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APPROXIMATE BOUNDARY CONDITIONS FOR ELECTROMAGNETIC FIELDS IN ELECTRODMAGNETICS

The results of an analytical review of literature sources on the use of approximate boundary conditions for electromagnetic fields of impedance type in solving boundary value problems of electromagnetism for more than 80 recent years are presented. During this period, the impedance approach was generalized to various electrodynamic problems, in which its use made it possible to significantly expand the limits of mathematical modeling, which adequately considers the physical properties of real boundary surfaces. More than eighty years have passed since the publication of approximate boundary conditions for electromagnetic fields. The meaning and value of these conditions lies in the fact that they allow solving diffraction problems about fields outside well-conducting bodies without considering the fields inside them, which greatly simplifies the solution. Since then, numerous publications have been devoted to the application of impedance boundary conditions, the main of which (according to the authors) are presented in this paper. Particular attention is paid to the characteristics of electrically thin impedance vibrators and film-type surface structures as a personal contribution of the authors to the theory of impedance boundary conditions in electromagnetism. The ***subject*** *of research in this article is the analysis of the limits and conditions for the correct application of impedance boundary conditions. The* ***goal*** *is to systematize the results of using the concept of approximate impedance boundary conditions for electromagnetic fields in problems of electrodynamics based on an analytical review of literature sources. The following* ***results*** *were obtained. The types of metal-dielectric structures are presented, for which methods of theoretical determination of the values of surface impedances for film-type structures are currently known, which are the most promising for creating technological control elements on their basis in centimeter and millimeter wavelength devices.* ***Conclusions****. The materials of this paper do not pretend to be a complete reference book covering all the results and aspects of the development of the concept of approximate impedance type boundary conditions in problems of electromagnetism over the past decades. Simultaneously, the authors hope that the information presented in this paper will be useful to specialists in the field of theoretical and applied electrodynamics, as well as graduate students, young scientists and students who are just mastering radiophysics and radio engineering specialties.*

Keywords: *impedance approach; impedance-type boundary conditions; surface impedance; effective impedance; impedance surface.*

Introduction

Eighty years have passed since the publication of the approximate boundary conditions for electromagnetic fields [1–3]. The meaning and value of these conditions lies in the fact that they make it possible to solve diffraction problems about fields outside of well-conducting bodies without considering the fields inside them, which greatly simplifies the solution. Since then, a large number of publications have been devoted to the application of impedance boundary conditions, the main of which (according to the authors) are presented in this paper (see References). Particular attention is paid to the characteristics of electrically thin impedance vibrators and surface structures of the film type, as the personal contribution of the authors to the theory of impedance boundary conditions in electrodynamics. The general information and specialized information presented in the paper will allow the reader to use the materials of the paper in their work, without resorting to searching for special hard-to-reach literary sources.

1. Impedance boundary conditions and the limits of their correct application

The one-sided impedance boundary conditions allow to reduce the number of interfacing electrodynamic volumes which should be taken in the problem solution. Eliminating the need to determine fields inside the adjacent metal-dielectric elements at the problem formula-

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tion level is the main advantage of the impedance approach. The Shchukin-Leontovich impedance condition on the boundary surface S can be written in the following form [1–3]

$$[\vec{n},\vec{E}]\big|_S = \bar{Z}_S[\vec{n},[\vec{n},\vec{H}]]\big|_S \,, \qquad (1)$$

where $\vec{E}$ and $\vec{H}$ are the vectors the electric and magnetic harmonic fields, $\vec{n}$ is the impedance surface normal, directed inside the impedance region, $\bar{Z}_S = Z_S / Z_0$ is the normalized surface impedance, and $Z_0 = 120\pi$ Ohm is the resistance of free space.

If the $\bar{Z}_S = 0$, i.e., interface surface is perfectly conducting, the formula (1) is reduces to $\left[\vec{n},\vec{E}\right]\Big|_S = 0$. It should be noted that the boundary condition (1) is approximate, since, in this case, the solution of the electrodynamic problem represents the first term of the asymptotic expansion of the exact solution [3,4] in powers of the small parameter

$$|\bar{Z}_S| << 1 \,. \qquad (2)$$

Since only the tangential components are included in the boundary condition (1), there exists some restrictions on the surface S geometry. It is evident that condition (1) holds if the surface curvature radius is much greater than the length of the incident wave. The conditions which take into account the interface curvature can be read as [2, 5]:

$$\begin{aligned} E_{\tau 1} &= \bar{Z}_S\left(1+\frac{\chi_1-\chi_2}{2ik\sqrt{\varepsilon_1\mu_1}}\right)H_{\tau 2}\Bigg|_S \,, \\ E_{\tau 2} &= -\bar{Z}_S\left(1+\frac{\chi_2-\chi_1}{2ik\sqrt{\varepsilon_1\mu_1}}\right)H_{\tau 1}\Bigg|_S \,, \end{aligned} \qquad (3)$$

where χ_1 and χ_2 are the main Gaussian curvatures of the surface, E_τ and H_τ are the tangential components of the electromagnetic fields on the interface surface, $k = 2\pi/\lambda$, λ is the wavelength of free space, ε_1, μ_1 are the material parameters of the environment.

The surface impedance of an electromagnetic field is usually interpreted as relationships determining links between the tangential components of the complex amplitudes on the surface S. If the impedance value does not depend on the incidence angle and incident wave polarization, it is known as extraneous impedance [4]. If the impedance value does not depend on the wave incidence angle, but depends on the wave polarization and spatial orientation of the surface S, the surface impedance is a two-dimensional second-rank tensor which components are extraneous impedances. In the general case, a concept of anisotropic surface impedance is introduced as matrix

$$\hat{\bar{Z}}_S = \begin{Vmatrix} \bar{Z}_{S11} & \bar{Z}_{S12} \\ \bar{Z}_{S21} & \bar{Z}_{S22} \end{Vmatrix}, \ \bar{Z}_{Sjk} = \bar{R}_{Sjk} + i\bar{X}_{Sjk}, \ j,k \in \{1,2\} \,, \qquad (4)$$

under conditions that inequalities

$$\bar{R}_{S11} \ge 0, \ \ \bar{R}_{S22} \ge 0, \ \ 4\bar{R}_{S11}\bar{R}_{S22} \ge \left|\bar{Z}_{S12} + \bar{Z}^*_{S21}\right|^2 \qquad (5)$$

hold. In (4) and (5) $\bar{Z}^*_{S21}$ is complex conjugate of $\bar{Z}_{S21}$. The inequalities (5) ensure that additional energy sources on the surface S and energy flows through this surface are absent. Of course, the impedance $\bar{Z}_S$ in (1) and (4) must be replaced by the tensor $\hat{\bar{Z}}_S$. It should be emphasized that the surface S, on which the impedance boundary condition should be satisfied, does not have to coincide with the real impedance boundary surface and can be considered as a conditional boundary surface. A spectral analysis of complex structures and media should require introduction of a partial impedance, the value of which in the general case depends on the frequency and the number of spatial harmonics in the electromagnetic field representation. Such impedance problems are beyond the scope of this paper.

First, let us consider possible formulations of the impedance conditions and the solution accuracies they can provide. According to results obtained in [3], the boundary condition (1) is applicable when the following requirements are met: a penetration depth of electric fields into an impedance material and a field wavelength should be small compared to an incident wave wavelength, a distance from a field source, and curvature radii of a boundary surface S. In addition, variations of the material parameters of the impedance layer at distances comparably with the field wavelength or penetration depth should be small. In the general case, the accuracy of the formula (1) was estimated to be proportional to $\sim|\bar{Z}_S|^2$, since only the first term of the solution obtained as asymptotic series with respect to the normalized impedance $\bar{Z}_S$ was used. Leontovich obtained a similar estimate by comparing the plane wave reflection obtained in the impedance approximation with that of the exact Fresnel solution [2]. However, for a certain class of propagation models, corrections to (1) are proportional to cubic but not quadratic terms in the small parameter $|\bar{Z}_S|$ [3].

Even though the accuracy estimates of the condition (1) were obtained on the basis of the skin-effect theory for of conducting body surfaces [2, 3], they can be uniquely extended to the more general case of impedance domains [4]. All the above requirements can be integrated into one of a purely physical nature: the field at the impedance surface must be a plane wave propagating in a direction normal to a boundary S. This condition is always fulfilled for electrically thin impedance structures, including film coatings.

However, the presented requirement cannot be waves at small incident or Brewster angles. In the first case, the reflected and refracted rays are gliding near the surface, while in the second case they must be mutually perpendicular. In both cases, the directions of the refracted rays do not coincide with the direction of the boundary surface normal. Therefore, it is customary to distinguish between three separate cases of the impedance conditions depending upon the wave incidence angle: 1) normal incidence, when the Shchukin-Leontovich condition are valid, 2) Brewster angle incidence, and 3) tangential incidence. Apparently for the first time, the condition (1) was corrected and used in [6] where problem of the reflection of electromagnetic waves from the surface of real soil at angles close to the sliding incidence angle was solved. Subsequently, similar cases were studied for a number of other media including inhomogeneous plasma. The analysis of various variants of the approximate impedance boundary conditions are presented in [7–9].

The accuracy justification of the impedance boundary condition (1) cannot give a complete answer to the question: with what accuracy the specific characteristics of the wave fields can be calculated for arbitrary angles of a plane wave incidence on the media interface. General conclusions concerning for the smallest errors relative to exact values for the whole range of incidence angles were formulated in [10-12] for perpendicular and parallel polarizations relative to the interface surface. For the perpendicular polarization, the reflection coefficients should be calculated based on the Shchukin-Leontovich approximate boundary conditions. For a wave of parallel polarization, the formulas valid for the Brewster angle are preferable. These conclusions were made based on the exact formulas obtained in [13].

The above analysis of the boundary condition (1) accuracy was performed under condition that surface impedance $\bar{Z}_S$ presented by a power series only terms proportional to the first degree of a small parameter were taken into account. However, this simplification permits only the small $\bar{Z}_S$, and secondly, and does not provide the necessary accuracy for solving the diffraction problem when the wave is incident at the Brewster angle or tangential to the surface interface. These shortcomings can be eliminated by the method proposed in [14], where a generalized impedance approximation was formulated as:

$$\vec{E}_\tau + \bar{Z}_S\left[\vec{H}_\tau, \vec{n}\right] + \frac{1}{2}\bar{Z}_S^3 n^2\left(N_1 + \sum_{s=1}^{\infty}\left(\bar{Z}_S n\right)^{2s}\frac{(2s-1)!!}{2^s(s+1)!}N_{2s+1}\right)\vec{H}_\tau = 0, \quad (6)$$

where the matrices $N_m\left(m = 2s+1\right)$ are defined as

$$N_m = \begin{Vmatrix} 0 & 1 \\ m & 0 \end{Vmatrix}, \quad (7)$$

$n = k/k_0$ is a dimensionless refraction parameter, k and k_0 are wavenumbers in the impedance medium and external space, respectively. In the first approximation, the equation (6) coincides with the boundary condition (1), but the vectors $\vec{E}_\tau$ and $\vec{H}_\tau$ are related through the refraction factor n, which in a reflection problem is uniquely connected to a wave incidence angle.

Thus, the approximate Shchukin-Leontovich condition (1) valid for a small surface impedance $\bar{Z}_S$, is generalized for arbitrary $\bar{Z}_S$ as the series expansion (6), which expand the applicability of the impedance approach. Since the exact boundary condition (6) is decomposed in a series in odd powers of the parameter $\bar{Z}_S$, the Shchukin-Leontovich condition linear in $\bar{Z}_S$ differs from the exact only by terms proportional to $\sim|\bar{Z}_S|^3$. That is, the fields obtained using condition (1) are correct up to $\sim|Z_S|^2$. Thus, the accuracy of condition (1) application turns out to be higher than it could be supposed based on the results obtained in [3].

2. Surface impedance of metal-dielectric structures

The key stage of the impedance approach application is the problem of determining the surface impedance for a specific spatial structure. In this subsection, we analyze metal-dielectric structures, which theoretical estimates of surface impedance are well-known. Let us first consider a problem of a plane electromagnetic wave incidence on a flat interface between two media [5, 6, 13], to demonstrate a general approach for obtaining surface impedance formulas.

Let a plane in a rectangular coordinate system $X0Y$ be the interface between two media with parameters $(\varepsilon_1, \mu_1, \sigma_1)$ and $(\varepsilon_2, \mu_2, \sigma_2)$, and a conductivity of

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the second medium $\sigma_2 \gg 1$. Consider reduced electromagnetic fields in the two media:

$$\tilde{\vec{E}}_1 = \sqrt{\varepsilon_1'}\vec{E}_1 ,\ \tilde{\vec{H}}_1 = \sqrt{\mu_1'}\vec{H}_1 ,$$
$$\tilde{\vec{E}}_2 = \sqrt{\varepsilon_2'}\vec{E}_2 ,\ \tilde{\vec{H}}_2 = \sqrt{\mu_2'}\vec{H}_2 , \qquad (8)$$

where $\{\vec{E}_1, \vec{H}_1\}$ and $\{\vec{E}_2, \vec{H}_2\}$ are the true fields, $\varepsilon_1 = \varepsilon_1' + 4\pi i\sigma_1/\omega$ and $\varepsilon_2 = \varepsilon_2' + 4\pi i\sigma_2/\omega$ are complex permittivity's of the media. Here, the periodic dependence of the fields on time t is preserved, as in [5], in the form $e^{-i\omega t}$. A plane wave incident at an angle θ_1 measured from the normal to the interface can be represented as $\tilde{\vec{E}}_1 = \left(\tilde{E}_1, 0, 0\right)$, where $\tilde{E}_1 = E_0 e^{ik_1(y\sin\theta_1 - z\cos\theta_1)}$. Therefore, we can write

$$\tilde{\vec{H}}_1 = \frac{1}{ik_1}\mathrm{rot}\tilde{\vec{E}}_1 = \frac{1}{k_1}[\vec{k}_1, \tilde{\vec{E}}_1] , \qquad (9)$$

where $\vec{k}_1 = \left(0, \sin\theta_1, -\cos\theta_1\right)$, $k_1 = \frac{\omega}{c}\sqrt{\varepsilon_1'\mu_1'}$, $\left|\tilde{\vec{E}}_1\right| = \left|\tilde{\vec{H}}_1\right|$.

Since a plane wave of the same polarization is excited in the second medium, and the density of the surface current is zero,

$$\tilde{\vec{E}}_2 = \left(\tilde{E}_2, 0, 0\right),\ \tilde{\vec{H}}_2 = \frac{1}{k_2}[\vec{k}_2, \tilde{\vec{E}}_2], \qquad (10)$$

where $\vec{k}_2 = \left(0, \sin\theta_2, -\cos\theta_2\right)$, $k_2 = \frac{\omega}{c}\sqrt{\varepsilon_2'\mu_2'}$, $\left|\tilde{\vec{E}}_2\right| = \left|\tilde{\vec{H}}_2\right|$. The tangential components of the electromagnetic field in the second medium are equal to $\tilde{E}_{2\tau} = \tilde{E}_{2x} = \tilde{E}_2$ and $\tilde{H}_{2\tau} = \tilde{H}_{2y} = \tilde{H}_2\cos\theta_2$, where θ_2 is the wave propagation angle in this medium. Then, the ratio of the tangential components of the electromagnetic field in the second medium can be immediately determined as

$$\frac{\tilde{E}_{2x}}{\tilde{H}_{2y}} = \frac{\tilde{E}_2}{\tilde{H}_2\cos\theta_2} = \frac{1}{\cos\theta_2} . \qquad (11)$$

The cosine of the angle θ_2 can be easily fined by using the Snell law $\frac{\sin\theta_1}{\sin\theta_2} = n = \sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}$, where n is the refractive index between the two media. Since $\sigma_2 \gg 1$, $\left|\varepsilon_2'\right| \gg 1$ and $n \gg 1$. Then

$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - \frac{1}{n^2}\sin^2\theta_1} . \qquad (12)$$

If the inequality $\left|\frac{\sin\theta_1}{n}\right| \ll 1$ holds, the $\cos\theta_2 \approx 1$, and we obtain the Shchukin-Leontovich condition, i.e., according to (11), $\frac{\tilde{E}_{2x}}{\tilde{H}_{2y}} \approx 1$. For the true fields we obtain

$$\frac{\tilde{E}_{2x}}{\tilde{H}_{2y}} = \sqrt{\frac{\mu_2}{\varepsilon_2}} . \qquad (13)$$

Further, surface currents are absent, we will use the continuity of the tangential components of the electric and magnetic fields and take into account the boundary condition (1). The relation for the fields only in the first medium can be written as

$$E_{1x} = \bar{Z}_S H_{1y} , \qquad (14)$$

therefore, the surface impedance of the second medium is equal to

$$\bar{Z}_S = \sqrt{\mu_2/\varepsilon_2} . \qquad (15)$$

Since the value of the surface impedance $\bar{Z}_S$ is determined as the square root of the complex value, the branch of the root, for which the imaginary part $\mathrm{Im}\bar{Z}_S < 0$ should be selected. In this case $\mathrm{Im}\sqrt{\varepsilon_2'} > 0$, and the waves propagating in the second media are damping. For another polarization of the incident wave, we obtain the expression

$$E_{1y} = -\bar{Z}_S H_{1x} . \qquad (16)$$

If the angle $\theta_1 \approx \pi/2$, we obtain using the expression (12) that $\cos\theta_2 = \sqrt{1 - \frac{1}{n^2}\sin^2\theta_1} = \sqrt{1 - \frac{1}{n^2}}$ valid for the arbitrary refractive index n. The surface impedance for the sliding waves can be easily obtained based on this result and expression (11). Analogously, the impedance for the wave incident at the Brewster angle can be obtained based on expression (12). Below in this section, the impedances of various structures are considered only for the normal incidence of the excitation wave on the flat interface between impedance surface and free half-space. The formula (16) can also be used to determine the impedance value if the material parameters of a medium filling the second region have been previously determined.

2.1. Real Metals

As is known, electromagnetic waves penetrate into metals at a depth small compared to a free space wavelength λ. For superconductors and normal metals, the penetration depth at high and microwave frequencies is equal to about $(10^{-2} \div 10)$ micron. Due to the small penetration depth, the fields components normal to the surface are much greater than their tangential components. The penetration depth Δ^0 can be determined using the expression [15]

$$\Delta^0 = \omega / k\sqrt{2\pi\sigma\omega\mu} . \quad (17)$$

Since the phenomenon of the electromagnetic field concentration near the surface of the body is related to the skin-effect, it is argued that the impedance boundary condition (1) occur when strong skin-effect is present, i.e., the skin layer thickness Δ^0 is small compared to all values with length dimensions that characterize the electrodynamic structure. First of all, the inequalities $\Delta^0 \ll \lambda/(2\pi)$ and $\Delta^0 \ll R$ hold, where R is the distance from the impedance surface to the source. The skin layer thickness should be small as compared with to the body dimensions, $\Delta^0 \ll l$, in all directions, and to the curvature radii of the body surface, $\Delta^0 \le D$.

In the general case, for the time dependence of the fields $e^{i\omega t}$, the complex depth of field penetration into the metal is introduced [7, 16]:

$$\delta = \frac{1}{H_{\tau 1}\big|_{z=0}} \int_0^\infty H_{\tau 1}(z)dz = \delta_1 - i\delta_2 , \quad (18)$$

where $\{0, z\}$ is the axis directed inside the metal along the surface normal. The symbols δ_1 and δ_2 denote the resistive and inductive skin layer depth. In this case, the surface impedance can be written as

$$\bar{Z}_S = \bar{R}_S + i\bar{X}_S = k\delta_2 + ik\delta_1 , \quad (19)$$

where $\bar{R}_S$ and $\bar{X}_S$ are the normalized surface resistance and reactance.

Let us analyze the case when the metal is located in an electromagnetic field at room temperature. A current in any point inside the metal is defined by two factors: first electrons are accelerated under the action of the electric field $\vec{E}$, and, second, the path between two successive collisions with the lattice is limited by the free path l of the electrons. When the current is forming, the fields existing on the length l should be taken into account. Since the free path l of electrons in metals at room temperature is much less than the depth of the skin layer, the field $\vec{E}$ in the process of current formation can be considered to be constant. Hence, in this case, the current density $\vec{j}$ is determined only by the magnitude of the field at that point. Under these conditions, the skin-effect is called by the classic skin-effect. To find a local relationship between the quantities $\vec{j}$ and $\vec{E}$, a simple model of free electrons can be used to obtain [16]:

$$\vec{j} = \sigma_2 \vec{E}/(1 + i\omega\tau) , \quad (20)$$

where $\tau = l/v_F$, v_F is the Fermi velocity.

When relaxation effects can be neglected, i.e., when the condition $\omega\tau \ll 1$ is fulfilled, the formula (20) transfers into the traditional Ohm's law, $\vec{j} = \sigma_2 \vec{E}$. Then, for an isotropic homogeneous metal, the formulas defining the normalized surface impedance and penetration depth can be determined as [17]

$$\bar{Z}_S^{cl} = (1+i)\sqrt{k/2\sigma_2 Z_0} , \quad (21)$$

$$\delta_{cl} = \sqrt{2/(kZ_0\sigma_2)} = 2\delta_1 . \quad (22)$$

As can be seen, the essential feature of the formulas is equality of surface resistance and reactance, $\bar{R}_S^{cl} = \bar{X}_S^{cl} = \sqrt{k/2\sigma Z_0}$.

If the electron mean free path l is comparable to or greater than the penetration depth, then the formation of a current in the vicinity any point of the metal is be determined by collision processes in an area where the electric field differs markedly from the field at that point. In this case, the current density $\vec{j}$ depends on the fields defined in the vicinity of this point with the radius l. If $l \approx \delta_{cl}$ or even $l \gg \delta_{cl}$, the effect becomes typical for pure metals at low temperatures and is known as the anomalous skin-effect. Really, when the temperature is decreasing, the average free-path length l increases as σ_2, while decreases δ_{cl} as $\sigma_2^{-1/2}$. Of course, with such free path lengths, the theory of the classical skin-effect is no longer applicable and a more general consideration is required. For example, for the pure copper $l \sim 5 \cdot 10^{-2}$ μm, the ratio $l/\delta_{cl} = 3 \cdot 10^{-2}$ at 300^0K and 10 GHz. The pure copper conductivity σ_2 at helium temperatures can increase by a factor of 10^5 [18], while the ratio l/δ_{cl} can be about 10^6. Of course, in this case the formula for the classical skin-effect is no

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longer applicable and a more general consideration is required.

A rigorous theory of the anomalous skin-effect based on the free electron model was developed by Reuter and Sondheimer [19]. The theory assumes that if external perturbations are absent, the electrons at some point in the metal are distributed in the momentum space sphere of radius mv_F, where m is the electron mass. If, for some reason, the sphere is deformed, the total electron momentum arises appearing, which determines the current in the metal. When the anomalous skin-effect is present, the free path length l is comparable to or greater than the field penetration depth, the fields at an arbitrary point will be defined by the fields of other region where electrons were located before entering the considered place. To take into account this effect, the effective field $\vec{E}_{eff}$ in the metal is included in the expression for current density, similar to (20). This approach resembles accounting for the secondary fields due to induced currents on the scatterer the diffraction problems.

However, the question arises: how to correctly define the current $\vec{j}$ at any point, if it lies at a distance less than l from the metal boundary? To so the boundary conditions for the reflection of electrons on the metal surface should be taken into account. One of the possible assumptions consists in that the electrons colliding with the interface completely lose information about the field in which they were before the collision, and are reflected equiprobable in all directions, i.e., the reflection is diffuse. Moreover, in the absence of external influences outside the metal, the field $\vec{E}=0$. Another assumption consists in that the electrons collides with a surface, the reflection can be specular. In this case, an electron moving to a flat boundary and reflected back to the observation point after the collision with the boundary to can be considered as moving from free space in the field which is the mirror-symmetric with respect to the interface. That is, the field outside the metal surface is assumed to be a mirror-symmetric field inside the metal. In the intermediate case of the two regimes, when a part of electrons p is mirror reflected and the remaining part $(1-p)$ is diffuse reflected. The specularity coefficient p is equal to zero or one for diffuse or mirror reflections.

As a result of a rather complicated solution of the general problem the correct expressions for impedances in specular $\bar{Z}_S^{mir}$ and $\bar{Z}_S^{dif}$ mode of electron reflections can be found in [16]

$$\bar{Z}_S^{mir}=\frac{2ikl}{\pi}\int_0^\infty \frac{d\tau}{\tau^2+i\alpha k(\tau)}, \qquad (23)$$

$$\bar{Z}_S^{dif}=ikl\pi\Big/\int_0^\infty \ln\left(1+i\alpha k(\tau)/\tau^2\right)d\tau, \qquad (24)$$

where

$$k(\tau)=\frac{2}{\tau^3}[(1+\tau^2)\mathrm{arctg}\tau-\tau],$$

$$\alpha=\frac{3}{4}kZ_0\left(\frac{l}{\sigma_2}\right)^2, \sigma_2^{mir}=\frac{3}{2}\left(\frac{l}{\delta_{cl}}\right)^2. \qquad (25)$$

The plots of expressions (23) and (24) as functions of σ_2 are similar. For small α, i.e., when mean free path of electrons is small, the formulas $\bar{Z}_S\sim\sigma_2^{-1/2}, \bar{R}_S=\bar{X}_S$ are valid for the diffuse and specular reflection, and are consistent with the classical skin-effect (21). For the large α the impedances $\bar{Z}_S^{mir(dif)}$ tend to the limits

$$\bar{Z}_S^{mir(dif)}\Big|_\infty=$$

$$=\tilde{k}_{mir(dif)}\left(\frac{\sqrt{3}}{4\pi}\right)^{1/3}\left(\frac{l}{\sigma_2}\right)^{1/3}\left(\frac{kZ_0}{2}\right)^{2/3}Z_0(1+i\sqrt{3}), \qquad (26)$$

where the coefficients $\tilde{k}_{mir}=8/9$ and $\tilde{k}_{dif}=1$. As can be seen, the limiting values for diffuse and specular reflections differ only by the coefficients. In this case, the surface resistance and reactance are related as

$$\bar{X}_S^\infty=\sqrt{3}\bar{R}_S^\infty. \qquad (27)$$

Of course, values of the impedances can only be calculated by numerical integration of formulas (23) and (24) valid for anomalous skin-effect. However, in, Chambers [20] have obtained simple interpolation formulas that allow to quickly calculate the values for the intermediate region between the classical and anomalous limits

$$R_S=R_S^\infty(1+F_R\alpha^{-G});\ X_S=X_S^\infty(1+F_X\alpha^{-G}), \qquad (28)$$

where the values of the constant $F_{R(X)}$ and G for are given in Table 1.

Table 1

The values of the coefficients in the formulas (28)

p	F_R	F_X	G
0	1.157	0.473	0.2757
1	1.376	0.416	0.3592

The resistance R_S and reactance X_S calculated by the interpolation formulas (28) and by expressions (23) and (24) with an accuracy of 0.1% [16]. For the arbitrary p, Hartman and Luttinger in [21] have obtained the highly accurate solution for the extremely anomalous region in the form

$$\bar{Z}_S^{\infty}(p) = 2\bar{Z}_S^{dif\infty}\left[1-\cos\left(\frac{2}{3}\arccos p\right)\right]\Big/(1-p). \quad (29)$$

The surface impedance of superconductors is of a separate fundamental interest for the researchers. As known that the electrical resistance of many pure metals, alloys and compounds at the DC disappears sharply at a critical temperature T_{cr}, which for all known superconductors are in the region of low temperatures. The highest critical temperature for pure metals, $9.3^0 K$, has niobium, while for compound Nb_3Ge it equals $22.3^0 K$. Perfect conductivity, $\sigma \to \infty$, i.e., total absence of resistance at the DC is considered the only fundamental property of superconductors. Meissner and R. Oxenfeld in [22] have found that the magnetic flux is pushed out of the conductor when it goes into the superconducting state. This effect cannot be explain by the perfect conductivity directly from the ideal conductivity and is another important fundamental property of superconductors. It should also be noted that the surface resistance of superconductors in alternating fields, in contrast to the DC resistance, is not zero, since transitions between adjacent quasiparticle excitations that occur in a superconductor at temperatures $T > 0$ can be induced.

Phenomenological theories of superconductivity (see a overview in [16]) describe a number of superconductor properties, but in many cases, they give only an approximate description and often they cannot adequately describe the specific superconductor parameter. Therefore, the microscopic theory of superconductivity, the BCS theory [23] is of great importance. This theory is based on a fact established by Cooper in [24] that an arbitrarily weak attraction between two electrons can lead to the formation of a bound state, which energy is less than the sum of the energies of the individual electrons.

The attraction can be explained by the polarization of the ion lattice by one electron, which leads to the attraction of the second electron. If attraction exceeds the repulsive Coulomb interaction, the bound pairs of electrons, the Cooper pairs are formed. Electron pairs do not exist independently of each other, but form a condensate that provides a single quantum state of the superconductor. If the Cooper pairs are exposed to external forces, for example, created by an electric field, their momentum is increased due to accelerating acquire a pulse, the same for all pairs. Thus a continuous electric current is arising. By acquiring momentum the kinetic energy of the couples is increased, but when it excides a binding energy, the Cooper pairs are destroyed.

The electrodynamics of superconductors, based on the microscopic theory [23], makes it possible to obtain expressions for the normalized impedances specular reflection and diffuse modes in the temperature range $0 < T < T_{cr}$

$$\bar{\bar{Z}}_S^{mir} = \frac{2ik}{\pi}\int_0^{\infty}\frac{d\tau}{\tau^2 + kZ_0\tilde{Q}(\tau,\omega)/\omega}, \quad (30)$$

$$\bar{\bar{Z}}_S^{dif} = ik\pi\Big/\int_0^{\infty}\ln(1 + kZ_0\tilde{Q}(\tau,\omega)/\omega\tau^2)d\tau. \quad (31)$$

The most difficult problem related to formulas (30) и (31) consists in determining the integral kernel $\tilde{Q}(\tau,\omega)$. However, it should be noted that the integral kernel can be represented as $\tilde{Q}(\tau,\omega) = \tilde{Q}_A + \tilde{Q}_P + \tilde{Q}_C$ where the first two terms are real and only the third term $\tilde{Q}_C$ have both real and imaginary part, which determine the losses . Since the real parts of the impedances $\bar{\bar{Z}}_S^{mir}$ and $\bar{\bar{Z}}_S^{dif}$ in most external problems are small, they can be neglected, and the more convenient formulas for the surface reactance can be used for calculations [16].

2.2. Rough and Corrugated Metal Screens

Above, we considered the surface impedance of real metals and superconductors with absolutely smooth surfaces on which the Schukin-Leontovich boundary condition (1) are valid. However, surface roughness, i.e. imperfections associated with the surface deviation from geometrically perfect form is unavoidably existing on solid material surfaces. The surface imperfections can be caused by a corpuscular structure of matter, technological defects as result of the surface treatment, ets.

There exist many approaches to study the effect of roughness upon the surface impedance. In several publications (for example, [25]), increase of the surface resistance and reactance was associated with a proportional increase of the actual area of a rough surface as compared to a flat one. However, this technique is valid only if characteristic dimensions noticeably exceed the penetration depth.

The most common techniques to study rough surfaces is statistical approach (for example, [26]), when a real surface is described by random function. Small deviations of the boundary shape from the plane are described by a set of random functions describing the boundary deviation from the plane $z=0$ at the point ρ ($\vec{\rho}$ is a two-dimensional vector in the plane $z=0$). This approach is often used for studying electromagnetic waves scattering by rough surfaces, when the effective surface scattering impedance is introduced [26]. In the common sense, the impedance is a characteristic of metal surfaces, determined by energy accumulated in the metal and its losses [16], while the scattering impedance describes the scattering properties of the surface and characterizes the energy loss by the coherent field component due to its transformation into a scattered component. In this case, the scattering processes are determined by diffraction effects which depend on the ratio between a wavelength and irregularity dimension. This effect can be significant, for example, in guide systems of large length, when, due to scattering, the energy of the fundamental mode is transformed into non-fundamental modes.

Naturally, the impedance boundary conditions are not satisfied at each point of the surface, and only the effective surface impedance $Z_S^{eff} = R_S^{eff} + iX_S^{eff}$ can be defined. It is usually assumed that characteristic dimensions of the surface irregularities (average height h and average horizontal dimension d) are much smaller than the free space wavelength and distance at which the incident wave field varies considerably. In other words, it is assumed that the impedance boundary conditions are valid on some plane surface that corresponds to a rough surface and is determined by the shape of the object at a given place. If the surface roughness is isotropic and the average dimensions and radii of curvature of the surface elements are much larger than the penetration depth, the impedance can be defined in the following form [16]:

$$R_S^{eff} = \tilde{\tilde{k}} R_S, \; X_S^{eff} = \tilde{\tilde{k}} X_S, \tag{32}$$

where $\tilde{\tilde{k}} = \frac{1}{\Delta S_0 \left|H_{0\tau}\right|^2} \int\limits_{\Delta S} \left|H_\tau\right|^2 ds$, ΔS_0 is the flat surface part corresponding to the area of the rough surface ΔS. The parameter $\tilde{\tilde{k}}$ is known as the roughness coefficient [27], which is a ratio between the surface resistance or reactance of the rough surface and that for the flat surface. The general approach for determining the parameter $\tilde{\tilde{k}}$ can be found in the monograph [18], where explicit formulas were obtained for some types of rough surfaces.

In the case of rectangular, periodically located notches (corrugations) in a conducting screen, the effective surface impedance can be determined in a different way, based on electrodynamic methods of diffraction grating analysis. If the corrugations are small, the equivalent boundary conditions can be written in the following form [28]: $E_z = Z_S^{eff} H_x, E_x = 0$ or $E_z = 0, E_x = Z_S^{eff} H_z$ for transverse or longitudinal notches with respect to the axis $\{0, z\}$. The axis $\{0, y\}$ is supposed to be directed vertically to the corrugated surface. Then according to [28]

$$\bar{Z}_S^{eff} = i(2g / L)\text{tg}kc, \tag{33}$$

where g is the notch width, c is the notch height, and L is the period of the corrugations.

2.3. Layered Dielectric Structures

Among of layered magnetodielectric structures which material parameters of the medium are piecewise constant functions of one coordinate, dielectric materials are used most frequently, since the layers are nonmagnetic materials in most applications. Electromagnetic waves propagation outside such structures (for example, above the underlying surface), can be analyzed by introducing the surface impedance, as for the air-dielectric interface over the half-space (formula (14)). Initially, such a formulation of the problem was caused by the interest in simulating of radio waves propagation over real layered soils. In some cases, it turns out to be methodically expedient to classified the layered dielectrics into natural and artificial structures.

Now, the method of radioimpedance sounding allowing to define the physical properties of inhomogeneous structures by application of interpretation models for experimental frequency dependences of surface impedance. This method applies to studies of the surface layer structure of the earth's crust (for example, [29]), and to a bio-impedance analysis of human body composition (for example, [30]). However, these questions are beyond the scope of this paper.

As for as the artificial structures are concerned, the most interesting from the point of view of practical applications are multilayer plane-parallel systems. Multilayer interference structures (MIS) are a set of various dielectric layers of small thicknesses which is of order or less than an operating wavelength. The MIS operation is based on interference effects occurring inside the system with multiple wave reflections at the interfaces between layers with different wave parameters. The material of the individual layers, layer numbers, sequence order and thickness are chosen depending on the spectral characteristics of the system as a whole.

The MIS are widely used in optics; however, these structures are increasingly being used in microwave techniques for creating matching devices, filters, wave energy absorbers, rejection elements and other waveguide sections. An important advantage of the MIS waveguide elements is absence of the wave mode conversion at the flat interface between media with different permeability. This is especially important in the millimeter wavelength range, since diffraction losses increase when the frequency is increasing and the waveguide dimension is decreasing. This effect can be explained by the wave front distortion at inhomogeneities of the waveguide path and the main mode transformation into rapidly damping higher modes, which are the source of the main wave energy losses.

The MIS are effective phase shifters: phases of the waves reflected by and transmitted through the structure can be easily controlled by variation of the structure parameters. Therefore, layered dielectric structures can be used as mirrors, band-pass and single-frequency filters, impedance matchers, absorbing materials, power dividers, and high Q resonant elements. As is known, it is very difficult to manufacture conventional diaphragm-pin impedance transformers for the millimeter and submillimeter wavelengths, while production of the high quality layered dielectric structures is quite simple.

Let an area occupied by the layered structure coincides with the half-space $-\infty < z < 0, (-\infty < x, y < +\infty)$ in the Cartesian coordinate system and the properties of the structure itself can vary only along the coordinate z. Then the electromagnetic field as function of $\vec{R}(x,y,z)$ can be represented as the discrete or continuum superposition of spatial inhomogeneous plane waves

$$\vec{E}(\vec{R}) = \vec{E}(\vec{k},z)e^{-i\vec{k}\vec{r}}, \ \vec{H}(\vec{R}) = \vec{H}(\vec{k},z)e^{-i\vec{k}\vec{r}}, \quad (34)$$

where $\vec{k} = (k_x, k_y, 0)$ is a spectral parameter, $\vec{E}(\vec{k},z)$ and $\vec{H}(\vec{k},z)$ are the vector amplitude of electric and magnetic wave, and $\vec{r} = (x,y,0)$. Then, the impedance boundary condition (1) can be written as

$$\vec{E}_\tau(\vec{k},z) = \hat{\bar{Z}}_S(\vec{k})[\vec{z}_0, \vec{H}_\tau(\vec{k},z)], \quad (35)$$

which establishes a relationship between the tangential components of the vector amplitudes on the plane $z = 0$. More precisely, limit values of the tangential components when $z \to -0$ or $z \to +0$ coincide in the formula (35) by virtue of the boundary condition defining continuity of the tangential field components. In the above formula, $\vec{z}_0$ s the unit vector of the axis $\{0,z\}$ considered to be directed upwards normal to the interface directed vertically upwards. The non-local impedance $\hat{\bar{Z}}_S(\vec{k})$ known also as frequency-dependent impedance is a dyadic function of the spectral parameter $\vec{k}$ that fully describes the interaction of the electromagnetic field with the medium in the lower half-space.

For an isotropic layered medium, the tensor $\hat{\bar{Z}}_S(\vec{k})$ is expressed in terms of two scalar values, which have the meaning of scalar impedance for waves of vertical and horizontal polarization, respectively. The scalar impedances can be calculated numerically using the Riccati equation. For a piecewise homogeneous medium, it can be determined analytically using recurrent formulas [13]. As an example, we present the result of problem solving for a layer of a magnetodielectric with material parameters (ε, μ) located on an perfectly conducting screen, when a layer is excited by a normally incident monochromatic plane wave

$$\bar{Z}_S = i\sqrt{\frac{\mu}{\varepsilon}} \mathrm{tg}(kh_d\sqrt{\varepsilon\mu}), \quad (36)$$

where h_d is the layer thickness. More detailed formulas for this case are presented in subsection 2.6.

For an arbitrarily anisotropic layered medium, the dyad $\hat{\bar{Z}}_S(\vec{k})$ is determined by four scalar quantities. In the general case, the dyad can be constructed only numerically by solving the matrix Riccati equation [31].

Application of the constant surface impedance (36) is usually limited by a condition associated with the provision of a single-mode wave propagation (including surface wave propagation) in the layered dielectric structures. For more general conditions, the frequency-dependent impedance (35) should be used. However, in some cases, even under the multimode regime of the layered structure, a constant surface impedance could be used. For example, if a dielectric layer on a metal screen is excited by a vertically oriented electric dipole, then the amplitudes of the reflected field can be correctly determined by using the constant impedance [32]. This is explained by the fact that of the possible for propagation of two or three types of waves inside a layer, in the cases considered, only the highest one will be effectively excited, thus providing the conditional single-mode mode.

Let us now consider a dielectric structure in which one of the layers is the planar volume of a cold plasma [33]. The equivalent permeability ε_{eff} and conductivity σ_{eff} of such layer is determined by the relations, $\varepsilon_{eff} = \varepsilon_0\left[1 - \omega_p^2/(\omega^2 + \nu^2)\right], \sigma_{eff} = \varepsilon_0\nu\omega_p^2/(\omega^2 + \nu^2),$ where ω_p is the angular plasma frequency, $\omega_p^2 = n_e e^2/(\varepsilon_0 m_e)$, ε_0 is the vacuum permittivity, e is the electron charge, m_e is the electron mass, n_e is the

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electron concentration, and ν is the frequency of electron-neutral collisions.

At present, interest in the studies of magnetoelectric materials and multiferroics, which are characterized by the interrelation of magnetic and electrical properties, has increased significantly. The magnetoelectric effect of such substances, i.e., to polarize in a magnetic field and to magnetized in an electric field, opens up a whole series of new areas for their practical use. The properties of magnetoelectric are described in the extensive overview [34], where a special attention is paid to media that have magnetoelectric properties at room temperature, for example, materials based on bismuth ferrite, films of garnet ferrites, etc., since these materials are most promising for practical applications. However, it turns out that the greatest magnetoelectric effects can be observed for composite multilayer structures. For example, the coefficient of the magnetoelectric effect in composites based on a piezoelectric-ferromagnetic, reaches $\sim 10^{-1}$, which is almost three orders of magnitude grater as compared to the best samples of single-phase multifferoics. Of course, these multi-layered structures can be described, as mentioned above, using the impedance approach. It is important, that the effective surface impedance can be varied by external electric and magnetic fields.

2.4. Thin Dielectric Frequency-Selective and Chiral Layers

Flat metal-dielectric frequency-selective surfaces (FSS) are now actively used to create microwave circuits and antennas, high-quality filters and resonators, wave-guiding structures, etc. The electrodynamic theory of the FSS based on metal screens have already being formed. For example, in monographs [35, 36], the effective transmission coefficients ware determined for several FSS structures. The transmission coefficients can be used to defined the effective surface impedance of the FSS. For doubly periodic dielectric structures [37], new problems have arisen associated with applicability of various cell and array geometries, the cell material parameters and metallic inclusions.

The mechanism of frequency-selectivity in dielectric frequency-selective structures differ from that in ordinary metal frequency-selective surfaces. Thin metal screens separated by a layer of a uniform dielectric with thickness of about a quarter wavelength can function as filters only for main waves. If a ratio of distances between layers and an operating frequency is large, higher modes can be excited in dielectric plates. The selectivity of the dielectric FSS is based on the behavior of higher modes excited in the plate, when these modes interfere with the main mode. At high frequencies, the characteristics of such structures strongly depend on a wave incidence angle. However, the dielectric structure at frequencies significantly lower than a cutoff frequency behaves as a homogeneous anisotropic material at a fundamental mode [38]. In this case, the properties of the artificial layer can be described by using the effective dielectric constant tensor, which, of course, allows us to introduce into consideration the effective surface impedance. Similar assertion can be made for the case of plane wave scattering at a two-periodic gyrotropic layer [39].

Among flat structures that are being actively investigated at the present time, thin chiral layers, structures with the property of chirality or enantiomorphism (from the Greek “χερ” – “hand”), should also be singled out. The chirality is a property of a living or non-living object that cannot be superposed on its image in a flat mirror for any movement and rotation. At microwave frequencies, the chiral properties can have only artificial media. The chiral inclusions at microwave frequencies are artificial conductive one-, two-, or three-dimensional microelements having a mirror-asymmetric shape, which dimensions are significantly smaller than the length of an excitation wave [40–42]. The chiral medium has a spatial dispersion; therefore, mirror-asymmetric microelements should be placed periodically at distances commensurate with a wavelength. The orientation of the geometrical axes of microelements must be chaotic, therefore, the chiral medium is biisotropic at the macroscopic level. If all the chiral microelements are oriented in one direction, then the structure becomes anisotropic [43, 44].

Various elements can be used in the chiral structures: three-dimensional objects (right-and-left handed metallic spirals [45, 46], spherical particles with spiral conductivity [47], open rings with protruding ends [48]) and two-dimensional microscopic objects (S-strip elements and their mirror equivalents [49, 50], flat multi-thread spirals, Möbius tapes [51], and others). The chiral layer is called planar if its elements are conductive microscopic strips of mirror-asymmetric shapes that are uniformly distributed on a dielectric or ferrite substrate. From the point of view of technical implementations of the chiral structure, the planar model is more preferable, however, its degree of chirality is less than that of the three-dimensional chiral structure. An increase in the chirality structure can be achieved by creating multi-layer chiral meta-structures, at the layer interfaces on which the stripe chiral microelements are distributed. The electrodynamic properties of single-layer arrays based on chiral strip elements are detailly studied [49–51].

Microwave chiral media can be described with the help of three material parameters: the relative permittivity ε, permeability μ, physical chirality parameter χ.

The material equations for the chiral medium can be presented in the CGS unit system as

$$\vec{D} = \varepsilon\vec{E} \mp i\chi\vec{H}\,,\;\; \vec{B} = \mu\vec{H} \pm i\chi\vec{E}\,, \tag{37}$$

where $\vec{D}$ and $\vec{B}$ are the electric and magnetic inductions. The upper plus or minus signs in the formulas (37) correspond to the physically chiral medium based on the right mirror forms of chiral elements, while the lower signs correspond to that based on the left mirror forms. Since the physically chiral medium is gyrotropic, the material equations include the tensor permittivity $\hat{\varepsilon}^{(\pm)}$ and permeability $\hat{\mu}^{(\pm)}$ [44, 52],

$$\vec{D} = \hat{\varepsilon}^{(\pm)}\vec{E}\,,\;\; \vec{B} = \hat{\mu}^{(\pm)}\vec{H}\,,$$
$$\hat{\varepsilon}^{(\pm)} = \begin{bmatrix} \varepsilon & \pm i\chi_E & 0 \\ \mp i\chi_E & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix},$$
$$\hat{\mu}^{(\pm)} = \begin{bmatrix} \mu & \mp i\chi_H & 0 \\ \pm i\chi_H & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}, \tag{38}$$

where $\chi_E = \chi/\eta$, $\chi_H = \chi\eta$, $\eta = \sqrt{\mu/\varepsilon}$. The upper plus or minus signs correspond to the chiral medium based on right spirals, while the lower signs correspond to the medium based on left spirals. The chirality parameter is included into the off-diagonal elements of tensors for the biotropic medium. The gyrotropic axis is directed along the axis $\{0z\}$. The determinants of matrices in (38) $\mathrm{Det}\hat{\varepsilon}^{(\pm)} = \varepsilon(\varepsilon^2 - \chi_E^2)$ and $\mathrm{Det}\hat{\mu}^{(\pm)} = \mu(\mu^2 - \chi_H^2)$ depend on the medium chirality parameters.

The parameter of physical chirality is determined by the dimension, shape and concentration of the microparticles in the medium, that is, it takes into account the resonant properties of the chiral element itself. Therefore, a physically chiral medium must be created for a specific, fairly narrow range of incident wave frequency, near the resonant frequency of the chiral element. This extremely complicates the theoretical and experimental study of the chirality parameter for practical media implementation. Strictly speaking, the description of wave processes in a bounded chiral medium is currently an intractable problem. Therefore, there is a need to develop approximate methods and algorithms for calculating the characteristics and parameters of limited chiral structures. We will not analyze geometric-chiral structures classified in a separate class, since they cannot be implemented in a thin-film format, consisting of a large number of layers, each of which is an ordered chiral composition.

2.5. Surface Impedance of Electrically Thin Vibrator

The surfaces of radiating or scattering vibrator antennas may have similar characteristics, with some of the impedance surfaces discussed above, i.e., they may differ from a perfectly conducting surface, hence the vibrators to be characterized as impedance ones.

Linear impedance vibrators are widely used in radio engineering and radio electronic complexes for various purposes, as stand along receiving and transmitting structures, elements of antenna systems, and devices of antenna-feeder paths. Wide application and multi-functional use of the impedance vibrators, including multi-element structures, is an objective prerequisite for theoretical studies of electrodynamic characteristics of such systems. Since longitudinal dimensions of the vibrators are comparable with the operating wavelength in the surrounding space, asymptotic long-wave or short-wave (quasi-optical) approximations for their analysis cannot be used. A correct mathematical modeling of real vibrator structures without increasing the complexity of formulating a corresponding electrodynamic problem, an impedance concept is successfully used (for example, the monographs [53–55] and references in it).

To determine the electrodynamic characteristics of electrically thin impedance vibrators, formulas for the numerical evaluation of the vibrator surface impedance should be obtained. With this purpose in mind, we consider an auxiliary problem of axially symmetric excitation of an infinite two-layer cylinder (external and internal radiuses are r and r_i) by a converging cylindrical wave. Let us introduce a cylindrical coordinate system (ρ, φ, z) so that the axis $\{0z\}$ is directed along the cylinder axis. Then, due to the problem symmetry, the electromagnetic field has only components E_z and H_φ, which depend only on the coordinate ρ. Permittivity and permeability of the media are ε, μ and ε_i, μ_i in the region $\rho \in [r_i, r]$ and $\rho \le r_i$, respectively.

The normalized surface impedance $\bar{Z}_S = E_z/H_\varphi$ can be found according to (1), under the condition as a solution of Maxwell equations in terms of the Bessel $I_{0,1}$ and $N_{0,1}$ Neumann functions

$$\begin{Bmatrix} iE_z \\ H_\varphi \end{Bmatrix} = \begin{Bmatrix} I_0(k\sqrt{\varepsilon\mu}r) + N_0(k\sqrt{\varepsilon\mu}r) \\ I_1(k\sqrt{\varepsilon\mu}r) + N_1(k\sqrt{\varepsilon\mu}r) \end{Bmatrix} \sqrt{\frac{\mu}{\varepsilon}} \times$$
$$\times \frac{\sqrt{\frac{\varepsilon}{\mu}} N_1(k\sqrt{\varepsilon\mu}r_i) I_0(k\sqrt{\varepsilon_i\mu_i}r_i) - \sqrt{\frac{\varepsilon_i}{\mu_i}} N_0(k\sqrt{\varepsilon\mu}r_i) I_1(k\sqrt{\varepsilon_i\mu_i}r_i)}{\sqrt{\frac{\varepsilon_i}{\mu_i}} I_0(k\sqrt{\varepsilon\mu}r_i) I_1(k\sqrt{\varepsilon_i\mu_i}r_i) - \sqrt{\frac{\varepsilon}{\mu}} I_1(k\sqrt{\varepsilon\mu}r_i) I_0(k\sqrt{\varepsilon_i\mu_i}r_i)}. \tag{39}$$

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Assuming that $r_i = 0$ and $|\varepsilon| >> 1$ $(\varepsilon = \varepsilon' + 4\pi\sigma / i\omega)$, we obtain the well-known formula the surface impedance of a cylindrical conductor which takes into account the skin-effect [56, 57]

$$\overline{Z}_S = \frac{k'}{120\pi\sigma} \frac{I_0(k'r)}{I_1(k'r)}, \quad (40)$$

where $k' = (1-i)/\Delta^0$, $\Delta^0 = \omega / k\sqrt{2\pi\sigma\omega\mu}$ is the skin layer thickness, and σ is metal conductivity.

It is possible to average the impedances along the cell period for the corrugated ($L_1 \sim L_2$) or the ribbed ($L_1 << L_2$) conductors (here L_1 is the ridge thickness with $\overline{Z}_S = 0$, L_2 is the cavity width with $\overline{Z}_S \neq 0$) with the cells period $(L_1 + L_2) << \lambda / \sqrt{\varepsilon\mu}$ and at $|\varepsilon_i| >> 1$. Then, taking into account (39), we have:

$$\overline{Z}_S = -i \frac{L_2}{L_1 + L_2} \sqrt{\frac{\mu}{\varepsilon}} \times$$
$$\times \frac{I_0(k\sqrt{\varepsilon\mu}r)N_0(k\sqrt{\varepsilon\mu}r_i) - I_0(k\sqrt{\varepsilon\mu}r_i)N_0(k\sqrt{\varepsilon\mu}r)}{I_1(k\sqrt{\varepsilon\mu}r)N_0(k\sqrt{\varepsilon\mu}r_i) - I_0(k\sqrt{\varepsilon\mu}r_i)N_1(k\sqrt{\varepsilon\mu}r)}, \quad (41)$$

which is also true for conductors with an insulating coating of an electrically-magneto-electrician $(L_1 = 0)$, as well as for metal cylinders $(r_i = 0)$ with transverse dielectric inserts $(L_2 << L_1)$.

Of particular practical interest is the case of thin vibrators, $|(k\sqrt{\varepsilon\mu}r)^2 \ln(k\sqrt{\varepsilon\mu}r_i)| << 1$, when the surface impedance does not depend on the excitation method of a conductor, and the corresponding boundary conditions become external [4], i.e. they coincide for exciting fields of any structure. Then, according to (39) – (41), we obtain expressions for the vibrator impedance in the thin wire approximation

$$\overline{Z}_S = \frac{1+i}{120\pi\sigma\Delta^0} \quad (42)$$

– the solid metallic cylinder of the $r >> \Delta^0$ radius ($\overline{Z}_S = 0$ for the case of the perfect conductivity, when $\sigma \to \infty$);

$$\overline{Z}_S = \frac{1}{120\pi\sigma h_0 + ikr(\varepsilon - 1)/2} \quad (43)$$

– the dielectrical metalized cylinder with covering, made of the metal of the $h_0 << \Delta^0$ thickness, hence at $\varepsilon = 1$;

$$\overline{Z}_S = \frac{1}{120\pi\sigma h_0} \quad (44)$$

– the tube metallic cylinder of the $r >> \Delta^0$ radius (for the “nano-radius” vibrator [57–59] $h_0 = r$, $r << \Delta^0$), and at $h_0 = 0$ in (43);

$$\overline{Z}_S = -i \frac{2}{kr(\varepsilon - 1)} \quad (45)$$

– the dielectrical cylinder;

$$\overline{Z}_S = -i \frac{L_2}{L_1 + L_2} \frac{2}{kr\varepsilon} \quad (46)$$

– the metal-dielectrical cylinder;

$$\overline{Z}_S = \frac{1}{120\pi\sigma h_0 - i / kr\mu \ln(r / r_i)} \quad (47)$$

– the magnetodielectrical metalized cylinder with the inner conducting cylinder, hence (44) at $r = r_i$, and at $h_0 = 0$;

$$\overline{Z}_S = ikr\mu \ln(r / r_i) \quad (48)$$

– the metallic cylinder with covering, made of magnetodielectric of the $r - r_i$ thickness or the ribbed cylinder.

The surface impedance for a vibrator with spiral conductivity, i.e., for the kiral objects [42], in a particular case of a monopillar metal helix can be found using the formula (31) as

$$\overline{Z}_S = (i/2) kr \,\mathrm{ctg}^2\psi, \quad (49)$$

where $r\,(kr << 1)$ is helix radius and ψ is an angle of helix.

The formulas (42) – (49) obtained within the framework of a general impedance concept are valid for thin infinite and finite cylinders, located in free space. If the vibrator is located in a material medium with parameters ε_1 and μ_1, a multiplier $\sqrt{\mu_1 / \varepsilon_1}$ should be included in the formulas. If the medium parameters ε and μ can be smoothly varied by a static electric or magnetic field, then the radiation characteristics of the system with fixed geometry can be controlled by these fields. As can be seen from formulas (48) and (49) the characteristics of the vibrators with the purely inductive surface impedance can be described by using a concept of effective vibrator length, defined by the formula

$$2L_{eff} = \left[1 + \frac{\mu \ln(r/r_i)}{2\ln(2L/r)}\right] 2L,$$

$$2L_{eff} = \left[1 + \frac{ctg^2\psi}{4\ln(2L/r)}\right] 2L. \qquad (50)$$

Thus, the electrodynamic characteristics of the impedance vibrator with the length $2L$ is equivalent to a perfectly conducting vibrator with a length $2L_{eff}$, so that $2L_{eff} > 2L$.

Separately, it is necessary to emphasize the possibility of calculating the surface resistance of a carbon nanotube [57–59]. For example, if the nanotube is located entirely in a dielectric with permittivity ε and permeability $\mu = 1$, the surface resistance can be determined using the following relation

$$\rho_S = i\pi^2 a\hbar^2(\omega - iv)/(2e^2 v_F), \qquad (51)$$

where a is nanotube radius, v_F is Fermi velocity, $v_F = 9.71 \cdot 10^5$ m/s, ω is cyclic frequency, v is relaxation frequency, $v = 3.33 \cdot 10^4$ Hz, e is electron charge, $\hbar$ is Planck constant.

2.6. Surface Impedance of a Magnetodielectric Layer on a Metal Substrate Surface

In this subsection, we present formulas determining the impedance $\bar{Z}_S$ for some specific examples of the physical implementation of impedance surfaces, First, we consider an auxiliary problem concerning a normal incidence of a plane electromagnetic wave on a dielectric layer that separates two half-spaces. The layer thickness is h_d, complex permittivity and permeability are ε_1, and μ_1, and the wave number $k_1 = k\sqrt{\varepsilon_1\mu_1}$. The incidence wave propagates in the free upper half-space $(\varepsilon = \mu = 1)$, while the lower half space is characterized by material parameters ε_2, μ_2.

The boundary value problem solution can be easily obtained by taking into account the boundary conditions for the electric and magnetic fields on both surfaces of the dielectric layer. Comparing this solution with the requirements of the Shchukin-Leontovich impedance boundary condition (1) on the upper boundary of the dielectric layer, we obtain the rigorous expression for the distributed surface impedance

$$\bar{Z}_S = \bar{Z}_1 \frac{i\bar{Z}_1 tg(k_1 h_d) + \bar{Z}_2}{\bar{Z}_1 + i\bar{Z}_2 tg(k_1 h_d)}, \qquad (52)$$

where $\bar{Z}_1 = \sqrt{\mu_1/\varepsilon_1}$ and $\bar{Z}_2 = \sqrt{\mu_2/\varepsilon_2}$.

If the layer of magnetodielectric is on the perfectly metal surface, the formula (52) after substitution $\bar{Z}_2 = 0$ is reduced to the relation similar to (36)

$$\bar{Z}_S = i\sqrt{\mu_1/\varepsilon_1}\, tg(k_1 h_d), \qquad (53)$$

where h_d is magnetodielectric thickness. In the case of an arbitrary incident field, formula (53) is approximate and becomes more accurate if the inequality $|\varepsilon_1\mu_1| >> 1$ better inequality holds (approximation of geometrical optics [4]).

As shown in the references [60 – 62], real parts of permittivity and/or permeability of magnetodielectric metamaterials can become negative. Therefore, the formula (53) can be written as [63] (first received by the authors):

$$\bar{Z}_S = \pm i\sqrt{\mu_1/\varepsilon_1}\, tg(k_1 h_d), \qquad (54)$$

where the sign plus or minus correspond to the cases $\mathrm{Re}\,\varepsilon_1 > 0$ or $\mathrm{Re}\,\varepsilon_1 < 0$.

If a layer is electrically thin ($|k_1 h_d| << 1$), when the quasi-stationary approximation [4, 64]), the relations (53) and (54) allow to obtain that $\bar{Z}_S \approx ik\mu_1 h_d$ [64 – 66], i.e. the normalized surface impedance does not depend on the material permittivity, as is the case for thin impedance vibrators (see formula (48)).

If a thin conductive film with a thickness h_R ($(h_R/\Delta^0) << 1$), is deposited on a layer of a magnetodielectric metamaterial located on a metal plane, the surface impedance defined by the formula (52) is equal to

$$\bar{Z}_{SR} = \frac{\bar{R}_{SR}}{1 + \bar{R}_{SR}/\bar{Z}_S}, \quad \bar{R}_{SR} = \frac{1}{Z_0\sigma_1 h_R}, \qquad (55)$$

where $\bar{Z}_S$ is defined by formulas (53) and (54).

Conclusion

The purpose of the paper was to systematize the results of using the concept of approximate impedance boundary conditions for electromagnetic fields in electrodynamics problems based on an analytical review of literature sources. The limits and conditions for their correct application are analyzed, including the

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requirements for the geometry of the boundary surfaces on which it is performed. The types of metal-dielectric structures are presented, for which methods of theoretical determination of the values of surface impedances are currently known. For most of the considered cases, the article contains formula expressions for calculating the values of surface impedances (except for those where the formulas are too cumbersome and contain a large number of different structural parameters that require a separate description). At the same time, attention is paid to the structures of the film type, which are the most promising for the creation of technological control elements on their basis in devices of the centimeter and millimeter wavelength ranges. The results for surface impedances, which can characterize electrically thin vibrators, are also presented quite fully, and used in the publication of monographs [53 – 55]. Of course, the materials of the paper do not claim to be a complete reference book covering all the results and aspects of the development of the concept of approximate boundary conditions of the impedance type in problems of electrodynamics over the last 80 years. At the same time, the authors hope that the information given in the paper will be useful for specialists in the field of theoretical and applied electrodynamics, as well as for graduate students, young scientists and students who are just mastering radio physical and radio engineering specialties.

Contribution of authors: development of subsections impedance boundary conditions and the limits of their correct application and real metals – **Sergey Berdnik**; development of subsectionsrough and corrugated metal screens and layered dielectric structures – **Andrey Gomozov;** development of subsectionsrough thin dielectric frequency - selective and chiral layers – **Dmitriy Gretskih;** development of an introduction to section 2 and subsection surface impedance of a magnetodielectric layer on a metal substrate surface and surface impedance of electrically thin vibrators – **Viktor Kartich;** analysis of the literature, setting and substantiation of the purpose and objectives of the study, development of conceptual provisions, research methodology and presentation of results. Section development introduction and conclusion – **Mikhail Nesterenko**.

All the authors have read and agreed to the published version of the manuscript.

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НАБЛИЖЕНІ ГРАНИЧНІ УМОВИ ДЛЯ ЕЛЕКТРОМАГНІТНИХ ПОЛІВ В ЕЛЕКТРОМАГНЕТИЗМІ

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Наведено результати аналітичного огляду літературних джерел щодо використання наближених граничних умов для електромагнітних полів імпедансного типу при вирішенні крайових завдань електромагнетизму за більш ніж 80 останніх років. У цей період імпедансний підхід був узагальнений на широке коло електродинамічних завдань, у яких його використання дозволило суттєво розширити межі математичного моделювання, що адекватно враховує фізичні властивості реальних граничних поверхонь. Минуло понад вісімдесят років з моменту публікації наближених граничних умов для електромагнітних полів. Сенс і цінність цих умов полягає в тому, що вони дозволяють вирішувати завдання дифракції про поля поза тілом, що добре

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проводять, без урахування полів усередині них, що значно спрощує рішення. З того часу застосуванню імпедансних граничних умов присвячено велику кількість публікацій, основні з яких (на думку авторів) представлені у цій статті. Особливу увагу приділено характеристикам електрично тонких імпедансних вібраторів та поверхневих структур плівкового типу, як особистому внеску авторів у теорію імпедансних граничних умов в електромагнетизмі. **Предметом** дослідження у статті є аналіз меж та умов коректного застосування імпедансних граничних умов. **Метою** є систематизація результатів використання концепції наближених імпедансних граничних умов електромагнітних полів у завданнях електродинаміки з урахуванням аналітичного огляду літературних джерел. Отримано такі **результати**. Представлені типи металодіелектричних структур, для яких в даний час відомі методи теоретичного визначення значень поверхневих імпедансів для конструкцій плівкового типу, які є найбільш перспективними для створення на їх основі технологічних елементів управління в приладах сантиметрового та міліметрового діапазонів довжин хвиль. **Висновки**. Матеріали статті не претендують на звання повного довідника, що охоплює всі результати та аспекти розвитку концепції наближених граничних умов імпедансного типу у завданнях електромагнетизму за останні десятиліття. Водночас автори сподіваються, що інформація, викладена у статті, буде корисною фахівцям у галузі теоретичної та прикладної електродинаміки, а також аспірантам, молодим вченим та студентам, які тільки освоюють радіофізичні та радіотехнічні спеціальності.

Ключові слова: імпедансний підхід; граничні умови імпедансного типу; поверхневий імпеданс; ефективний імпеданс; імпедансна поверхня.

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