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TASK SOLVING ORGANIZATION OF INVERSE PROBLEMS OF THERMOELASTICITY FOR A THIN RING

The research of stress-strained conditions of aviation and power industrial facilities refers to the problems of mechanics of a solid deformed body. In this case the ultimate goal of the research may be different: determination of the stress-strain state of specific objects; identification of mechanical characteristics of materials. But if we assume that the study of the stress-strain state of a body with known all other initial data is the subject of a direct problem, then the definition of any parameter of the system by known characteristics stress-strain state can be attributed to the class of inverse problems of the mechanics of a deformed solid. New inverse thermoelasticity problems for thin ring have been formulated in which unknown thermal loading (temperature of boundary surface and intensity of frictional heat flux) has been determined using additionally given vertical displacements of one of the outer boundary surfaces. The functional spaces, for which the problems are well-posed, have been found. The method for solving the problems has been suggested and numerically verified with the use of the solution of the direct problem. This paper deals with the determination of heating temperatures and temperature distributions on the upper surface of a thin ring. The expressions of the heating temperatures and temperature distributions have been obtained in series form, involving Bessel's functions with the help of the integral transform technique. Thermoelastic deformations have been discussed and illustrated numerically with the help of temperature and determined. In the framework of the nonlinear theory of thin rings on the basis of the inverse problem method was formulated a model of deformation of the observed thin-walled ring with loads, boundary conditions, geometric parameters to be determined. The basis of the model is the parametrization of a direct problem of nonlinear theory thin-walled elements using the boundary elements method г and the variational formulation of the identification problem, which provides for minimization of the residual functional reflecting the deviation of stress-strain state parameters obtained as a result of observationfrom those calculated on the basis of an approximate solution.

Keywords: *inverse problem***,** *inverse transient function, thermoelastic deformation, thin ring.*

Introduction

Today the transition from expensive experimental methods to cheaper ones becomes more and more relevant. Experiments with thin rings, which are used in propulsion systems, aerospace engineering and airplanes, involve using a multitude of specialists, equipment, working space (laboratories) and time. The method of inverse problems proposes to solve the problem much more cheaply and without loss of efficiency. It is especially important to use the inverse problem method in cases when an repeated enough times for statistical representativeness experiment may be expensive or dangerous.

It should be remembered that quality is fundamentally unstable, since it is determined by randomly changing factors and tends to a constant deviation from given levels. Under the influence of production and operational factors, the quality changes during the experiment. This means that even a representative results of the data can result in a significant error due to a change in the quality of the equipment during the experiments. Therefore it is obvious that the method of inverse problems is the most optimal for solving such questions as the thermoelasticity of a thin disk.

1. Statement of the problem

Let's suppose that in a circular ring $a \le r \le b$ there is no heat source, the inner side of it is heated to temperature T_a and the external side is heated temperature to T_b. Then the temperature distribution in the ring is described by the stationary heat conduction equation with prescribed temperatures at its boundaries

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right]=0,\quad a<r<b,\ T(a)=T_a,\ T(b)=T_b. \quad (1)$$

As a result of heating in the area of a ring formed thermoelastic stresses, which in this case are represented by the following components: radial σ_r and circumferential σ_ϕ. To define the data component we will use the

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differential equation for radial displacement u, which has the following form [1]

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(ru)}{dr}\right] = (1+\nu)\alpha\frac{dT}{dr}, \quad a < r < b. \qquad (2)$$

Let's suppose that at borders of this area are given displacements

$$u_a(a) = u_a, \; u_b(a) = u_b. \qquad (3)$$

Need to find the temperature distribution in the ring $T(r)$, $a \le r \le b$, if the values of the radial stress at certain points of the ring are known with an error characterized by a random amount distributed according to the normal law with zero mathematical expectation and variance δ^2 [2]

$$R[\sigma_r(r_k)] = \sigma_k^e, \quad a < r_k < b, \qquad (4)$$

where R is the operator representing this error.

2. Exact solutions for direct problems of heat conduction (1) and thermoelasticity (2)

The solution in closed form for the Dirichlet boundary value problem of the form (1) can be obtained after performing successive double integration

$$r\frac{dT}{dr} = D_1, \; \frac{dT}{dr} = \frac{D_1}{r}, \; T = D_1 \ln r + D_2.$$

Constants and are determined from the boundary conditions of the problem (1)

$$\left.\begin{aligned} T_a &= D_1 \ln a + D_2 \\ T_b &= D_1 \ln b + D_2 \end{aligned}\right\}; T_b - T_a = D_1 \ln\frac{b}{a}; \; D_1 = \frac{T_b - T_a}{\ln\frac{b}{a}}.$$

Thus, the obtained temperature distribution in the ring

$$T = \frac{(T_b - T_a)\ln r + T_a \ln b - T_b \ln a}{\ln b - \ln a}. \qquad (5)$$

For the solution of the boundary value problem (2), (3) will do the same.

$$\frac{d(ru)}{dr} = (1+\nu)\alpha T r + C_1 r;$$

$$ru = (1+\nu)\alpha\int_a^r T r dr + C_1\frac{r^2}{2} + C_2.$$

And finally, for the radial stress we obtain the following

$$u = (1+\nu)\frac{\alpha}{r}\int_a^r T r dr + C_1\frac{r}{2} + \frac{C_2}{r}. \qquad (6)$$

Now need to determine the unknown coefficients C_1 and C_2 from boundary conditions (3) for problem (2). Write the formulas for the component stresses [1]

$$\sigma_r = \frac{E}{1-\nu^2}\left[\varepsilon_r + \nu\varepsilon_\phi - (1+\nu)\alpha T\right],$$

$$\sigma_\phi = \frac{E}{1-\nu^2}\left[\varepsilon_r + \nu\varepsilon_r - (1+\nu)\alpha T\right],$$

where the corresponding deformations are determined through radial displacement $\varepsilon_r = \frac{du}{dr}$ $\varepsilon_\phi = \frac{u}{r}$. Inserting the expressions for the displacements (6) and temperature (5) into the formula for σ_r, we get the following dependence for the radial stresses

$$\varepsilon_r = \frac{du}{dr}, \varepsilon_\phi = \frac{u}{r}.$$

Taking the integral in the last expression, we obtain finally

$$\sigma_r = -\frac{\alpha E}{r^2}\int_a^r (D_1 \ln r + D_2) r dr + \frac{E}{1-\nu^2}\left[\frac{C_1}{2}(1+\nu) - \frac{C_2}{r^2}(1-\nu)\right],$$

$$\sigma_r = -\frac{\alpha E}{r^2}\left[D_1\left(\frac{r^2\ln r - a^2\ln a}{2} - \frac{r^2 - a^2}{4}\right) + D_2\frac{r^2 - a^2}{2}\right] + \frac{E}{1-\nu^2}\left[\frac{C_1}{2}(1+\nu) - \frac{C_2}{r^2}(1-\nu^2)\right]. \qquad (7)$$

Similarly, we can obtain the expression for tangential stress σ_ϕ.

We determine the unknown coefficients in (7) from the system resulting from the boundary conditions (3), C_1, C_2,

$$\text{here} \; \left.\begin{aligned} \sigma_r(a) &= 0 \\ \sigma_r(b) &= 0 \end{aligned}\right\} \left.\begin{aligned} &\frac{C_1}{2}(1+\nu) - \frac{C_2}{a^2}(1-\nu) = 0 \\ &\frac{C_1}{2}(1+\nu) - \frac{C_2}{b^2}(1-\nu) = \frac{F}{E}(1-\nu^2) \end{aligned}\right\}.$$

For the coefficients finally get

$$C_2\left[\frac{1}{b^2} - \frac{1}{a^2}\right](1-\nu) = -\frac{E}{F}(1-\nu^2),$$

$$C_2 = \frac{F(1+\nu)}{E\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}, C_1 = \frac{2C_2}{a^2}\frac{1-\nu}{1+\nu}.$$

The exact solution of both boundary value problems are obtained to verify approximate solutions of the same task by using one of the approximate methods (for example, the method of weighted residuals).

3. Regularizing algorithm A. N. Tikhonov

Regularizing algorithm A. N. Tikhonov constructing analytical solutions of linear inverse problems (2)-(4) is to minimize the following functional [3]

$$J = \int_a^b [\sigma_r(r) - \sigma_r^e]^2 dr + \alpha_r \Omega[T], \quad (8)$$

which relies on the method of least squares. Under the integral in the functional (8) is squared the difference between the voltages obtained in the measurement result and the simulated stresses are obtained by solving the boundary value problem in displacements (2), (3) with the subsequent calculation of the radial stresses. Here, the stabilizer or stabilizing functions that depend on an identifiable temperature – the regularization parameter. Let us write the expression for the radial stress, which deformation will introduce through the move

$$\sigma_r = \frac{E}{1-\nu^2}\left[\frac{du}{dr} + \nu\frac{u}{r} - (1+\nu)\alpha T\right]. \quad (9)$$

The stabilizer $\Omega[T]$ present in the form of a square of the second derivative from the desired temperature in the following form

$$\Omega[T] = \int_a^b \left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right]^2 rdr. \quad (10)$$

Substitute expressions (9) and (10) into the functional (8), then get

$$J = \int_a^b \left[\frac{du}{dr} + \nu\frac{u}{r} - (1+\nu)\alpha T - \sigma_r^e\right]^2 rdr + \alpha_r \int_a^b \left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right]^2 rdr.$$

In the last expression for the functional J, to simplify calculations when minimize, radial voltage divided by the multiplier $\frac{E}{1-\nu^2}$.

Due to the fact that the task of minimization of functional is infinite-dimensional task, as it is necessary to find the temperature function and the corresponding function of displacement. In turn, the temperature and radial displacement is the solution of corresponding boundary value problems of heat conduction (1) and thermoelasticity (2), (3), So we reduce the infinite dimensional problem of minimizing the functional to finite-dimensional, using with the solution of boundary value problems (1), (2) and (3) method of weighted residuals [4].

4. The method of weighted residuals in the form of Galerkin

For the solution of boundary value problems of heat conduction (1) select basis functions in spline of Schoenberg a third-degree defect 1 [5]. This choice of basis due to the fact that subsequent will need to get a radial voltage as the solution of the boundary problem (2), (3) where the temperature enters in the form of its first derivative and to achieve a good precision on the displacement, it is desirable to have a smooth first derivative from temperature function.

Construct the following grid on the interval, where is the parameter grid integer number.

Multiply equation (1) for the selected spline and will printeriem the obtained expression for the area of solving the boundary value problem, then get

$$\int_a^b \frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right]\phi_j(r) rdr = 0, \; j = \overline{-1, n+1}$$

or after integration by parts are

$$\int_a^b \frac{dT}{dr}\frac{d\phi_j}{dr} rdr - \frac{dT}{dr}\phi_j r\Big|_a^b = 0, \quad j = \overline{-1, n+1}.$$

In the future, Dirichlet boundary conditions will be presented in the form of the boundary conditions of heat transfer with large heat transfer coefficients and

$$\frac{dT}{dr} = \alpha_a (T - T_a), \; r = a; \; -\frac{dT}{dr} = \alpha_b (T - T_b), \; r = b.$$

After substitution of the boundary conditions in we obtain the following system of equations and now imagine the desired temperature function in the form of a linear combination of basis functions

$$T = \sum_{i=-1}^{n+1} T_i \phi_i(r), \quad (11)$$

where the coefficients $T_i, i = \overline{-1, n+1}$ of the uknown function at baseline.

The solution of equation (13) is a vector of the components, which confirms the reduction to a finite-dimensional problem. Similarly dealing with the solution of the boundary problem of thermoelasticity (2), (3) using the finite element method.

Following the method of weighted residuals [4], we multiply equation (2) to the element of the chosen basis $\phi_j(r)$ and will printeriem in the region.

After integration by parts in the first integral we get the following expression and imagine the move function is in the form of a sum where – the same splines of the third degree and with the same sampling region, as in the conduction problem (1). Then, given the boundary conditions (3), for the coefficients we get the following system of linear algebraic equations

$$\sum_{i=-1}^{n+1} u_i \int_a^b \left[\frac{d\phi_i}{dr}\frac{d\phi_j}{dr} + \frac{\phi_i \phi_j}{r^2} \right] rdr = -\int_a^b (1+\nu)\alpha \frac{dT}{dr}\phi_j rdr,$$
$$j = \overline{-1, n+1}.$$

5. The method of the influence functions in the inverse problem of thermoelasticity

With the aim to link both boundary value problems (heat conduction (1) and thermoelasticity (2), (3)) by one target vector, in this case, we use the method of influence functions [6]. Since we already have an expression for temperature in the form (13), then substitute it into the right side of equation (2), pre-differentiated with respect to spatial coordinates. If now the function of the radial stresses present in the form of the sum of the same number of components as the temperature and substitute into the equation of thermoelaticity

$$u = W(r) + \sum_{i=-1}^{n+1} T_i U_i(r).$$

Here the function "accumulates" a heterogeneous boundary conditions for the displacements on the boundaries of the region in which the radial tension is equal to zero. Due to the fact that this differential equation is linear and the boundary conditions for homogeneous functions, then it splits into linear differential equations of the form

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(rU_i)}{dr}\right] = (1+\nu)\alpha\frac{d\phi_i}{dr}, \quad a < r < b, \qquad (12)$$

with homogeneous boundary conditions $U_i(a) = 0$, $U_i(b) = 0$, $i = \overline{-1, n+1}$.

You can talk and Vice versa, using the principle of superposition. Have a first linear differential equation (16), multiply each of them on constant coefficient and fold, then given function, taking into account the inhomogeneous boundary conditions for the displacements on the boundaries of the ring will receive the original equation (12) with inhomogeneous boundary conditions.

Here for each effect we get the system response in the form of a $\frac{d\phi_i}{dr}$, i.e., a function of influence. Summarizing all the effects in the right part of (12), obtained as the sum of

$$u(r) = W(r) + \sum_{i=-1}^{n+1} T_i U_i(r) \qquad (13)$$

the desired movement.

Having said this $U_i, i = \overline{-1, n+1}$, we note that two functions for the temperature and radial displacement are included in the functional (11), was presented in the form of linear combinations of different bases. For each function your basis, that is for temperature – it's the splines of the third degree, a function of the radial displacement is a function of influence. Influence functions are found by solving the direct boundary value problems of thermoelasticity (13). The coefficients of both functions are the same. Here also it turned out that the functional (11) it is necessary to minimize the space dimensions, that is, in finite-dimensional space.

To compute the influence functions, it is necessary to solve ( ) boundary value problems of the form (13). Again we apply the method of weighted residuals with the same basis in the form of splines of the third degree, then get

$$\int_a^b \left\{ \frac{d^2U_i}{dr^2} + \frac{1}{r}\frac{dU_i}{dr} - \frac{U_i}{r^2} - (1+\nu)\alpha\frac{d\phi_i}{dr} \right\} \phi_j rdr = 0.$$

The integral of the second derivative, as before, write out, using the method of integration by parts. With the zero boundary conditions we have

$$\int_a^b \left\{ -\frac{dU_i}{dr}\frac{d\phi_j}{dr} - \frac{U\phi_j}{r^2} - (1+\nu)\alpha\frac{d\phi_i}{dr}\phi_j \right\} rdr = 0,$$
$$i = \overline{-1, n+1}.$$

According to FEM the influence function ("single" radial tension) will be presented in the following form $U_i(r) = \sum_{k=-1}^{n+1} U_{ik}\phi_k(r)$.

Below in Fig.1 and Fig.2 shows radial tension and temperature.

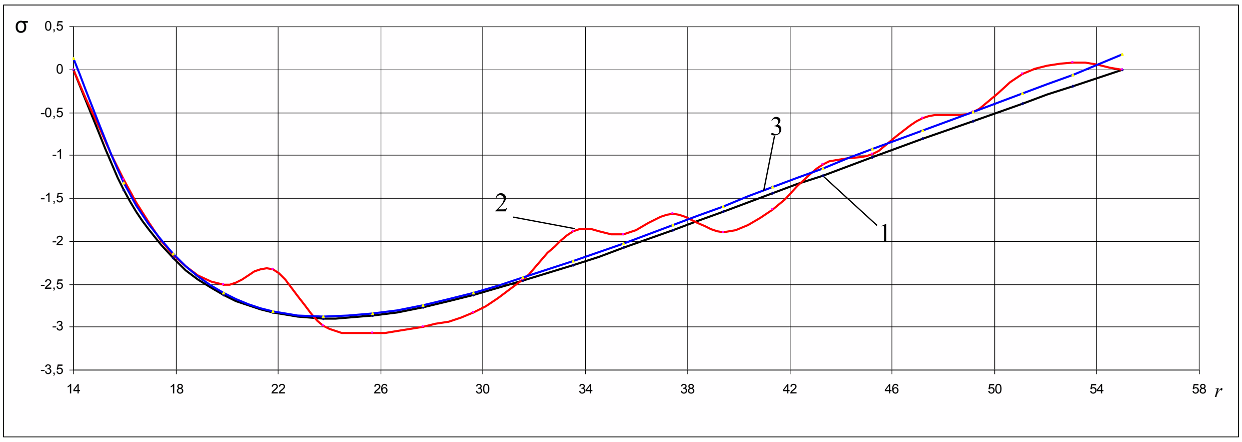

Fig. 1. Radial tension: 1 (black line) – exact, 2 (red line) – noisy, 3 (blue line) – identified

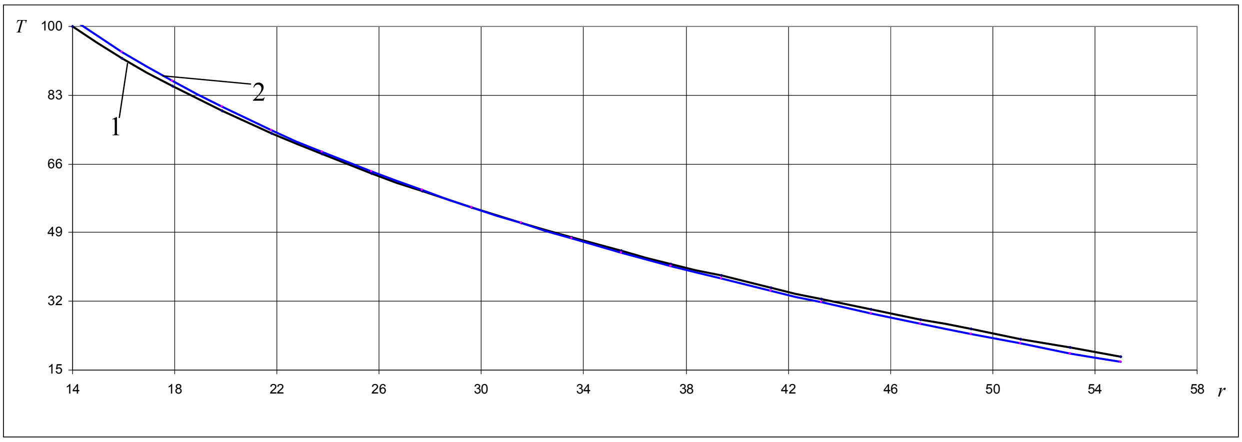

Fig. 2. Temperature: 1 (black line) – exact, 2 (blue line) – identified

6. Minimization of functional

Substitute in the expression for the functional (11) representation of temperature functions (13) and function of radial displacement (17)

$$J = \int_a^b \left[\frac{dW}{dr} + \sum_{i=-1}^{n+1} T_i \frac{dU_i}{dr} + \frac{\nu W}{r} + \frac{\nu}{r}\sum_{i=-1}^{n+1} T_i U_i - \right.$$

$$\left. -(1+\nu)\alpha \sum_{i=-1}^{n+1} T_i \phi_i - \sigma_r^e\right]^2 rdr + +\alpha_r \int_a^b \left[\frac{1}{r}\frac{d}{dr}\left(r\sum_{i=-1}^{n+1} T_i \frac{d\phi_i}{dr}\right)\right]^2 rdr .$$

After substitution functionality is a function of variables in the unknown coefficients. To determine these coefficients we use the necessary condition of minimum of function of several variables, namely

$$\frac{dJ}{dT_j} = 0, \quad j = \overline{-1, n+1} . \qquad (14)$$

Based on the fact that the experimental values of the radial stresses is known at specific points (4), the integrals in the right part of system (14) can be computed numerically by, for example, by the method of rectangles.

Based on the fact that the experimental values of the radial stresses is known at specific points (4), the integrals in the right part of system (14) can be computed numerically by, for example, by the method of rectangles

$$\int_a^b \left[\sigma_r^e - \frac{dW}{dr} - \frac{\nu W}{r}\right]\psi_j rdr \approx$$

$$\approx \sum_{k=1}^{m} \left[\sigma_{rk} - \frac{dW(r_k)}{dr} - \frac{\nu W(r_k)}{r}\right]\psi_j(r_k) r_k h_k ,$$

where is the point of a ring, in which the measured radial strain, $h_k = r_k - r_{k-1}$, $k = \overline{1, m}$, m is the number of measurements. The integrals in the left part can be calculated with prescribed accuracy, as function is the sum of influence functions and their derivatives of Schoenberg splines of the third degree. Influence functions can be calculated with reasonable accuracy by solving the boundary value problem (14).

7. A search of the regularization parameter

Following the algorithm for computing the regularization parameter, we write the norm of the deviation of the simulated and experimental radial stresses. For choice of the regularization parameter the following algorithm is used [7]. The parameter should be chosen in such a way that the values of and (15) would be as similar as possible because to make them equal is not possible. To do this, in practical calculations choose the "narrow" range of change values in the form $[\min I_{cp}, \max I_{cp}]$. The regularization parameter varies in such a way that the value was in the given numeric interval.

$$\min I_{cp} < I < \max I_{cp}. \qquad (15)$$

Once this occurs, the calculation stops.

For this variation you can use, for example, the method of dichotomy. First sets the two values and their arithmetical mean is taken. Next, with the same value minimizes the Tikhonov functional, i.e. solve the system. The obtained nodal factors are used when checking the conditions (15). If this is less, is taken as the same value. If more, take it as. Then calculated the arithmetic mean and continue the process until, until you run condition (15).

8. The results of identification

In Fig. 1-2 presents the results of the identification of the radial stresses and temperature with an error of measurement voltages of 1, 5, 10, 15 and 20%.

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ОРГАНИЗАЦИЯ РЕШЕНИЯ ОБРАТНЫХ ЗАДАЧ ТЕРМОУПРУГОСТИ ДЛЯ ТОНКОГО КОЛЬЦА

В. О. Повгородний, О. С. Буданова

Новые обратные задачи термоупругости для тонкого кольца были сформулированы и применяются при проектировании устройств аэрокосмической техники. В этих задачах неизвестная тепловая нагрузка (температура граничной поверхности и интенсивность теплового потока) была определена с использованием данных вертикального смещения одной из внешних граничных поверхностей. Функциональные пространства, для которых обратные задачи корректны, были найдены. Способ решения обратных задач, был предложен и проверен с использованием многократного решения прямой задачи. Эта статья посвящена определению температур нагрева и распределения температур на верхней поверхности тонкого кольца. Выражения температур нагрева и распределения температур были получены в виде ряда, включая функции Бесселя с помощью интегрального преобразования. Термоупругие деформации были обсуждены и проиллюстрированы численно с помощью численных методов определения температур.

Ключевые слова: обратная задача, обратная переходная функция, термоупругая деформация, тонкое кольцо.

ОРГАНІЗАЦІЯ ВИРІШЕННЯ ОБЕРНЕНИХ ЗАДАЧ ТЕРМОПРУЖНОСТІ ДЛЯ ТОНКОГО КІЛЬЦЯ

В. О. Повгородній, О. С. Буданова

Нові обернені задачі термопружності для взаємодіючих шарів були сформульовані та використовуються при проектуванні пристроїв аерокосмічної техніки. В цих задачах невідоме теплове навантаження (температура граничної поверхні та інтенсивність теплового потоку) було визначене з використанням даних вертикального зміщення однієї з зовнішніх граничних поверхонь. Функціональні пространства, для котрих обернені задачі коректні, були знайдені. Засіб використання обернених задач, було запропоновано та перевірено з використанням багатократного вирішення прямої задачі.

Цю статтю присвячено визначенню температур нагріву та розподіленню температур на верхній поверхні тонкого кільця. Вираження температур нагріву та розподілення температур були одержані у вигляді ряду, враховуючи функції Беселя за допомогою інтегрального перетворення. Термопружні деформації були розглянуті та проілюстровані чисельно за допомогою чисельних методів визначення температур.

Ключові слова: обернена задача, обернена перехідна функція, термопружна деформація, тонке кільце.

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