

## Center-of-Gravity Position Variation due to Fuel Utilization Influence on Airplane Aerodynamic and Thrust Performance

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Algorithm for aerodynamic and thrust performance calculation of transport category airplanes at cruise flight mode taking into account its center-of-gravity variation in the process of fuel utilization is presented. Center-of-gravity position graph calculation for the A-310-200 airplane at cruise flight mode was done for example. Adequacy of the developed algorithm is shown by means of the calculating values of cruise parameters comparison with the values at two fixed center-of-gravity positions.

**Key words:** center-of-gravity, center of pressure, aerodynamic center, fuel trim transfer, required thrust, aerodynamic drag, lift force coefficient, angle-of-attack, engine throttling coefficient, lift-to-drag ratio.

### Introduction

Now, the problem to minimize fuel consumption required for a transport category airplane is one of the most actual. One of the ways to decrease fuel consumption is increase of airplane lift-to-drag ratio at cruising mode. For this purpose, airplane center-of-gravity (CG) control by means of fuel trim transfer (FTT) is already applied in some foreign airliners (A-310, A-330, A-340, A-380, B-747 etc.), but in the domestic practice this way is not still used. Airplane CG calculation taking into account fuel migration, presence of anti-surge ribs, fuel burn schedule and FTT was considered in publications [1-5]. One more problem, which must be discussed during FTT system development, is accounting of the airplane CG variation caused by fuel utilization influence on airplane aerodynamic and thrust performance, such as angle-of-attack (AOA) at the cruise mode, required lift force coefficients, thrust and engine throttling coefficient, and also airplane lift-to-drag ratio.

**The aim** of the publication is airplane CG position variation due to fuel utilization influence accounting on its aerodynamic and thrust performances.

### 1. Problem Statement

The expression for the required engine thrust taking into account actual CG position was found in the publication [6]:

$$P_{req} = 0.7 p_H M^2 S C_{x0} + \frac{(mg)^2}{0.7 p_H M^2 S} \times \left[ A_{WHT} \left( \frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2 + A_{HT} \frac{S}{S_{HT}} \left( \frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2 \right], \quad (1)$$

where  $p_H$  — is the atmospheric pressure at the flight altitude;  $M$  — is the Mach flight number;  $S$  and  $S_{HT}$  — are the areas of wing and of the horizontal tail (HT), correspondingly;  $C_{x0}$  — is the airplane drag coefficient at zero lift;  $m$  — is the airplane current mass;  $A_{WHT}$  and  $A_{HT}$  — are the drag-due-to-lift factors of airplane without HT

and of separate HT;  $x_{CG}$  — is the airplane CG position relatively wing MAC leading edge;  $x_{pWHT}$  — is the center of pressure (CP) position of the airplane without HT relatively wing MAC leading edge;  $x_{pHT}$  — is the CP position of separate HT relatively HT MAC leading edge;  $\Delta L$  — is the distance between MAC leading edges of wing and HT.

CG coordinate is a known function of the current mass and pitch angle  $\nu$  [5] (which is equal to the airplane angle-of-attack  $\alpha$  at cruise flight)

$$x_{CG} = f(m, \nu). \quad (2)$$

Airplane flight performance  $C_{x0}$ ,  $A_{WHT}$  and  $A_{HT}$  are determined by the airplane shape and its flight mode (altitude and Mach number), and therefore they do not depend on airplane CG variation.

CP position of the airplane without HT and separate HT are determined by the airplane shape, flight mode and the angle-of-attack:

$$x_{pWHT} = b_a \left[ \bar{x}_{FaWHT} + \frac{m_{z0WHT}}{C_{yaWHT}^\alpha (\alpha - \alpha_{0WHT})} \right], \quad (3)$$

$$x_{pHT} = b_{aHT} \left[ \bar{x}_{FaHT} + \frac{m_{z0HT}}{C_{yaHT}^\alpha (\alpha - \alpha_{0HT})} \right], \quad (4)$$

where  $b_a$ ,  $b_{aHT}$  — are the MAC of wing and HT correspondingly,  $\bar{x}_{FaWHT}$ ,  $\bar{x}_{FaHT}$  — are the coordinates of aerodynamic centers of the airplane without HT and separate HT correspondingly,  $m_{z0WHT}$ ,  $m_{z0HT}$  — are the coefficients of pitching moment at zero lift of the airplane without HT and separate HT correspondingly,  $C_{yaWHT}^\alpha$ ,  $C_{yaHT}^\alpha$  — are the derivatives of lift coefficient of the airplane without HT and separate HT correspondingly,  $\alpha_{0WHT}$ ,  $\alpha_{0HT}$  — are the zero lift angles-of-attack of the airplane without HT and separate HT correspondingly.

Thus, to calculate the required thrust, it is necessary to find the airplane angle-of-attack under the current flight mass and CG position. Formulas for the required lift coefficient of the airplane without HT and separate HT correspondingly were found in publication [6]

$$C_{yaWHT} = \frac{mg}{0.7 p_H M^2 S} \left[ \frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right]. \quad (5)$$

$$C_{yaHT} = \frac{mg}{0.7 p_H M^2 S_{HT}} \left[ \frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right]. \quad (6)$$

In the same time, the lift coefficient is related with the angle-of-attack by known formula

$$C_{yaWHT} = C_{yaWHT}^\alpha (\alpha - \alpha_{0WHT}), \quad (7)$$

whence the cruise flight angle-of-attack can be determined

$$\alpha = \alpha_{0WHT} + \frac{mg}{0.7 p_H M^2 S C_{yaWHT}^\alpha} \left[ \frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right]. \quad (8)$$

As airplane CG and CP coordinates depending on its AOA enter into the right part of the formula (8), then the AOA computation is provided by the iteration procedure together with correction of CG position by formula (2) and CP by formulas (3) and (4) for each current flight mass value. After the calculation of CG and CP coordinates, their values are substituted into formulas (5) and (6) to determine coefficients of required lift force and into the formula (1) to calculate required engine thrust. Dividing the required thrust onto the available one ( $P_{av}$ ), we get engine throttling coefficient

$$\xi_{th} = \frac{P_{req}}{P_{av}}. \quad (9)$$

Airplane lift-to-drag ratio in the conditions of steady level flight is determined as the airplane current weight ratio to the required engine thrust

$$K = \frac{mg}{P_{req}}. \quad (10)$$

## 2. Calculation Example for A-310-200 Airplane

The considered algorithm is implemented by the author in calculation module of the Power Unit 11.7 software. Aerodynamic and thrust performances calculations of the A-310-200 airplane were done as an example.

To calculate the airplane flight performance, the methodology of the Prof. V. I. Holiavko [7] was used as a basic one, which was updated to calculate wings with kinks at leading and trailing edges, and also with airfoil thickness ratio and its setting angle linearly varying spanwise.

CG position calculation results are shown in Fig. 1. It is clear from the figure that CG position for cruise pitch angles (curve 2) determined by the formula (8), differs from the CG position at given mean pitch angle ( $2^\circ$ ) (curve 3) not more than by 0.12 %.

Fig. 2 shows graphs of the cruise AOA (pitch) by the formula (8) (curve 1), and also calculating pitch angle at two fixed CG positions:  $\bar{x}_{CG} = 0.25$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass.

Fig. 3 shows graphs of the CG position (curve  $\bar{x}_{CG}$ ) and CP of airplane without HT (curve  $\bar{x}_p$ ) on the current flight mass. Horizontal lines of aerodynamic center of the airplane (curve  $\bar{x}_{FA}$ ) and the airplane without HT (curve  $\bar{x}_{FA WFT}$ ) are also shown here. It is clear from the figure, that airplane CG moves between the aerodynamic center of the airplane without HT and the airplane aerodynamic center, i. e. it is located in the stability zone. Thus, CP of the airplane without HT is also located in front of the CG, that provides positive lift on the HT.

Fig. 4 shows graphs of the lift coefficients of the airplane without HT for the cruise AOA (pitch) by the formula (8) (curve 1), and also their calculating values at two fixed CG positions:  $\bar{x}_{CG} = 0.5$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass.

Fig. 5 shows graphs of the lift coefficients of the separate HT for the cruise AOA (pitch) by the formula (8) (curve 1), and also their calculating values at two fixed CG positions:  $\bar{x}_{CG} = 0.25$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass. As it was mentioned, the HT lifting force appeared positive, but the minus sign in

the figure is caused by the fact, that during the formula derivation, the HT lifting force was directed downwards. It is clear from the figure, that the HT lifting force depends on the flight mass in a complicated manner, due to analogous CG position variation.

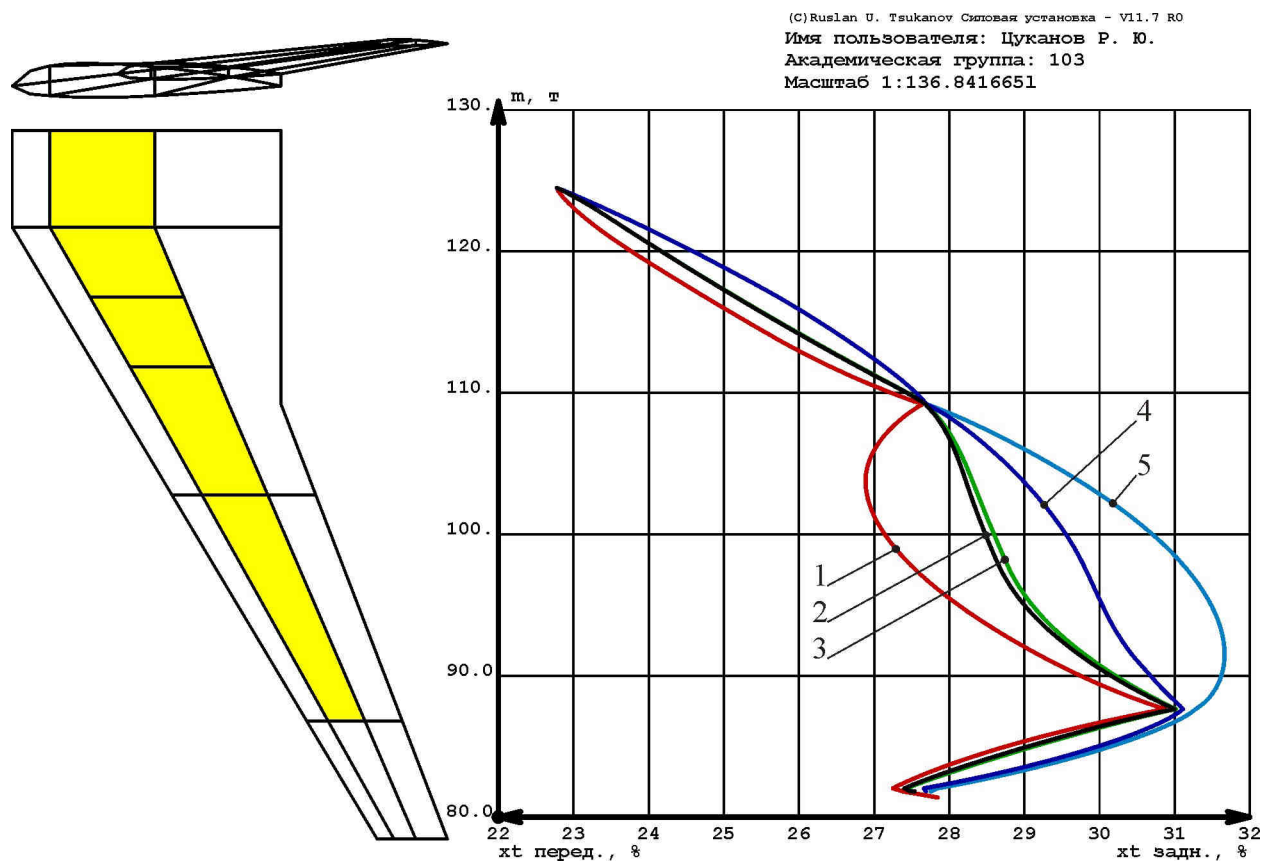


Fig. 1. CG position graphs of A-310-200 airplane:  
1 — at minimal pitch angle; 2 — at cruise pitch angle, determined by formula (8); 3 — at mean cruise pitch angle ( $2^\circ$ ); 4 — at maximum pitch angle accounting anti-surge ribs;  
5 — at maximum pitch angle not accounting anti-surge ribs

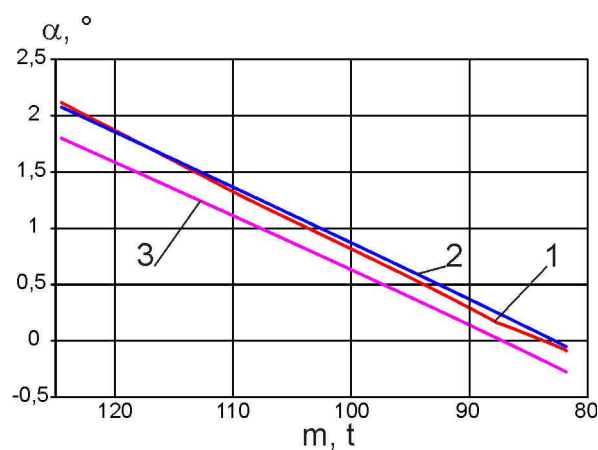


Fig. 2. Cruise AOA (pitch) vs. current flight mass

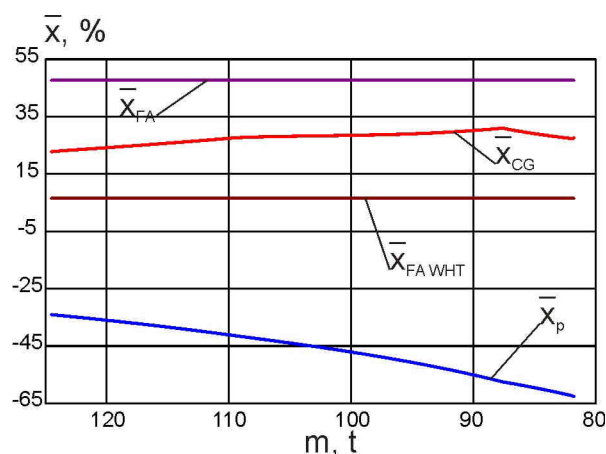


Fig. 3. CG and CP of airplane without HT, aerodynamic center of airplane and airplane without HT vs. current flight mass

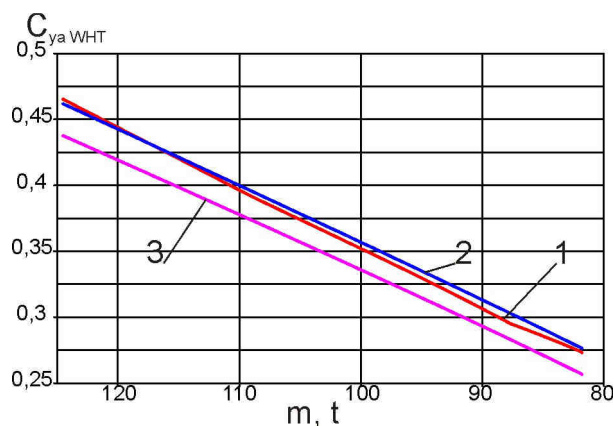


Fig. 4. Lift coefficients of the airplane without HT vs. current flight mass

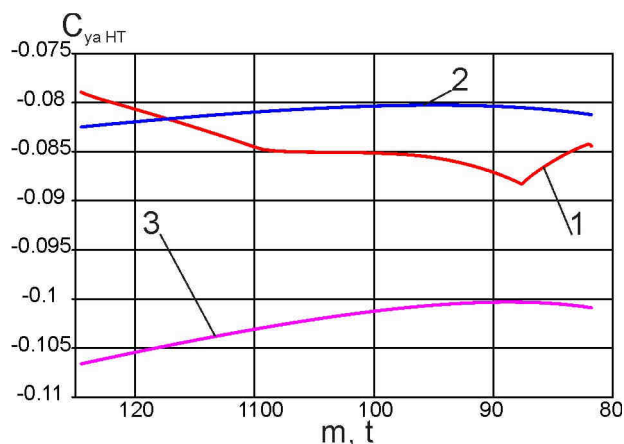


Fig. 5. Lift coefficients of separate HT vs. current flight mass

Fig. 6 shows graphs of the required thrust for the cruise AOA (pitch) by the formula (8) (curve 1), and also its calculating values at two fixed CG positions:  $\bar{x}_{CG} = 0.25$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass.

Fig. 7 shows graphs of the engine throttling coefficient for the cruise AOA (pitch) by the formula (8) (curve 1), and also its calculating values at two fixed CG positions:  $\bar{x}_{CG} = 0.25$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass.

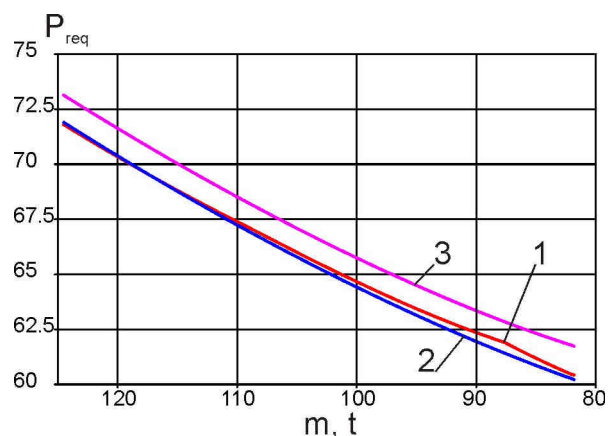


Fig. 6. Required thrust vs. current flight mass

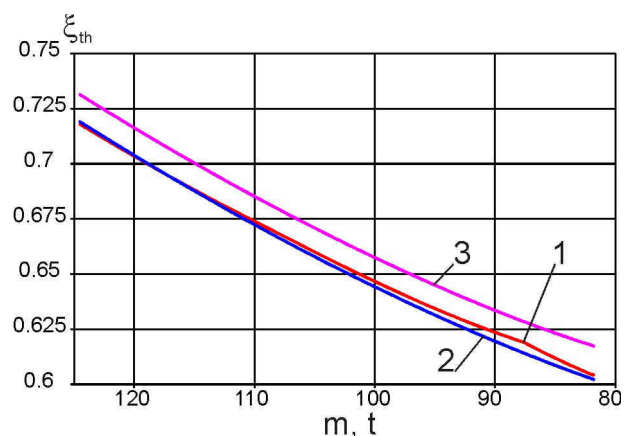


Fig. 7. Engine throttling coefficient vs. current flight mass

Fig. 8 shows graphs of the lift-to-drag ratio for the cruise AOA (pitch) by the formula (8) (curve 1), and also its calculating values at two fixed CG positions:  $\bar{x}_{CG} = 0.25$  (curve 2) and  $\bar{x}_{CG} = 0.4$  (curve 3) dependence on the current flight mass.

It is clear from Figs 2, 4, 6, 7, 8, that the values corresponding to the cruise AOA by the formula (8) (curves 1) are practically located within the intervals limited by the curves (2) and (3) that indicates adequacy of the developed algorithm.

## Conclusions

1. Algorithm for aerodynamic and thrust performance calculation of transport category airplanes at cruise flight mode taking into account its center-of-gravity variation in the process of fuel utilization is created.

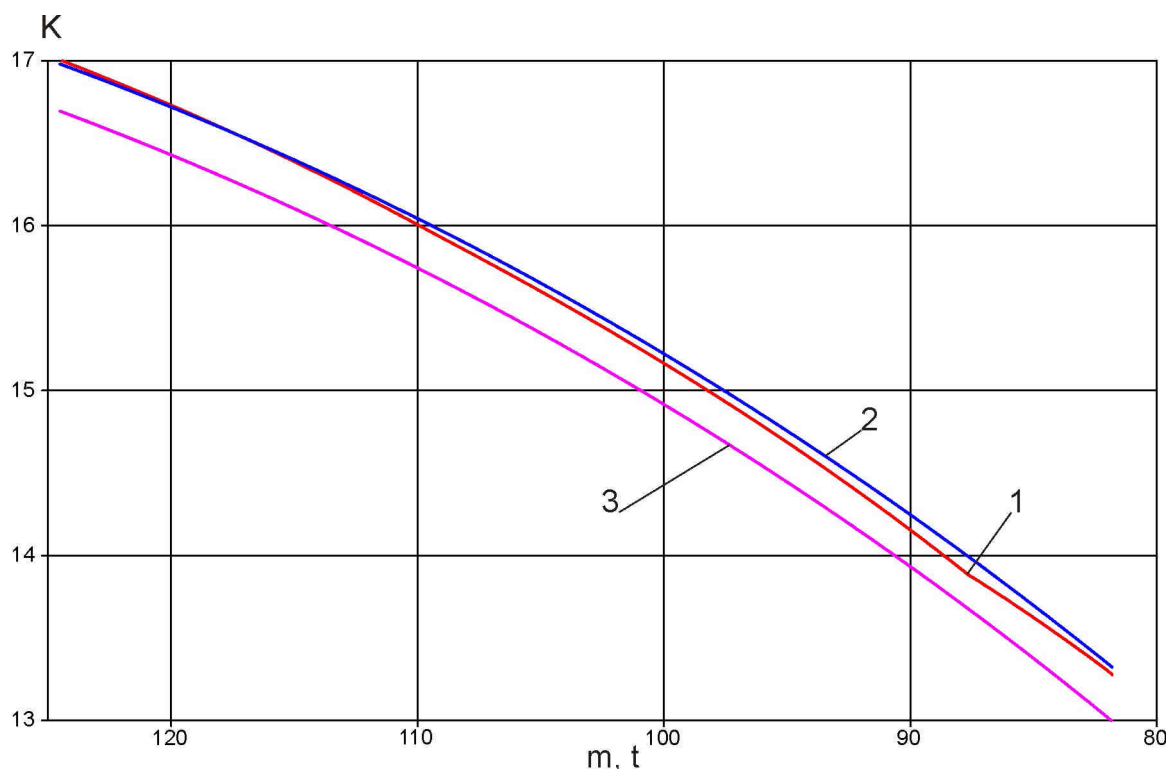


Fig. 8. Lift-to-drag ratio vs. current flight mass

2. Adequacy of the developed algorithm is shown by means of the calculating values of cruise parameters comparison with the values at two fixed center-of-gravity positions.

3. The developed procedure can be used to estimate fuel trim transfer influence on transport category airplanes' flight range and fuel efficiency.

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## **Влияние изменения центровки, вызванного выработкой топлива, на аэродинамические и тяговые характеристики самолёта**

Представлен алгоритм расчёта аэродинамических и тяговых характеристик самолёта транспортной категории на крейсерском режиме полёта с учётом изменения положения его центра масс по мере выработки топлива. Для примера выполнен расчёт графика центровки самолёта А-310-200 на крейсерском режиме полёта. Путём сравнения расчётных крейсерских значений параметров с их значениями при двух фиксированных положениях центра масс показана адекватность разработанного алгоритма.

**Ключевые слова:** центр масс, центр давления, балансирующая перекачка топлива, потребная тяга, аэродинамическое сопротивление, коэффициент подъёмной силы, угол атаки, коэффициент дросселирования двигателя, аэродинамическое качество.

## **Вплив зміни центрування, спричиненого виробленням палива, на аеродинамічні та тягові характеристики літака**

Наведений алгоритм розрахунку аеродинамічних і тягових характеристик літака транспортної категорії на крейсерському режимі польоту з урахуванням зміни положення його центру мас у міру вироблення палива. Для прикладу виконано розрахунок графіка центрування літака А-310-200 на крейсерському режимі польоту. Шляхом порівняння розрахункових крейсерських значень параметрів із їхніми значеннями при двох фіксованих положеннях центру мас показано адекватність розробленого алгоритму.

**Ключові слова:** центр мас, центр тиску, балансувальне перекачування палива, потрібна тяга, аеродинамічний опір, коефіцієнт піднімальної сили, кут атаки, коефіцієнт дроселювання двигуна, аеродинамічна якість.

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