aeroelastic damping and stiffness characteristics, all of which can be significantly affected by nonlinear aerodynamic effects and other structural nonlinearities. Comprehensive discussions on the mechanisms of nonlinear aeroelastic response can be found in references [35, 32, 42]. In the context of free play, the magnitude, shape, and location of the nonlinearity directly impact different characteristics of the observed LCO response as parameters of the aeroelastic system are varied. Free play-related effects encompass both predictable and unpredictable changes in the amplitudes, stability, and periodicity of the LCOs. Contrary to early historical understanding, the starting velocity of LCOs and the size of free play are found to be independent of each other. Conner et al. [43] were the first to comprehensively demonstrate many of these ideas through both theory and experiments.

Fig. 10 depicts experimental results revealing two significant observations. Firstly, the magnitude of LCO amplitudes scales with increasing free play gap sizes. Secondly, the starting velocity (approximately 0.18) of the LCO remains unaffected by the varying free play gap sizes in the flap. This conclusion is drawn from observing that the same numerical solution predicts the aeroelastic response three times for the three different magnitudes of free play, with the LCO amplitudes normalized by the free play gap size. Therefore, only one "universal" numerical prediction for the aeroelastic system is evident in Fig. 10. As the predicted onset velocity of the LCO consistently occurs at 18% of the linear flutter velocity for all three free play gaps, we can conclude independence between the magnitude of free play and the starting velocity of the LCO. This conclusion is supported by experimental data, which aligns fairly accurately with numerical predictions. These findings are generalizable to various aeroelastic configurations utilizing linear unsteady aerodynamics and similar models of free play.

4. Hopf Bifurcations and Stability of LCOs

A closer analysis of Fig. 10 reveals additional intriguing and intricate behavior in the predicted response of the nonlinear system. Specifically, it is evident that the mapping of the peak LCO amplitudes exhibits several discontinuous jumps at specific velocity ratios. These jump phenomena arise from the nonlinear aeroelastic system encountering both Hopf and fold bifurcations. Understanding these bifurcation phenomena is crucial for unraveling the complex behavior and stability of limit cycles, which arises due to the presence of free play in the aeroelastic system.

Hopf bifurcations, as illustrated in Fig. 11, are responsible for generating limit cycles. They occur when the complex conjugate eigenvalues of a system's state-space fixed point become purely imaginary. Indeed, the stability characteristic of the fixed point undergoes a change. This alteration in stability can occur for both increasing and decreasing airspeeds. Image (a) illustrates both supercritical and subcritical Hopf bifurcations. Image (b) depicts the subcritical Hopf bifurcation, which is detrimental to both increasing and decreasing airspeed. In the forward direction, it can lead to the jump phenomena observed in Fig. 10. Conversely, in the reverse direction, LCO may persist at airspeeds below the identified flutter speed.

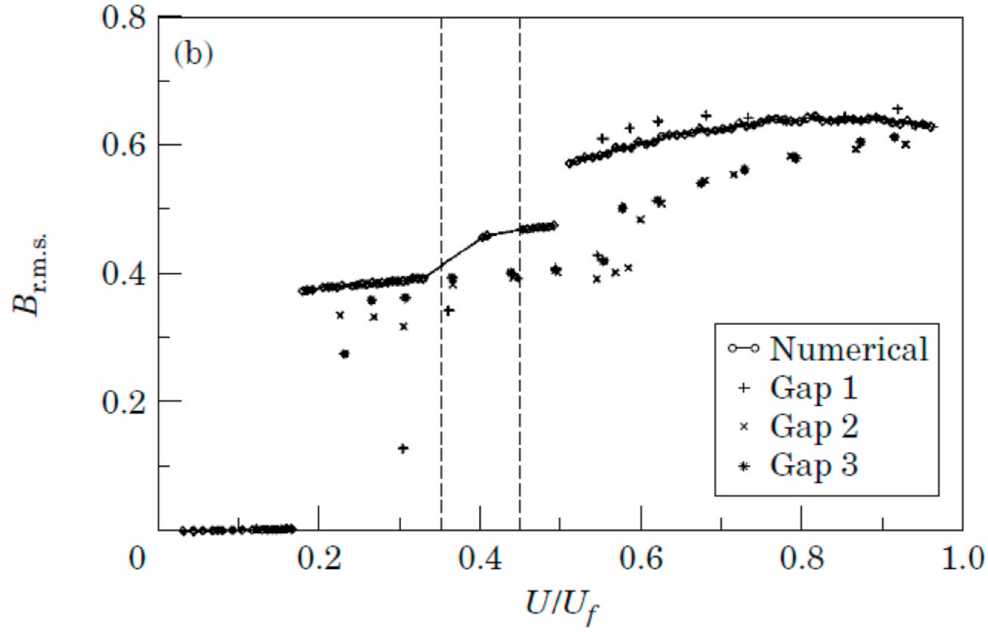
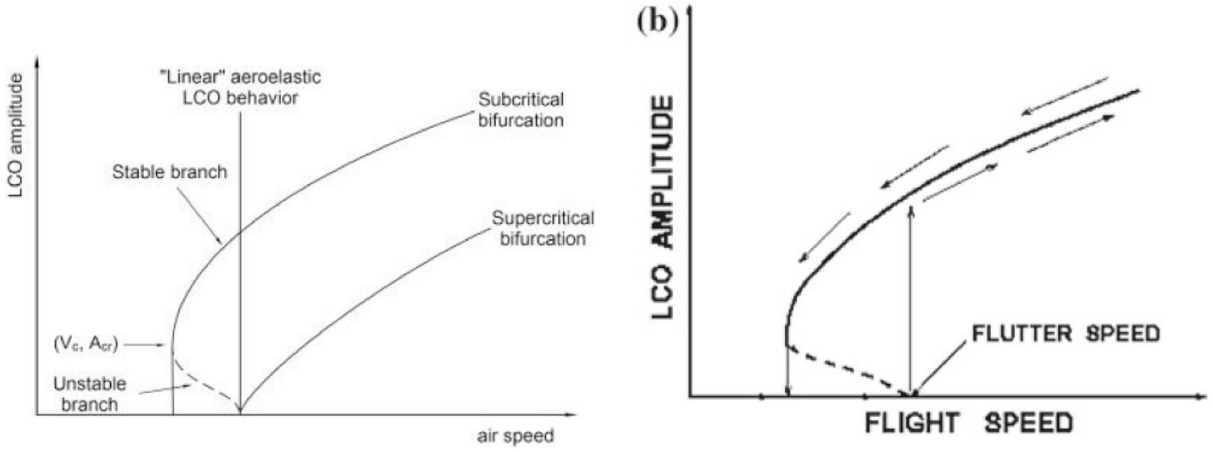


Fig. 10. Experimental results LCO in the flap.

In a "hard-flutter" scenario, reducing the airspeed to the pre-flutter flight speed doesn't promptly halt the LCO vibration. Instead, as depicted in Fig. 11b, the system will persist on the stable branch of the subcritical Hopf bifurcation, extending considerably below the flutter airspeed. Reducing the airspeed diminishes the LCO amplitude, but doesn't eradicate it until the system reaches a fold bifurcation. At this point, the stable branch becomes unstable, as indicated by the dashed line. Further reduction in airspeed causes the system response to eventually return to the stable zero fixed point, attenuating the vibration.



(a) (b)

Fig. 11. (a) supercritical and subcritical Hopf bifurcations; (b) subcritical Hopf bifurcation with increasing and decreasing airspeed.

Bifurcation diagrams serve as valuable tools for examining the evolving characteristics of a system's LCO response. An example of a bifurcation diagram is depicted in Fig. 12, where steady-state fixed points intersecting in the state-space for multiple oscillation cycles are plotted at discrete velocities. Fig. 12 illustrates a bifurcation diagram plotting the local minima and maxima of α for a two-degree-of-freedom aeroelastic system (Theodorsen's unsteady aerodynamics) across a range of discrete non-dimensional velocities. Periodic motion at a specific velocity will result in a few fixed points on the vertical axis, indicating repeated constant amplitude oscillations. If the motion is periodic, its periodicity can be estimated as half of the total number of fixed points.

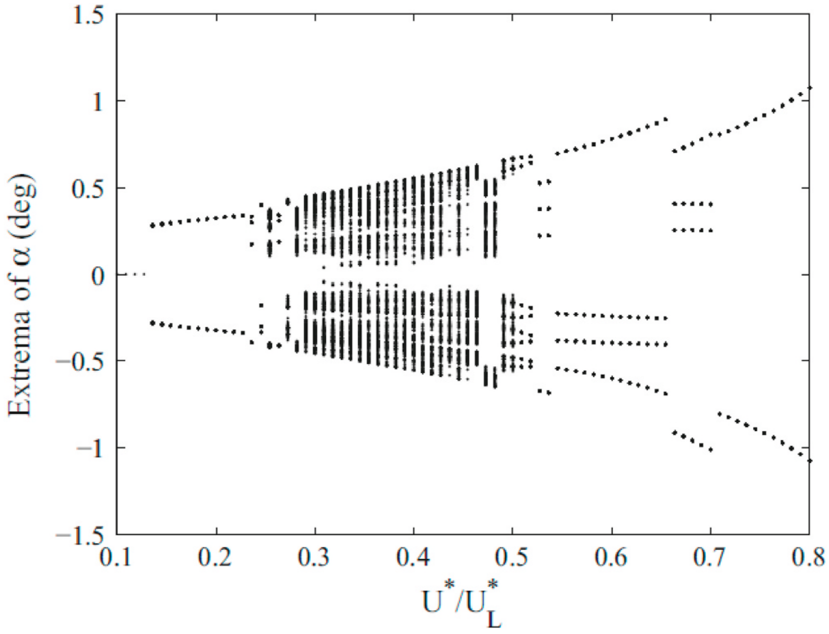


Fig. 12. Bifurcation diagram with the local min. and max. of α

Conclusions

To summarize free play gives rise to a diverse range of periodic, aperiodic, and chaotic responses across various types of aeroelastic systems. Nonlinearities can generate multiple coexisting stable and unstable limit cycles, various bifurcations at different airspeed ranges. These phenomena can result in asymmetry and intermittent shifts in the response structure. A rigorous mathematical assessment is essential to understand the theoretical mechanisms underlying the intricate and complex free play responses. This includes conducting bifurcation analyses, LCO branch calculations, stability analyses, and time-series analyses. The literature on free play exhibits a comprehensive understanding of the diverse types of vibration and aeroelastic responses stemming from free play. Even across different aeroelastic systems, including those involved in transonic and supersonic flight, there are qualitative similarities in the predicted responses. While implicitly acknowledged, the anticipated effects of free play in modern aircraft are typically transient. Due to the dynamic nature of aircraft flight, which introduces varying preload, the implementation of numerous control laws designed for stable flight, and established industry maintenance practices, sustaining the effects of free play for extended durations during flight is nearly impossible. Despite its transient nature, free play-induced vibration, as outlined in this section, carries both short-term and long-term consequences in aircraft flight. One of the shortcomings in the literature is that the findings from existing free play studies are not directly applicable to assessing ongoing aircraft flight operations. While these studies effectively predict various negative behaviors that a wing or control surface with free play may experience, most existing research fails to illustrate how these effects can escalate into more severe consequences during aircraft flight. Many researchers cite this gap as the original motivation for studying the problem.

The objectives of further research are to develop methods for mathematical modeling of aeroelastic vibrations of a wing in the presence of free play in the mechanical wiring of control surfaces taking into account a greater number of degrees of freedom, in particular, changes in pitching and plunging along the wing span; analysing of aeroelastic stability of a wing with control surface play; developing of practical safety criteria and recommendations to demonstrate compliance with the provisions of the Aviation Regulations relating to the design standards for transport category aircraft in relation to the prevention of aeroelastic instability. In particular, new results can be expected from the application of the continuation method [44] to the study of the stability of wing oscillations.

References

1. Bisplinghoff, R. L. Aeroelasticity / R. L. Bisplinghoff, H. Ashley, R. L. Halfman. (2nd ed.). Dover Publications, 1996. - 860 p.
2. Chen F. Flutter Analysis of an Airfoil with Nonlinear Damping Using Equivalent Linearization / F. Chen, J. Liu, Y. Chen // Chin. J. Aeronaut. – 2014. – V. 27. P. 59–64. https://doi. org/10.1016/j.cja.2013.07.020.
3. Eller, D., Friction, Free Play and Flutter of Manually Controlled Aircraft, in: International Forum on Aeroelasticity and Structural Dynamics. Stockholm, Sweden. – 2007, P. 1–16.
4. Tan, T. Stability Analysis of an Aeroelastic System with Friction / T. Tan, M. Li, B. Liu // Chin. J. Aeronaut. – 2013. – V. 26. – P. 624–630. https://doi.org/10.1016/j.cja.2013.04.051
5. F. A. Administration, Means of Compliance with title 14 CFR Part 23, Section 23.629 flutter (AC 23.629-1B), 2004, – 239 p.
6. F. A. Administration, Aeroelastic Stability Substantiation of Transport Category Airplanes (AC 25.629-1B), 2014, - 19 p.
7. Rosenbaum, R., Vollmecke, A. A., Simplified Flutter Prevention Criteria for Personal Type Aircraft, Airframe and Equipment Engineering. Report 45, Federal Aviation Administration, 1955. – 27 p.
8. F.A. Administration, Airworthiness Directives; Boeing Model 737 Airplanes, Federal Register Document E6-16553, Federal Aviation Administration, 2006, P. 59368–59372.
9. Embraer-505 Revision 01 to Alert Service Bulletin 505-55-0004, Revision 01: 24/Jun/2020. – P. 3 –71.
10. F. A. Administration, Airworthiness Directives; Boeing Model 747 Airplanes, Federal Register Document 05-20268, Federal Aviation Administration, 2005, P. 58623–58626.
11. F. A. Administration, Airworthiness Directives; Boeing Model 767 Airplanes, Federal Register Document E7–18544, Federal Aviation Administration, 2007, P. 53701–53704.
12. Abdelkefi, A. Performance Enhancement of Wing-Based Piezoaeroelastic Energy Harvesting through Free Play Nonlinearity / A. Abdelkefi, Hajj M. R. //

Theoretical and Applied Mechanics Letters. – 2013. – V. 3 - P. 1–4. https//oi.org/10.1063/2.1304101.

13. Mannarino, A., Nonlinear Aeroservoelasticity: Reduced Order Modeling and Active Control, Doctoral Dissertation, Politecnico Di Milano, 2016. – P.2–6. URL, http://hdl.handle.net/10589/117844.

14. Shukla, H. Control of Limit Cycle Oscillation Amplitudes in Nonlinear Aerelastic Systems Using Nonlinear Normal Modes / H. Shukla, Patil M. // American Institute of Aeronautics and Astronautics, 2017. – P. 1–14. https://doi.org/10.2514/6.2016-2004.

15. Kiiskila, J. C. Investigation of the Dynamic Characterization of Aircraft Control Surface Free Play, in: Modal Analysis Conference; IMAC XXIV / J. C. Kiiskila, M. R. Duncan, D. M. Pitt: St. Louis, Missouri, USA. 2006, Society for Experimental Mechanics. – V. 1. – P. 31–42.

16. Urbano, S. Detection and Diagnostic of Free Play Induced Limit Cycle Oscillation in the Flight Control System of a Civil Aircraft, Doctoral dissertation, Universit´e de Toulouse, 2019.- 140 p. URL, https://oatao.univ-toulouse.fr/24352/.

17. Danowsky, B. Nonlinear Analysis of Aeroservoelastic Models with Free Play Using Describing Functions / B. Danowsky, P.M. Thompson, S. Kukreja // J. Aircraft. – 2013. V.– 50. – P. 329–336, https://doi.org/10.2514/1.C031370.

18. U.S. Military, Military Specification: Airplane Strength and Rigidity Vibration, Divergence, Flutter, 1993. – 52 p.

19. Chen, P. C. Nonlinear Flutter Analysis for the Scaled f-35 with Horizontal-Tail Free Play / P.C. Chen, E. Ritz, N. Lindsley // J. Aircraft. – 2014. – V. 51. – P. 883-889, https://doi.org/10.2514/1.C032501.

20. Anderson, W. D., Maximum Control Surface Free Play, Design and Flight Testing Approach on the F-22 / 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2007. – P. 1–12, https://doi.org/10.2514/6.2007-1767.

21. Frumusa, M. Investigation on the Effects of Control Surface Free Play on the Aeroelastic Characteristics of a Trainer Aircraft and Extension of Limits in Support of Maintenance Tasks, in: International Forum on Aeroelasticity and Structural Dynamics / M. Frumusa, V. Vaccaro: 2019, P. 1–12.

22. Asjes, D. C., Nonlinear Analysis of a Two- and Three-Degree-Of-Freedom Aeroelastic System with Rotational Stiffness Free-Play, Doctoral dissertation, Iowa State University, 2015. URL,-145 p. http://lib.dr.iastate.edu/etd/14317.

23. Chen, P. C., Lee, D. H., Flight-Loads Effects on Horizontal Tail Free-Play-Induced Limit Cycle Oscillation / P.C. Chen, D.H. Lee // J. Aircraft. – 2008. – V. 45. – P. 478–485, https://doi.org/10.2514/1.29611.

24. Kiiskila, J. C. Investigation of the Dynamic Characterization of Aircraft Control Surface Free Play, In: Modal Analysis Conference; IMAC XXIV / J. C. Kiiskila, M. R. Duncan, D. M. Pitt: St. Louis, Missouri, USA. 2006, Society for Experimental Mechanics. – V. 1. – P. 31–42.

25. Aircraft Maintenance Manual, P/N 2757, Rev.84, dated 23 Feb 2024, System Description Section, Subject 27-24-00 - Rudder Trim System,- P.1-16.

26. Pitt, D. M., Kiiskila, J. C., System for and Method of Monitoring Free Play of Aircraft Control Surfaces, 2011. United States Patent No. 7933691 B2,- P.1-12

27. Livne, E., Aircraft Active Flutter Suppression: State of the Art and Technology Maturation Needs, J. Aircraft. – 2018 – V. 55 – P. 410–450, https://doi.org/10.2514/1.

28. Pereira, D. A. Effects of Combined Hardening and Free-Play Nonlinearities on the Response of a Typical Aeroelastic Section / D. A. Pereira, R. M. G. Vasconcellos, M. R. Hajj, F. D. Marques // Aero. Sci. Technol. – 2016. – V. 50. – P. 44–54. https://doi.org/10.1016/j. ast.2015.12.022.

29. Asjes, D. C Development and System Analysis of a Two-Dimensional Rotational Free play Nonlinearity Model / D. C. Asjes , A. Diwadkar, U. Vaidya, A. Kelkar // J. Aircraft. – 2016. –V. 53. – P. 860–865, https://doi.org/10.2514/1.C033275.

30. Kholodar, D. B. Nature of Freeplay-induced Aeroelastic Oscillations / D. B. Kholodar // J. Aircraft – 2014. – V. 51. – P. 571–583. https://doi.org/10.2514/1.C032295.

31. Lee, B. H. K. Nonlinear Aeroelastic Analysis of Airfoils: Bifurcations and Chaos / B. H. K. Lee, S. J. Price, Y. S. Wong // Prog. Aero. Sci. – 1999. – V. 35. – P. 205–334. https://doi.org/10.1016/S0376-0421(98)00015-3

32. Dowell, E. H. A Modern Course in Aeroelasticity, Solid Mechanics and its Applications / E.H. Dowell: Springer International Publishing Switzerland, 2015. 828 p.

33. Hodges, D. H. Introduction to Structural Dynamics and Aeroelasticity / D.H. Hodges, G.A. Pierce: Cambridge University Press, New York, NY, 2002. 247 p. https://doi.org/10.1017/CBO9780511997112.

34. He, S. Limit Cycle Oscillation Behavior of Transonic Control Surface Buzz Considering Free-Play Nonlinearity / S. He, Z. Yang, Y. Gu // J. Fluid Struct. – 2016. – V. 61. –P. 431–449. https://doi.org/10.1016/j.jfluidstructs.2015.11.014.

35. Dimitriadis, G. Introduction to Nonlinear Aeroelasticity / G. Dimitriadis, P. Belobaba, J. Cooper, A. Seabridge: John Wiley &Sons Ltd, 2017. 592 p. https://doi.org/10.1002/9781118756478.

36. Liu, L. Harmonic Balance Approach for an Airfoil with a Free play Control Surface / L. Liu, E. Dowell // AIAA J. – 2005. – V. 43. – P. 802–815.

37. Padmanabhan, M. A. Store-Induced Limitcycle Oscillations Due to Nonlinear Wingstore Attachment / M.A. Padmanabhan, E.H. Dowell, J.P. Thomas, C.L. Pasiliao / J. Aircraft. – 2016. –V. 53. – P. 778–789, https://doi.org/10.2514/1.C033577.

38. Fichera, S. High Order Harmonic Balance Applied to an Aeroelastic T-Tail Model with a Control Surface Free play, in: 54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference / S. Fichera, S. Ricci : 2013, 11 p. https://doi.org/10.2514/6.2013-1704.

39. Yi, L. Uncertainty Quantification in Flutter Analysis for an Airfoil with Preloaded Free play / L. Yi, Y. Zhichun // J. Aircraft. – 2010. – V. 47. – P. 1454–1457. DOI: 10.2514/1. C031011.

40. Ibrahim, R. A. Nonlinear Stochastic Flutter of a Cantilever Wing with Joint Relaxation and Random Loading. Report / R. A. Ibrahim, R. F. Gibson: Wayne State University, 2008. 152 p.

41. Peng, C. Numerical Investigation of the Effects of Structural Geometric and Material Nonlinearities on Limit-Cycle Oscillation of a Cropped Delta Wing / C. Peng, J. Han // J. Fluid Struct. – 2011. – V. 27, – P. 611–622. DOI:10.1016/j.jfluidstructs.211.03.015.xz.

42. Wright, J. R. Introduction to Aircraft Aeroelasticity and Loads / J. R. Wright, J. E. Cooper: Aerospace Series, John Wiley & Sons, Ltd, Chichester, UK, 2007. 543 p. DOI:10.1002/9781118700440.

43. Conner, M. D. Nonlinear Behavior of a Typical Airfoil Section with Control

Surface Free play: a Numerical and Experimental Study / M. D. Conner, D. Tang, E. Dowell, L. N. Virgin // J. Fluid Struct. – 1997. – V. 11. – P. 89–109.

44. Seyde,l R. Nonlinear computation / R. Seydel // International Journal of Bifurcation and Chaos. – 1997. – V. 7, – N. 9. – P. 2105–2126.

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Огляд досліджень та обґрунтування нових математичних методів моделювання аеропружності крила з люфтом керуючих поверхонь

Явище нестійкості у польоті через взаємодію аеродинамічних, пружних та інерційних сил може виявитися катастрофічним. Тому належне значення надається аеропружному аналізу літака на стадії проектування. Вільний хід, званий люфтом, виникає в результаті зношування деталей, таких як вуха або рухомі кріплення, часто через старіння літака. Надмірний люфт шкідливий для безпеки літака. Це може призвести до катастрофічних аеропружних нестійкостей, таких як коливання граничного циклу (LCO), явище, пов'язане із стійкою вібрацією фіксованої амплітуди. LCO може прискорити накопичення втомних ушкоджень конструкції та поставити під загрозу керованість під час польоту. Системи управління літаком відіграють найважливішу роль у забезпеченні стійкості, маневреності та безпеки польотів. За останні кілька десятиліть нелінійна аеропружність привернула дедалі більшу увагу. Розширені обчислювальні можливості стимулювали відродження інтересу серед дослідників до розгляду проблеми люфту керуючої поверхні з метою удосконалення аналітичних моделей для більшої точності. Як сучасний теоретичний, так і експериментальний аналіз люфт відкрив нові можливості дослідження аеропружності. Незважаючи на розуміння фундаментальної проблеми, що здається, і наявність численних підходів до моделювання, дослідження люфта продовжують генерувати різноманітні прогнози щодо аеропружної динамічної поведінки і пов'язаних з ним властивостей. Люфт, який визначається як провисання або переміщення механічних зв'язків між входами керування та поверхнями керування літальним апаратом, є повсюдним аспектом проектування систем керування. Хоча невеликий люфт може бути терпимим, надмірний люфт може призвести до нелінійності та невизначеності, потенційно ставлячи під загрозу льотні характеристики та безпеку. Метою статті є огляд існуючих методів чисельних досліджень, аналіз нових математичних методів для моделювання аеропружних коливань та постановка завдання з розробки критеріїв безпеки літаків транспортної категорії в частині запобігання пружній нестійкості за наявності люфтів в механізмі управління.

Ключові слова: вільний хід (люфт), поверхня керування, флаттер, граничні циклічні коливання, ступінь вільності, нелінійна аеропружність.

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