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DISCRETE ATOMIC COMPRESSION OF DIGITAL IMAGES

The ***subject matter*** *of this paper is the discrete atomic compression (DAC) of digital images, which is a lossy compression process based on the discrete atomic transform (DAT). The* ***goal*** *is to investigate the efficiency of the DAC algorithm. We solve the following* ***tasks****: to develop a general compression scheme using discrete atomic transform and to compare the results of DAC and JPEG algorithms. In this article, we use the* ***methods*** *of digital image processing, atomic function theory, and approximation theory. To compare the efficiency of DAC with the JPEG compression algorithm we use the sets of the classic test images and the classic aerial images. We analyze compression ratio (CR) and loss of quality, using uniform (U), root mean square (RMS) and peak signal to noise ratio (PSNR) metrics. DAC is an algorithm with flexible parameters. In this paper, we use "Optimal" and "Allowable" modes of this algorithm and compare them with the corresponding modes of JPEG. We obtain the following* ***results****: 1) DAC is much better than JPEG by the U-criterion of quality loss; 2) there are no significant differences between DAC and JPEG by RMS and PSNR criterions; 3) the compression ratio of DAC is much higher than the compression ratio of JPEG. In other words, the DAC algorithm saves more memory than the JPEG compression algorithm with not worse quality results. These results are due to the fundamental properties of atomic functions such as good approximation properties, the high order of smoothness and existence of locally supported basis in the spaces of atomic functions. Since generalized Fup-functions have the same convenient properties, it is clear that such compression results can be achieved by application of a generalized discrete atomic transform, which is based on these functions. We also discuss the obtained results in the terms of approximation theory and function theory.* ***Conclusions****: 1) it is possible to achieve better results with DAC than with JPEG; 2) application of DAC to image compression is more preferable than JPEG in the case when it is planned to use recognition algorithms; 3) further development and investigation of the DAC algorithm are promising.*

Keywords: *atomic function; discrete atomic transform; lossy image compression; JPEG compression.*

Introduction

The amount of information has increased significantly due to the rapid development of technology now. A large part of the big data is digital images. For instance, over a trillion digital photos were taken in 2017 [1].

As it can be seen from the fig. 1, the number of digital photos taken annually increases significantly and will continue to grow. It is clear that this leads to an increase in the cost of storing, processing and transferring them over networks. Hence, the problem of digital image compression is of high importance.

There are many different algorithms for data compression. Foundations, basic principles and detailed description of the most used compression methods can be found in [2 - 4].

Image compression has its own specific features. One of them is that the human eye is the best criterion for assessing of processing quality. Therefore some loss of quality in the process of image compression is permissible. Using lossy compression algorithms, it is possible to obtain significant reduction in resources with invisible or almost invisible changes of the original image. The JPEG algorithm, which is the most popular image compression algorithm, is one of them. Its description can be found in [2 - 5] (see also [6] for the JPEG file format).

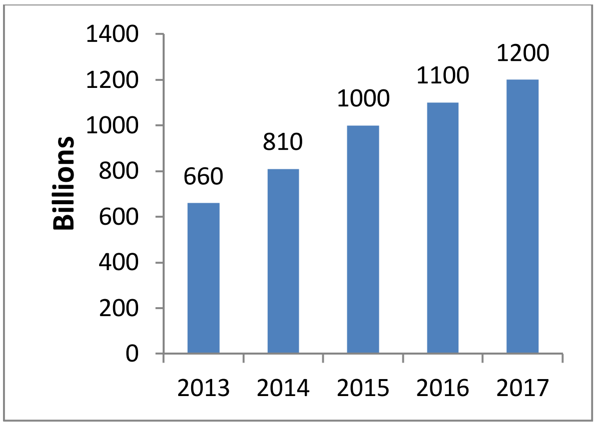

Fig. 1. Dynamics of changes in the number of digital photos [1]

In many cases, efficiency of data compression algorithm with loss of quality depends on the features of the applied mathematical tools. For example, the JPEG algorithm, which is based on a discrete cosine transform, is highly effective for compression of digital images due to the good approximation properties of the trigonometric polynomials.

Atomic functions have the same advantages. In particular, it was shown in [7] that spaces of linear combinations of V.A. Rvachev up-function are asymptotically extremal for approximation of periodic differentiable functions in the norm of the space C. Also, in the paper [8], it was prove that these spaces are extremal for approximation of periodic differentiable functions in L_2-norm. In addition, asymptotic extremeness of some other atomic space was obtained in [9]. In terms of approximation theory this means that atomic functions are as perfect mathematical tool as trigonometric polynomials.

One of the main disadvantages of trigonometric functions is the non-compactness of their support. For this reason, systems of functions with a compact support are widely used in different applications, in particular, in digital image processing and compression [10]. We note that the JPEG2000 compression algorithm, which is based on wavelets, was supposed to be a replacement for JPEG [11], but it did not become widely used.

Spaces of atomic functions combine good approximation properties with an existence of locally supported basis. Hence, an idea of their application to lossy image compression is quite natural. In [12], it was shown that atomic functions

$$\mathrm{up}_s(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\prod_{k=1}^{\infty}\frac{\sin^2\left(st(2s)^{-k}\right)}{s^2(2s)^{-k}\sin\left(t(2s)^{-k}\right)}dt$$

can be useful in compression of full color digital images. It should be also mentioned that application of atomic functions to compression of medical images was considered in [13].

In this paper, we consider application of discrete atomic transform, which is based on the atomic function $\mathrm{up}_s(x)$ and was introduced in [14], to lossy compression of full color digital images.

1. Formulation of the problem

A large number of lossy image compression algorithms contain the following steps: preliminary processing of data, discrete transform of the data, quantization and encoding of the quantized data (fig. 2).

In JPEG image compression algorithm, discrete cosine transform (DCT) is used. Since trigonometric polynomials have good approximation properties, data is well represented by a small number of DCT-coefficients. Therefore it is possible to reduce information redundancy and hence compress the image.

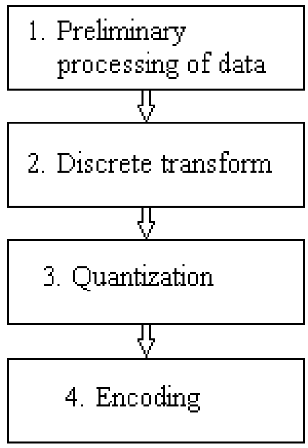

Fig. 2. Lossy compression scheme

In the paper [14], discrete atomic transform (DAT) was introduced. Also, it was shown that this procedure can be much more effective than DCT. Whence, application of DAT to lossy image compression is prospective.

The aim of this paper is to develop full color 24-bit image compression algorithm based on DAT and investigate its efficiency.

2. Discrete atomic compression

Consider some full color 24-bit bitmap image represented by RGB-matrix. Figure 3 shows the process of its compression using DAT.

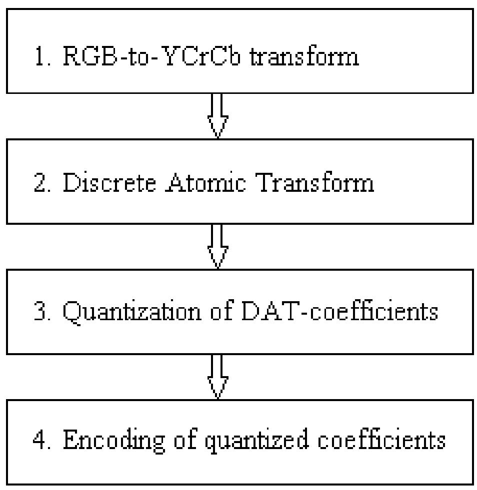

Fig. 3. Discrete atomic compression of full color 24-bit digital images

In the first step of this algorithm, transformation from RGB to YCrCb color space is used. After this step we obtain the matrix Y of luma components and two matrices Cr and Cb of chroma components. In the step 2, we apply DAT-procedure based on the space

$$UP_{s,n} = \left\{ \varphi : \ \varphi(x) = \sum_k c_k \, up_s \left(x - \frac{k}{(2s)^n} \right) \right\}$$

to each of these matrices [14]. We get three matrices Ω_Y, Ω_{Cr} and Ω_{Cb} of DAT-coefficients. After that elements of these matrices are quantized and rounded to integers. The human eye is more sensitive to loses of quality in luminance than in chrominance. Hence, larger quantization coefficients can be used for elements of Ω_{Cr} and Ω_{Cb}. Finally, quantized coefficients are encoded using lossless compression algorithms. Run-length encoding and statistical methods of data compression are suitable for this purpose. To obtain the decoded image, we perform the reverse steps.

We call this algorithm the **discrete atomic compression** (DAC) of digital images.

The combination of high quality and high compression ratio is supposed to be achieved as a result of convenient properties of DAT [14].

It can be seen that DAC is a lossy compression algorithm. Major loss of quality occur during the quantization stage. By varying the quantization coefficients, it is possible to obtain results of different size and quality.

3. Efficiency of lossy compression

The main efficiency indicator of data compression algorithm is the compression ratio (CR)

$$CR = \frac{\text{size of original image}}{\text{size of compressed image}}.$$

It is clear that higher value of CR means higher memory savings.

For lossy compression algorithms, another important indicator is the measure of quality loss. There are many numerical criteria for assessing loss of quality, but none of them is reliable. The human eye is the best criteria. In fact, this is one of the problems of digital image processing.

In this paper, we use the following criteria:

1. Uniform metric (U-metric)

$$d(X,Y) = \max_{i,j} \left| x_{ij} - y_{ij} \right|. \qquad (1)$$

Notice that high dependence on local changes is the key feature of this measure.

2. Root mean square metric (RMS-metric)

$$d(X,Y) = \sqrt{\frac{1}{nm} \sum_{i,j=1}^{n,m} \left(x_{ij} - y_{ij} \right)^2}. \qquad (2)$$

This measure characterizes averaged changes of all pixels. Low dependence on local changes and high dependence on global changes are key features of this measure.

Let us remark that higher value of U-metric and RMS-metric means worse quality of result.

3. Peak-to-peak signal-to-noise ratio (PSNR-metric)

$$d(X,Y) = 10 \log_{10} \frac{255^2 nm}{\sum_{i,j=1}^{n,m} \left(x_{ij} - y_{ij} \right)^2}. \qquad (3)$$

It differs from RMS only in another scale. Higher value of this measure means better quality of the result.

In (1) – (3), X and Y are before and after processing image matrices.

For the case of RGB matrices, we use

$$d(X,Y) = \max_{i,j} \{ |x.r[i][j] - y.r[i][j]|, |x.g[i][j] - y.g[i][j]|, |x.b[i][j] - y.b[i][j]| \}, \qquad (4)$$

$$d(X,Y) = \sqrt{\frac{S}{3nm}} \qquad (5)$$

and

$$d(X,Y) = 10 \log_{10} \frac{255^2 \cdot 3nm}{S}, \qquad (6)$$

where

$$S = \sum_{i,j=1}^{n,m} \Big((x.r[i][j] - y.r[i][j])^2 + (x.g[i][j] - y.g[i][j])^2 + (x.b[i][j] - y.b[i][j])^2 \Big),$$

instead of (1) – (3) respectively. Here each element of X and Y has red, green and blue components:

$$x[i][j] = (x.r[i][j], x.g[i][j], x.b[i][j]),$$
$$y[i][j] = (y.r[i][j], y.g[i][j], y.b[i][j]).$$

The choice of data compression algorithm is usually based on how effective and suitable this method is.

For the case of single image compression, it is natural to find optimal setting of the compressor. Note that optimality means the smallest size at a given quality level or the best quality at a given size limit. The procedure for finding the optimal settings can be quite complicated.

For the case of compressing a large number of images, the search for optimal parameters for each image may be too expensive. A compromise solution of this problem is the selection of such compressor mode that is suitable for a group of test images. In this case, a comparison of the effectiveness of different compression algorithms is reduced to a comparison of the results of their application with some fixed settings.

In the next section, we compare several modes of DAC and JPEG. For this purpose, the following procedure is used:

1) fix the modes of DAC and JPEG and compress test images;

2) calculate CR-value, U-metric, RMS-metric and PSNR-metric for each compression result;

3) compare the corresponding values of CR, U, RMS and PSNR for each test image.

4. DAC vs JPEG

To compare DAC with JPEG, we use a set of the classic test images and a set of the classic aerial images [15] (fig. 4, fig. 5).

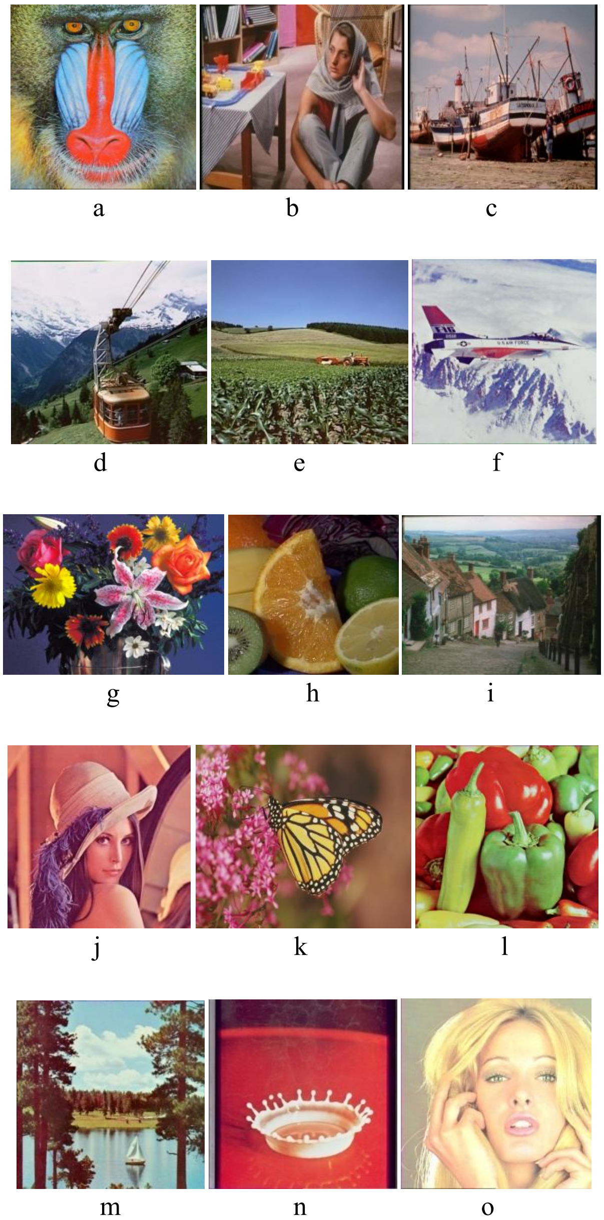

Fig. 4. Classic test images: a – baboon, b – Barbara, c – boats, d – cable car, e –corn field, f – f-16, g – flowers, h – fruits, i – hills, j – Lena, k – monarch, l – peppers, m – sailboat, n – splash, o – Tiffany

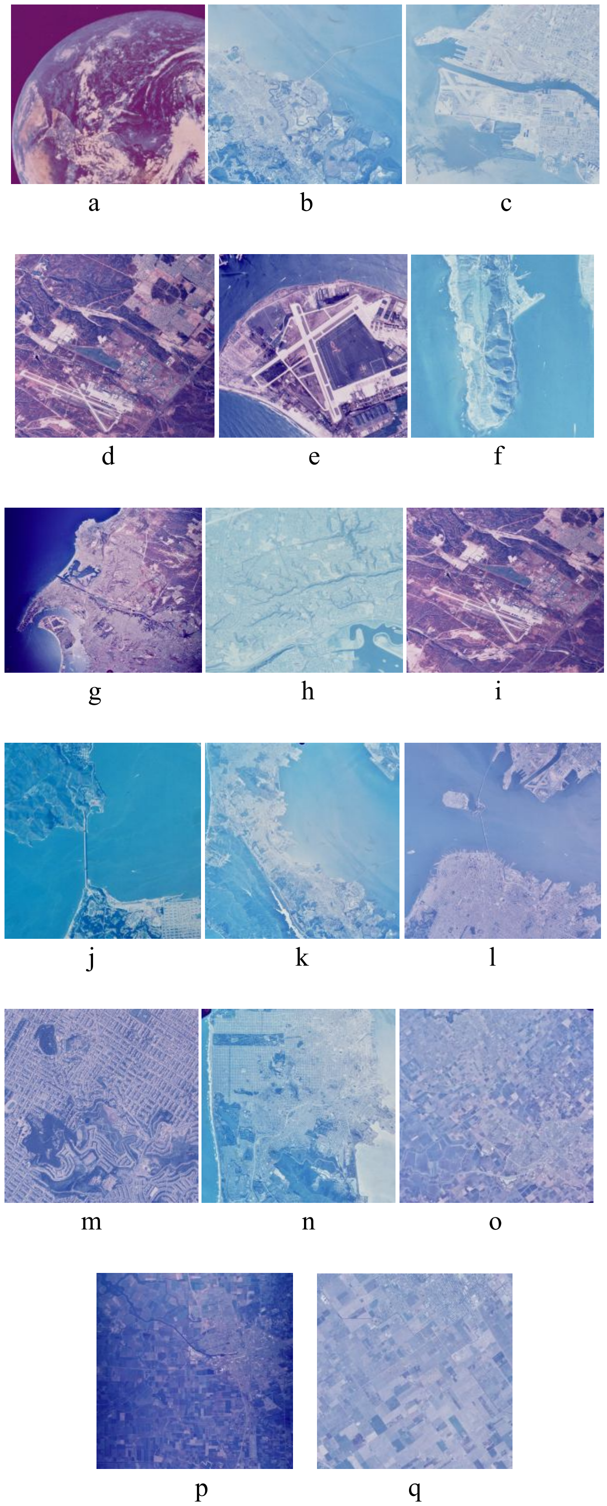

Fig. 5. Classic aerials: a – Earth, b – Foster City, c – Oakland, d – San Diego 1, e – San Diego 2, f – San Diego 3, g – San Diego 4, h – San Diego 5, i – San Diego 6, j – San Francisco 1, k – San Francisco 2, l – San Francisco 3, m – San Francisco 4, n – San Francisco 5, o – Stockton 1, p – Stockton 2, q – Stockton 3

There are different ways to compress an image by JPEG. As it was stated in the previous section, we should use the fixed settings of the codec for any test image. To obtain JPEG compression of the test images, we use computer programs MS Picture Manager and MS Paint, which are easily accessible to the average user. In the program MS Picture Manager, we use the function "Export…" to obtain JPEG compression with the best quality (for this purpose, we choose 100 for the compression parameter). We call this mode "JPEG 100". Further, in MS Paint, we use "Save as…" function. We call this compression "Paint JPEG". The results thus obtained almost correspond to JPEG compression in MS Picture Manager with the parameter of quality equal to 90. Note that we got almost the same results using other software and web-services.

To obtain DAC compression of the test images, we use the computer program "Discrete Atomic Compression: User Kit" with the modes "Optimal" and "Allowable" [16]. Using the first one, we get the result of DAC processing with a high compression ratio and no visible changes. Application of the second mode provides a higher compression ratio with more noticeable loss of quality.

Now we compare DAC "Optimal" with "JPEG 100" and DAC "Allowable" with "Paint JPEG". All original images and their compressed and decompressed versions are available on the link to Google drive: https://drive.google.com/file/d/1wcEYKSjIXRUM29tE66KXXE4yP-pmvx-F/view?usp=sharing.

4.1. DAC "Optimal" vs "JPEG 100"

In the table 1, results of the classic test images compression are presented.

Table 1

Results of compression of the classic test images by DAC "Optimal" and "JPEG 100"

No	**Original image**		**DAC "Optimal"**					**"JPEG 100"**				
	name	size (kB)	size (kB)	CR	loss of quality			size (kB)	CR	loss of quality		
					U	RMS	PSNR			U	RMS	PSNR
1	baboon	769	331	2,324	27	5,315	33,621	334	2,302	73	8,75	29,291
2	Barbara	1216	282	4,312	23	3,065	38,402	375	3,243	36	2,694	39,522
3	boats	1330	229	5,808	16	2,313	40,848	344	3,866	31	1,952	42,32
4	cablecar	721	163	4,423	25	2,831	39,093	199	3,623	57	3,48	37,298
5	cornfield	721	193	3,736	24	3,1	38,302	221	3,262	55	3,878	36,358
6	f16	769	151	5,093	21	2,671	39,596	206	3,733	93	4,123	35,827
7	flowers	531	172	3,087	26	3,568	37,082	188	2,825	90	5,087	34,002
8	fruits	721	157	4,592	27	2,775	39,265	197	3,66	66	3,483	37,293
9	goldhill	1216	272	4,471	22	3,284	37,804	377	3,225	50	2,818	39,132
10	Lena	769	164	4,689	29	3,436	37,411	222	3,464	93	3,409	37,477
11	monarch	1153	186	6,199	16	2,225	41,186	293	3,935	34	1,5	44,611
12	peppers	769	186	4,134	24	4,113	35,849	241	3,191	120	5,841	32,801
13	sailboat	769	241	3,191	27	4,668	34,749	271	2,838	96	6,799	31,482
14	splash	769	124	6,202	21	2,499	40,177	195	3,944	165	4,153	35,762
15	Tiffany	769	164	4,689	24	3,274	37,829	221	3,48	135	5,702	33,011

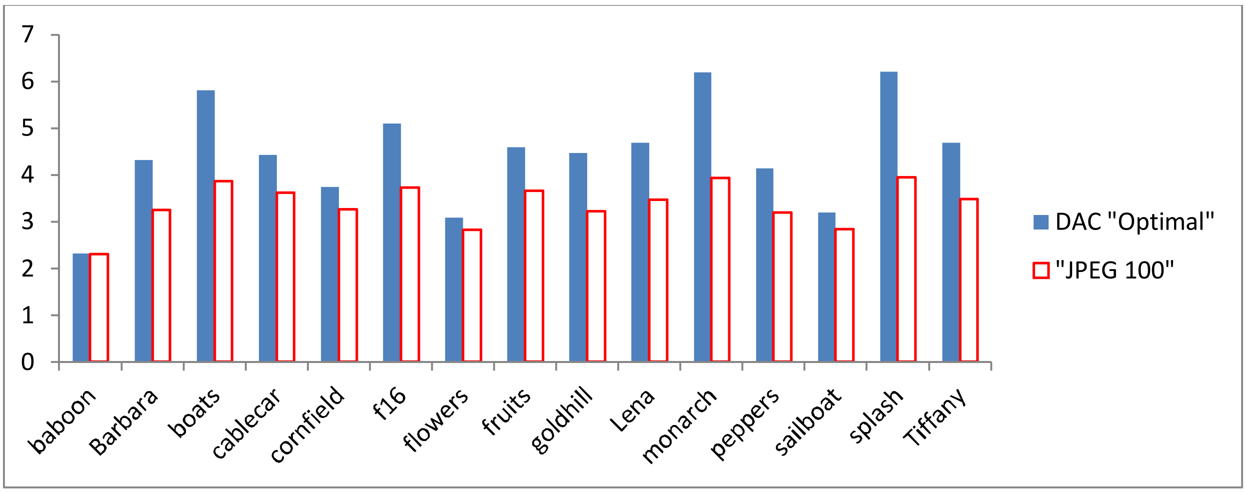

Fig. 6. Classic test images. Compression ratio

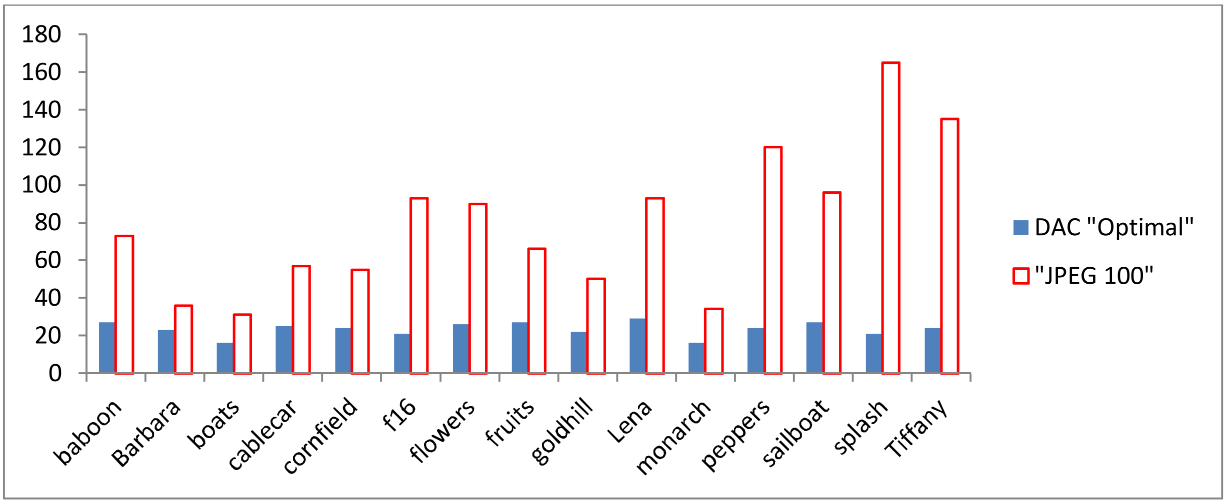

Fig. 7. Classic test images. Loss of quality: U-criterion

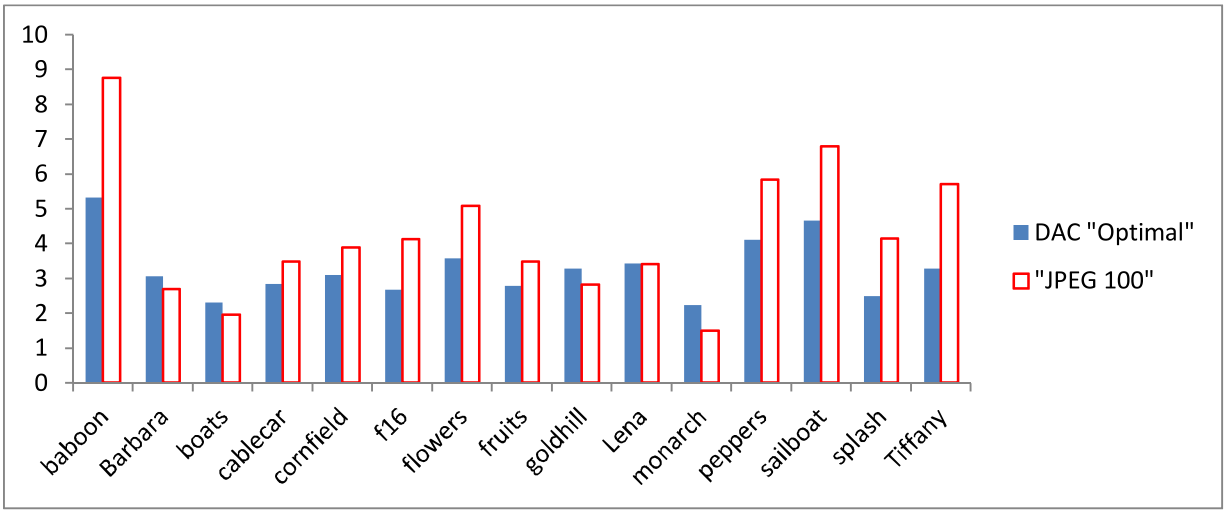

Fig. 8. Classic test images. Loss of quality: RMS-criterion

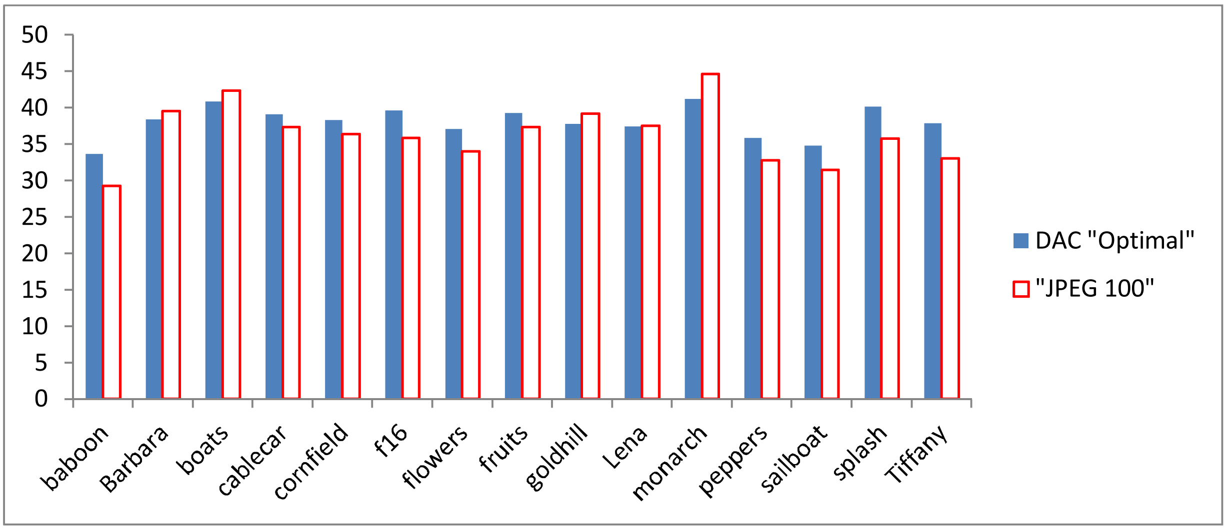

Fig. 9. Classic test images. Loss of quality: PSNR-criterion

From table 1 and figures 6 – 9, it follows that

1) DAC "Optimal" saves more memory than "JPEG 100" (actually, the size of DAC-file is smaller than the size of the corresponding JPEG-file by an average of 30,59 percent);

2) "JPEG 100" produces greater local loss of quality than DAC "Optimal";

3) DAC "Optimal" and "JPEG 100" have almost the same averaged loss of quality.

We can obtain more accurate comparison of quality loss as follows.

Consider any test image. Let $d(n)$ be the number of pixels that have absolute RGB changes equal to n after DAC "Optimal" processing. Also, denote by $j(n)$ the number of pixels that have absolute RGB changes equal to n after "JPEG 100" processing. By comparing these functions, we get more accurate difference between compression algorithms. The graphs of $d(n)$ and $j(n)$ for test images "Lena", "Baboon" and "Monarch" are presented on the figures 10 – 12.

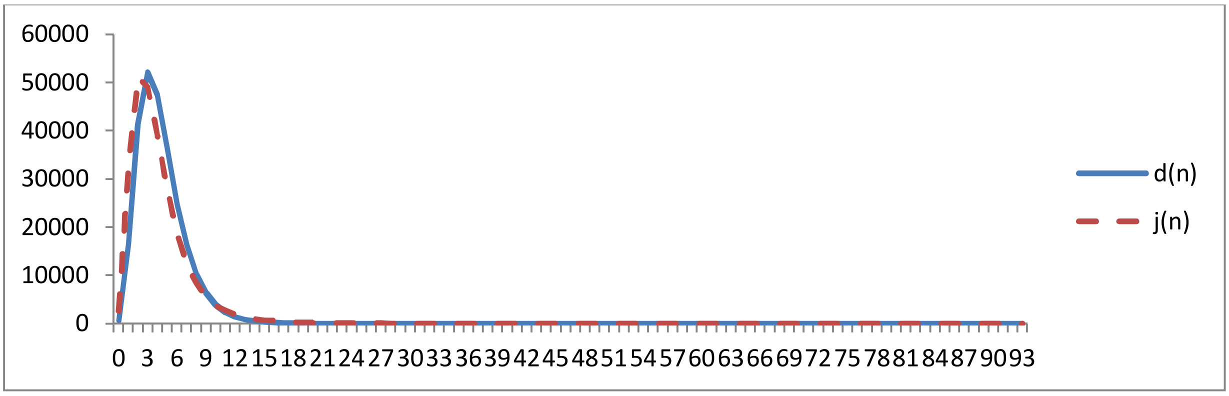

Fig. 10. Graphs of $d(n)$ and $j(n)$ for the image "Lena"

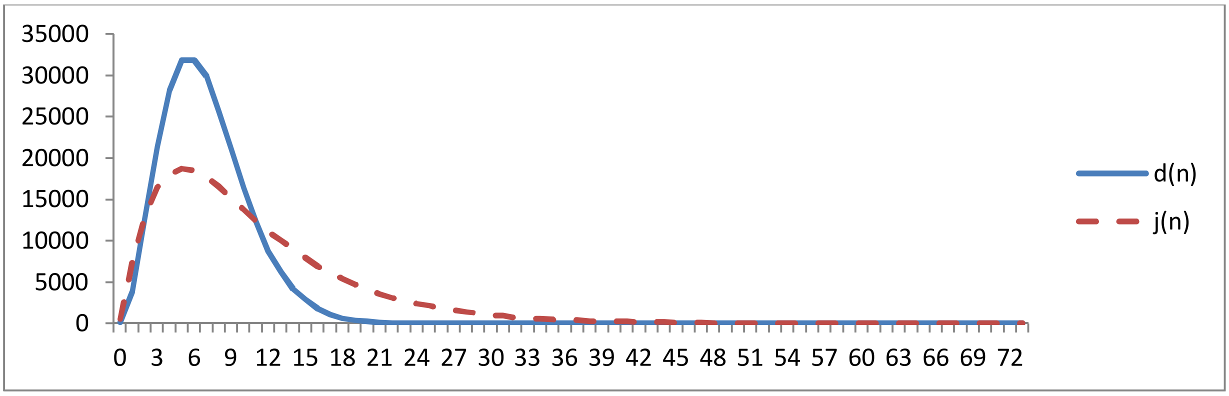

Fig. 11. Graphs of $d(n)$ and $j(n)$ for the image "Baboon"

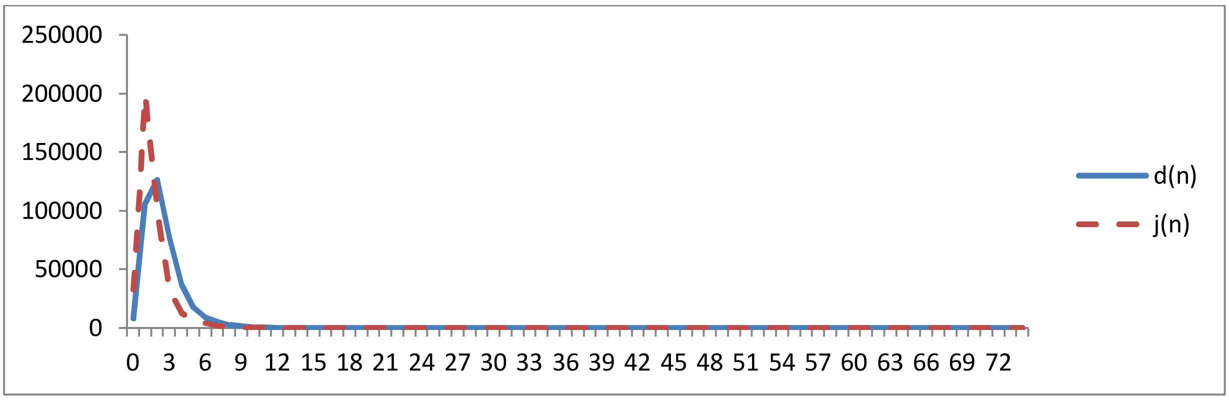

Fig. 12. Graphs of $d(n)$ and $j(n)$ for the image "Monarch"

We see that difference between the functions d(n) and j(n) significantly depends on the image. On average, these functions behave in the same way.

Consider now classic aerial images compression results, which are presented in the table 2 and figures 13 – 16. It follows that

1) on the average, U-value of "JPEG 100" loss of quality is greater than U-value of DAC "Optimal" by 20 (it should be mentioned that the minimal difference equals 5 and the maximal equals 45);

2) RMS-values and PSNR-values do not differ significantly;

3) DAC "Optimal" compresses better than "JPEG 100" (it can be seen that the size of DAC-file is smaller than the size of the corresponding JPEG-file at least 13 percent).

Table 2

Results of compression of the classic aerial images by DAC "Optimal" and "JPEG 100"

No	Original image		DAC "Optimal"					"JPEG 100"				
	name	size (kB)	size (kB)	CR	loss of quality			size (kB)	CR	loss of quality		
					U	RMS	PSNR			U	RMS	PSNR
1	Earth	769	193	3,997	21	3,703	36,76	247	3,122	32	3,594	37,018
2	Foster City	769	198	3,879	26	4,056	35,97	246	3,13	45	4,372	35,317
3	Oakland	3073	689	4,459	24	3,755	36,639	931	3,301	29	3,524	37,191
4	San Diego 1	769	277	2,773	27	4,694	34,701	323	2,381	50	4,67	34,744
5	San Diego 2	769	228	3,378	27	3,912	36,282	274	2,81	40	3,738	36,678
6	San Diego 3	769	163	4,715	24	3,599	37,007	224	3,44	31	3,351	37,628
7	San Diego 4	3073	1016	3,026	28	4,596	34,883	1222	2,515	70	5,103	33,975
8	San Diego 5	3073	702	4,377	21	3,349	37,632	940	3,271	28	3,065	38,403
9	San Diego 6	3073	988	3,11	27	4,453	35,158	1143	2,69	44	4,791	34,522
10	San Francisco 1	769	176	4,383	27	3,487	37,281	224	3,444	51	4,11	35,854
11	San Francisco 2	3073	706	4,354	26	3,587	37,036	908	3,385	54	3,727	36,704
12	San Francisco 3	3073	743	4,143	28	3,866	36,385	960	3,201	40	3,831	36,464
13	San Francisco 4	3073	1043	2,947	26	4,671	34,742	1200	2,562	58	4,489	35,088
14	San Francisco 5	3073	974	3,157	28	4,439	35,185	1139	2,697	73	4,899	34,329
15	Stockton 1	3073	779	3,947	26	3,988	36,115	1054	2,916	41	3,304	37,751
16	Stockton 2	3073	800	3,841	27	3,984	36,125	1039	2,957	62	4,266	35,531
17	Stockton 3	3073	766	4,015	27	4,079	35,919	1008	3,049	36	3,616	36,966

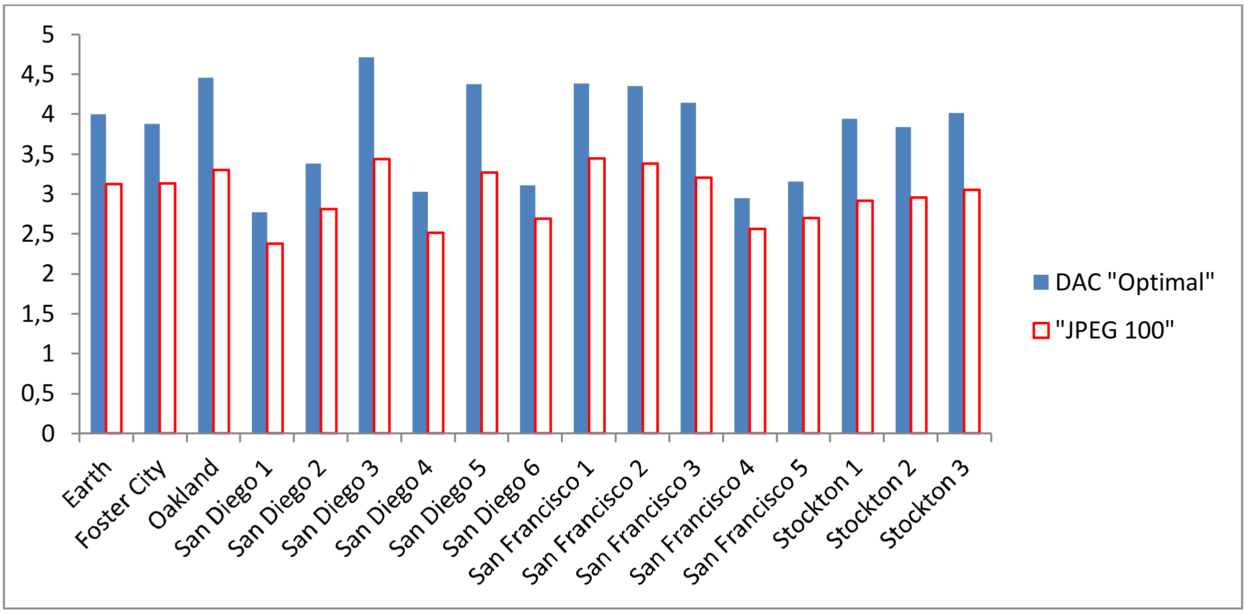

Fig. 13. Classic aerial images. Compression ratio

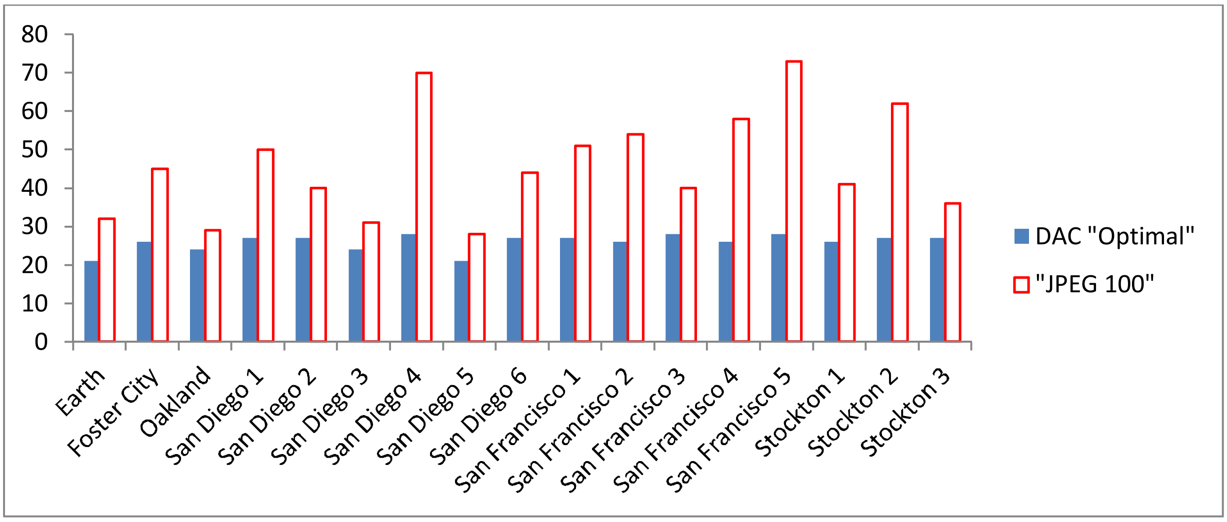

Fig. 14. Classic aerial images. Loss of quality: U-criterion

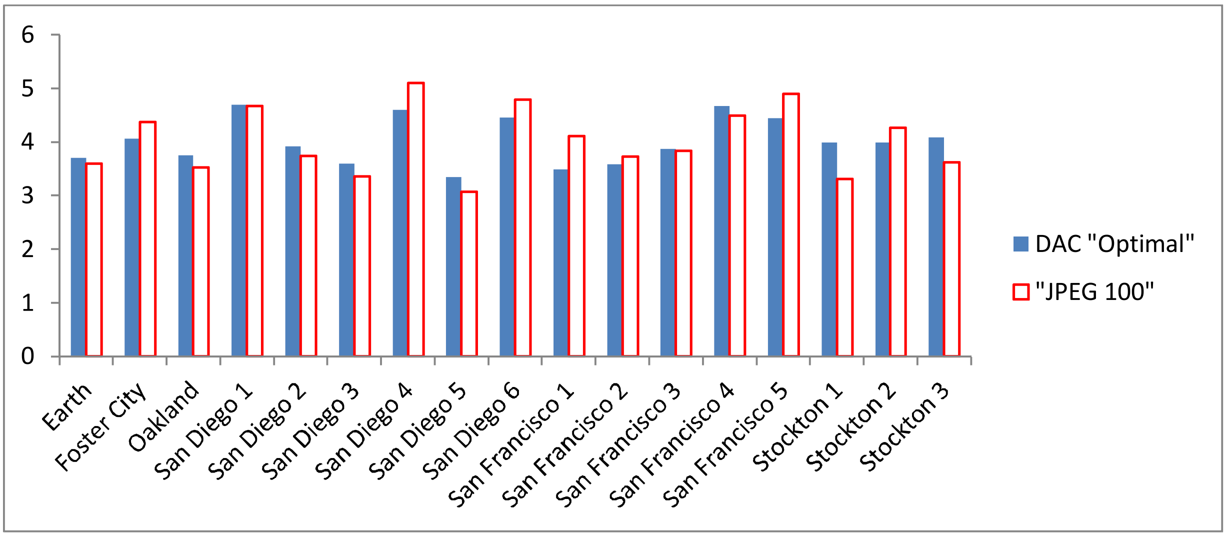

Fig. 15. Classic aerial images. Loss of quality: RMS-criterion

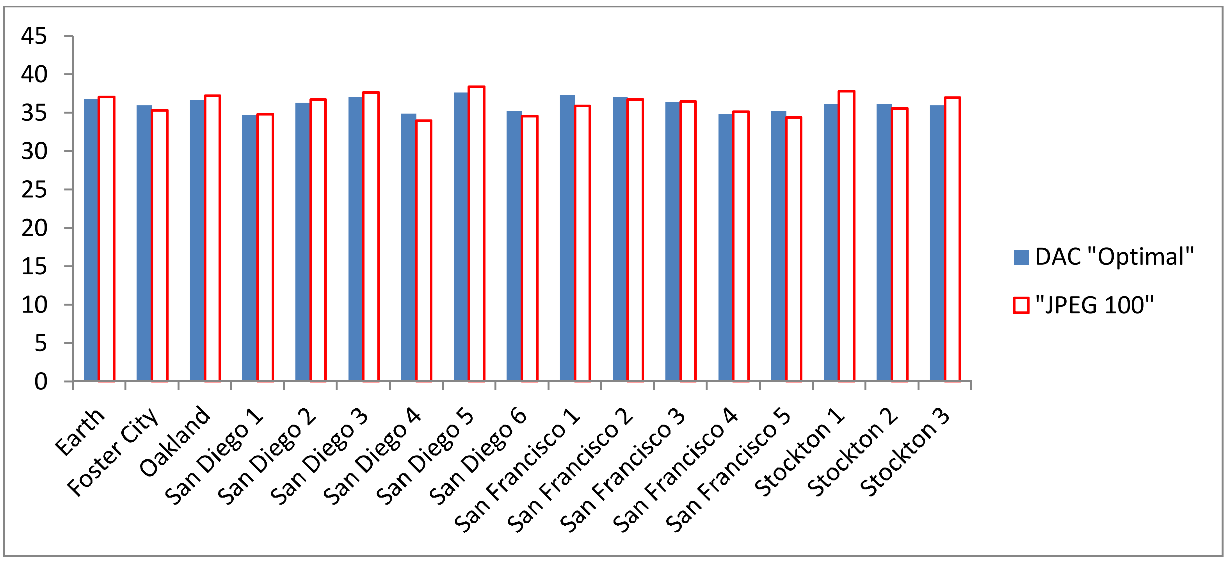

Fig. 16. Classic aerial images. Loss of quality: PSNR-criterion

Besides, if we compare visually results of DAC and JPEG compression, we see no visible differences.

Hence, **DAC "Optimal" and "JPEG 100" have almost the same loss of quality**. In addition, **DAC "Optimal" saves more memory than "JPEG 100"**.

Therefore, we claim that application of DAC "Optimal" is more preferable than "JPEG 100".

4.2. DAC "Allowable" vs "Paint JPEG"

Compare two other modes of DAC and JPEG.

First, it can be checked that visually results of compression are almost equal.

Secondly, the tables 3, 4 and figures 17 – 24 present numerical results.

Table 3

Results of compression of the classic test images by DAC "Allowable" and "Paint JPEG"

No	Original image		DAC "Allowable"					"Paint JPEG"				
	name	size (kB)	size (kB)	CR	loss of quality			size (kB)	CR	loss of quality		
					U	RMS	PSNR			U	RMS	PSNR
1	baboon	769	162	4,747	70	10,24	27,924	173	4,445	75	9,344	28,721
2	Barbara	1216	138	8,812	45	5,42	33,451	172	7,07	42	3,658	36,865
3	boats	1330	115	11,566	31	3,496	37,259	146	9,12	33	2,807	39,166
4	cablecar	721	83	8,688	55	4,608	34,87	90	8,011	60	4,152	35,765
5	cornfield	721	101	7,139	51	5,284	33,671	103	7	57	4,518	35,032
6	f16	769	76	10,118	68	4,05	35,982	90	8,544	81	4,663	34,758
7	flowers	531	90	5,9	64	6,335	32,102	91	5,835	97	5,767	32,912
8	fruits	721	77	9,364	47	4,409	35,244	83	8,687	64	4,178	35,711
9	goldhill	1216	127	9,575	39	5,001	34,15	168	7,238	48	3,961	36,174
10	Lena	769	73	10,534	63	4,898	34,33	97	7,928	98	4,226	35,613
11	monarch	1153	105	10,981	37	3,858	36,404	131	8,802	31	2,463	40,302
12	peppers	769	84	9,155	52	5,565	33,222	113	6,805	123	6,475	31,906
13	sailboat	769	116	6,629	68	7,121	31,08	133	5,782	100	7,428	30,713
14	splash	769	57	13,491	49	3,4	37,5	80	9,613	161	4,651	34,781
15	Tiffany	769	72	10,681	67	4,868	34,383	95	8,095	132	6,188	32,3

Table 4

Results of compression of the classic aerial images by DAC "Allowable" and "Paint JPEG"

No	Original image		DAC "Allowable"					"Paint JPEG"				
	name	size (kB)	size (kB)	CR	loss of quality			size (kB)	CR	loss of quality		
					U	RMS	PSNR			U	RMS	PSNR
1	Earth	769	87	8,895	32	5,165	33,87	113	6,813	35	4,605	34,866
2	Foster City	769	90	8,54	61	6,805	31,474	110	6,988	49	5,183	33,84
3	Oakland	3073	288	10,698	49	5,223	33,772	407	7,563	39	4,515	35,038
4	San Diego 1	769	135	5,726	57	8,712	29,328	166	4,648	50	5,777	32,897
5	San Diego 2	769	110	7,035	49	6,353	32,071	134	5,765	45	4,784	34,535
6	San Diego 3	769	68	11,438	43	4,881	34,361	96	8,061	38	4,304	35,453
7	San Diego 4	3073	490	6,28	63	9,113	28,938	599	5,133	77	6,006	32,559
8	San Diego 5	3073	309	9,947	38	4,83	34,452	409	7,514	35	4,16	35,749
9	San Diego 6	3073	458	6,721	56	7,162	31,03	560	5,492	51	5,8	32,863
10	San Francisco 1	769	81	9,597	53	5,366	33,538	96	8,073	55	4,809	34,49
11	San Francisco 2	3073	323	9,523	61	5,923	32,679	395	7,781	57	4,548	34,974
12	San Francisco 3	3073	330	9,315	59	6,716	31,589	428	7,18	44	4,71	34,67
13	San Francisco 4	3073	482	6,374	49	7,175	31,015	608	5,058	63	5,623	33,132
14	San Francisco 5	3073	449	6,851	58	7,233	30,945	553	5,56	69	5,858	32,776
15	Stockton 1	3073	353	8,719	57	6,676	31,641	500	6,151	41	4,495	35,077
16	Stockton 2	3073	365	8,436	58	6,466	31,918	482	6,379	73	5,212	33,79
17	Stockton 3	3073	327	9,366	54	6,268	32,188	468	6,576	37	4,7	34,687

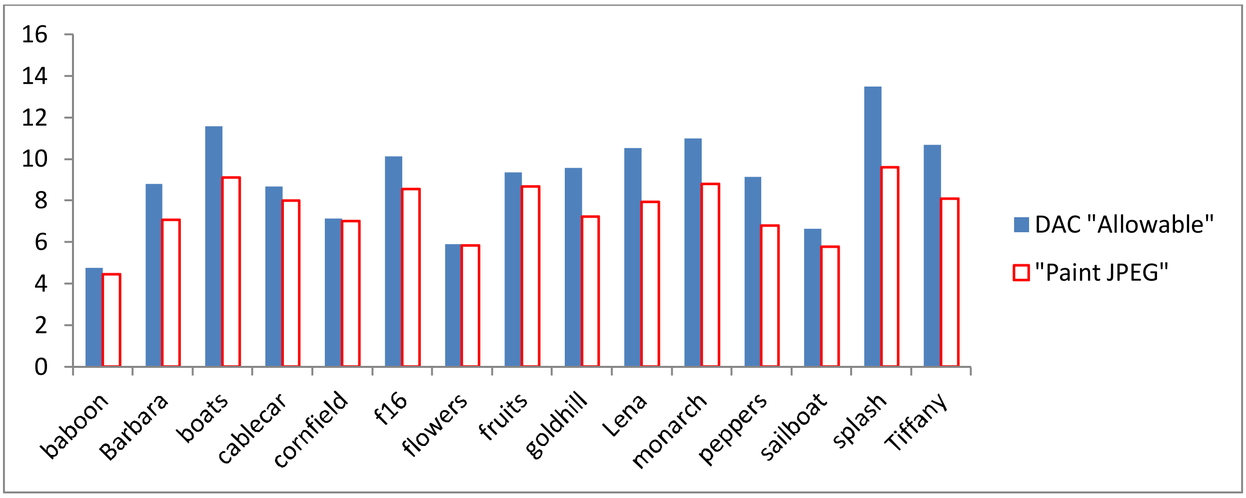

Fig. 17. Classic test images. Compression ratio

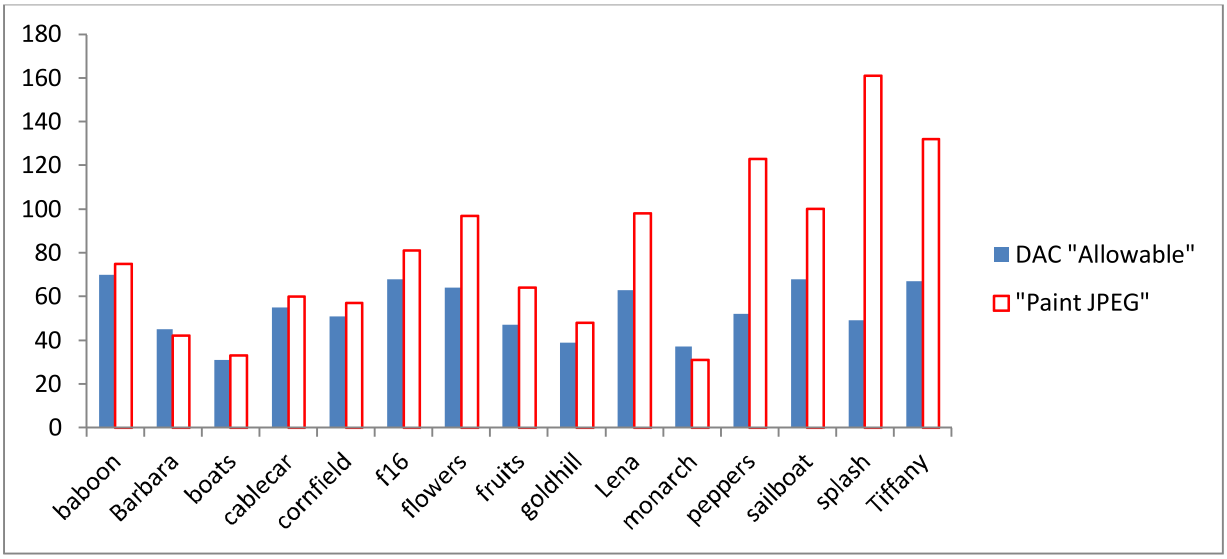

Fig. 18. Classic test images. Loss of quality: U-criterion

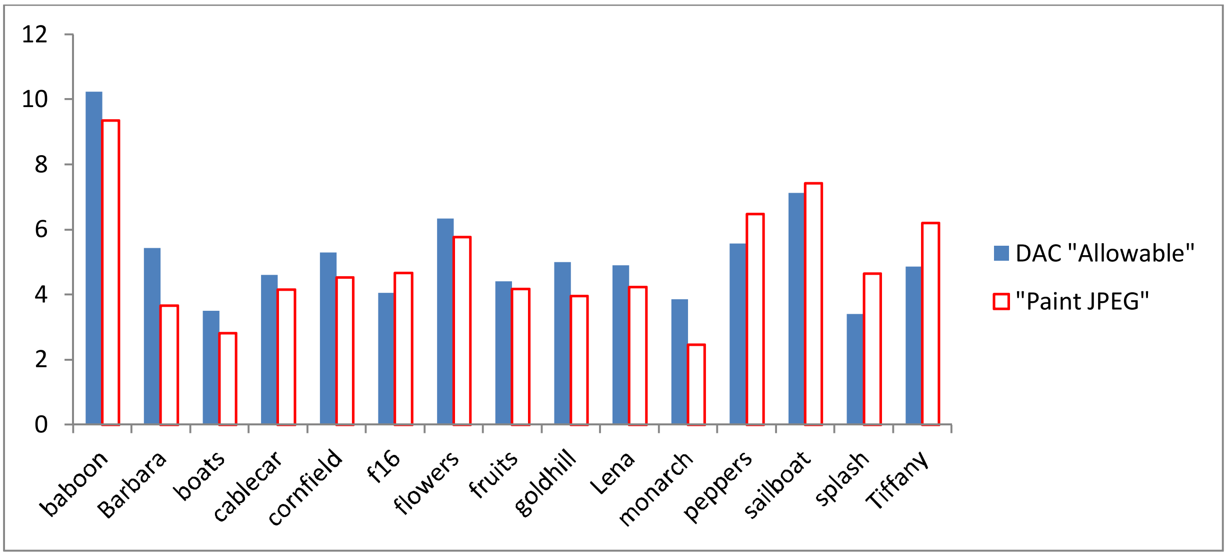

Fig. 19. Classic test images. Loss of quality: RMS-criterion

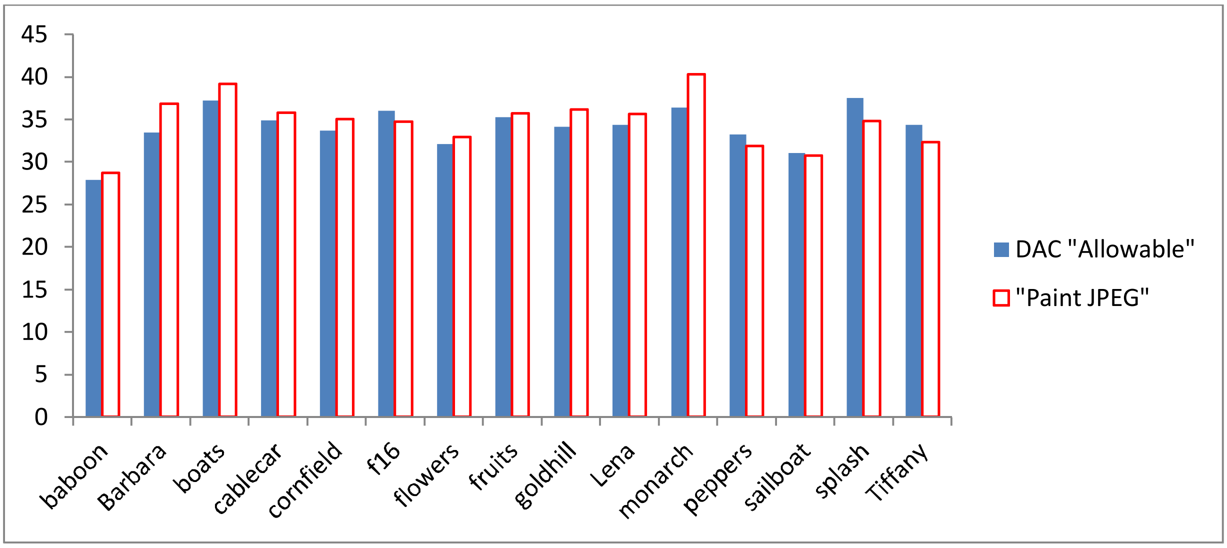

Fig. 20. Classic test images. Loss of quality: PSNR-criterion

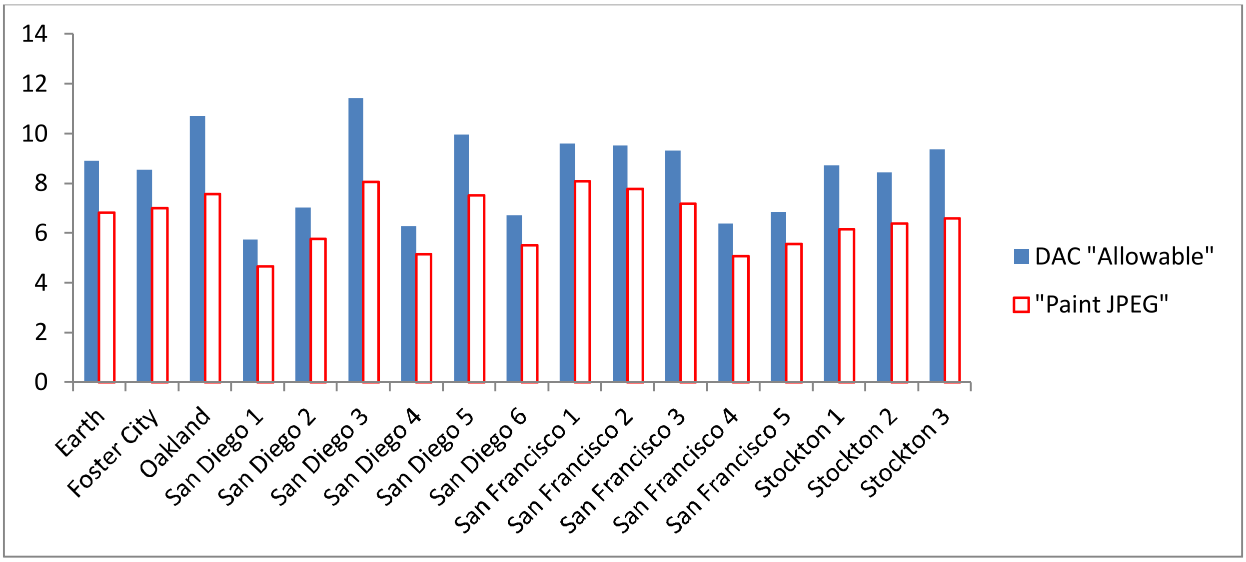

Fig. 21. Classic aerial images. Compression ratio

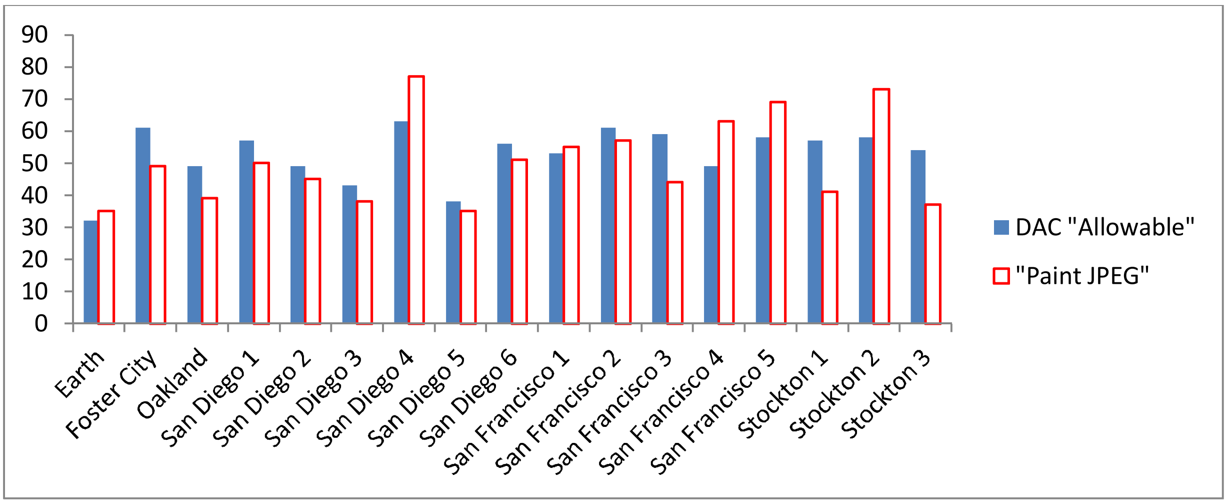

Fig. 22. Classic aerial images. Loss of quality: U-criterion

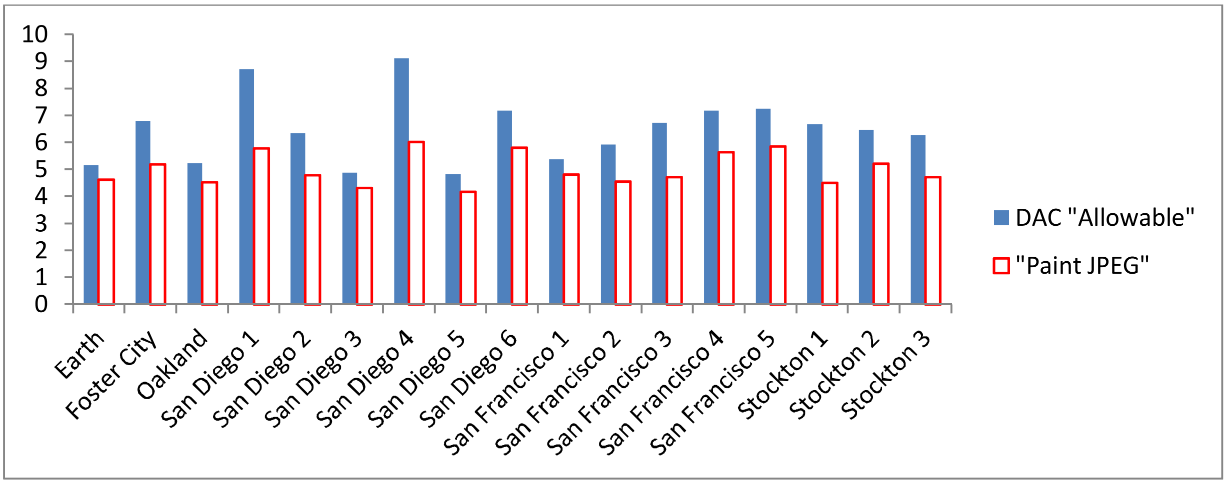

Fig. 23. Classic aerial images. Loss of quality: RMS-criterion

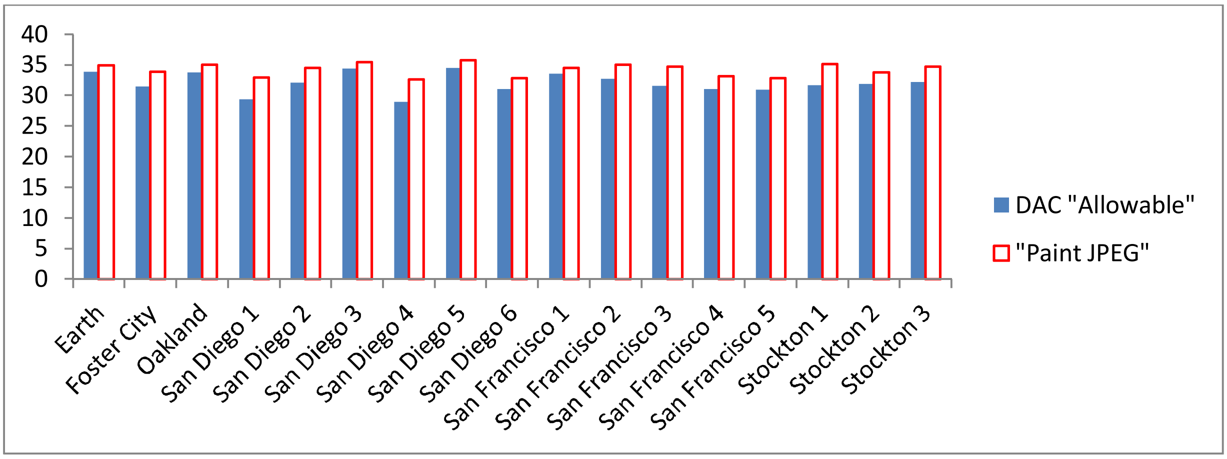

Fig. 24. Classic aerial images. Loss of quality: PSNR-criterion

We see that DAC “Allowable” is more “lossy” than “Paint JPEG”. But the difference is quite small and can be eliminated by changing of DAC settings. At the same time, the compression ratio of DAC “Allowable” is much higher than the JPEG’s one. This implies that DAC “Allowable” saves essentially more memory than “Paint JPEG” with a small deterioration in the quality (note that visual changes are hard to see). Also, if we reduce quantization coefficients of DAC and thus achieve almost similar values of U, RMS, and PSNR, we obtain such mode of DAC that still has higher compression ratio.

5. Discussion of the results

In this section, we discuss the results obtained in the previous one.

Generalized results are presented in the tables 5, 6.

First, we see that DAC provides less variation of CR, U, RMS and PSNR. In other words, using DAC, we can obtain more stable results.

Secondly, on the average, loss of quality obtained by DAC “Optimal” is not greater than loss of quality obtained by “JPEG 100”.

Thirdly, on the average, quality provided by “Paint JPEG” is insignificantly better than quality provided by DAC “Allowable”.

Finally, on the average, compression ratio of DAC is essentially higher than compression ratio of JPEG. Therefore, using DAC, we can save more memory than with usage of JPEG. For instance, total size of the initial image files is 51409 kB. Total size of DAC “Optimal” files is 13456 kB and total size of “JPEG 100” files is 16966 kB. Using DAC “Allowable”, we get 6221 kB. And, using “Paint JPEG”, we get 7889 kB.

Now we consider the results in terms of function theory and approximation theory.

Since spaces $UP_{s,n}$ are asymptotically extremal for approximation of periodic differentiable functions, DAT-coefficients are almost as perfect as DCT-coefficients for description of data in the case of sufficiently high dimension of the corresponding functional space. To obtain bigger compression ratio without changing the scheme of DAC, some coefficients of quantization should be increased. Indeed, we get higher compression and higher loss of quality. Also, in this case, DAT-coefficients become less perfect. Therefore the following hypothesis is quite reasonable: CR of JPEG is greater than CR of DAC with the same low quality of result. In other words, JPEG is seemed to be more effective than DAC if quality requirements are low. If we compare general results, which are shown in the tables 5 and 6, we get a partial proof of this statement. Currently, the requirements for quality of digital images are very high. Therefore, compression with high or even medium loss of quality may be unacceptable by most users. Hence, the advantage of JPEG over DAC described above may be useless in the future.

Table 5

General results: classic test images

Compressor	Compression ratio			U-metric			RMS-metric			PSNR-metric		
	average	min	max	average	min	max	average	min	max	average	min	max
DAC "Optimal"	4,463	2,324	6,202	23,467	16	29	3,276	2,225	5,315	38,081	33,621	41,186
"JPEG 100"	3,373	2,302	3,944	79,6	31	165	4,245	1,5	8,75	36,412	29,291	44,611
DAC "Allowable"	9,158	4,747	13,491	53,733	31	70	5,237	3,4	10,24	34,105	27,924	37,5
"Paint JPEG"	7,532	4,445	9,613	80,133	31	161	4,965	2,463	9,344	34,715	28,721	40,302

Table 6

General results: classic aerial images

Compressor	Compression ratio			U-metric			RMS-metric			PSNR-metric		
	average	min	max	average	min	max	average	min	max	average	min	max
DAC "Optimal"	3,794	2,773	4,715	25,882	21	28	4,013	3,349	4,694	36,107	34,701	37,632
"JPEG 100"	2,992	2,381	3,444	46,118	28	73	4,026	3,065	5,103	36,127	33,975	38,403
DAC "Allowable"	8,439	5,726	11,438	52,765	32	63	6,475	4,83	9,113	32,048	28,938	34,452
"Paint JPEG"	6,513	4,648	8,073	50,471	35	77	5,005	4,16	6,006	34,199	32,559	35,749

It should be mentioned that there is a dependence of the compression result on some special features of the initial test image. The smoothness of changing pixel colors is such a feature.

Consider in more detail the results of compression of the test images "Baboon", "Lena" and "Monarch" by DAC "Optimal" and "JPEG 100".

Above all, we see that U-value of DAC is significantly less than JPEG's one. One of the reasons for this is the application of chroma sub-sampling procedure in JPEG compression. This step can be changed or canceled, but we thus get a new mode of JPEG that should be compared with another mode of DAC.

One of the most important differences between "Baboon", "Lena" and "Monarch" is the number of sharp color changes. It is evident that "Baboon" contains significant number of such jumps (see fig. 25). In these terms, other two pictures are smoother (see figures 26 and 27). Besides, "Monarch" has large blurred areas.

Atomic functions $up_s(x)$ are nonquasianalytic. Hence, they are less smother than trigonometric polynomials, which are analytic. This implies that DAT is more preferable than DCT in case of processing of contrast images. Indeed, the figure 11 illustrates that loss of quality provided by JPEG is greater than loss of quality obtained after DAC processing. Also, an increase in image smoothness leads to an increase in the quality of the JPEG processing (see fig. 10 and 12).

Contrast increasing is often used, since this kind of image processing is visually perceived as an improvement in quality. Moreover, such a procedure as reducing the number of pixels makes significant changes that reduce the order of smoothness (it should be mentioned that this procedure is very often applied when uploading photos to Facebook, Instagram, etc). The application of DAC to compression of digital images, which were processed with such transformations, is reasonable.

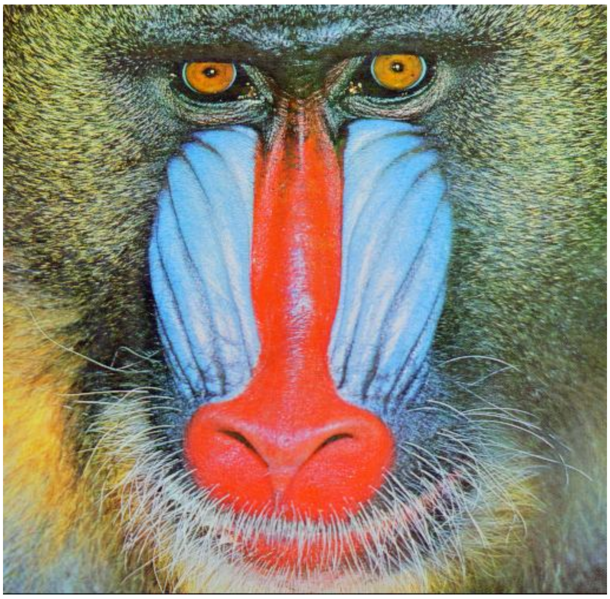

Fig. 25. Test image "Baboon"

Fig. 26. Test image "Lena"

Conclusions

In this paper, we have introduced the discrete atomic compression of full color 24-bits bitmap images. Also, we've compared it with the JPEG compression algorithm. The obtained results show that DAC saves significantly more memory than JPEG with the same loss of quality. This means that with DAC it is possible to achieve better results than with JPEG. It was also shown that application of DAC to image compression is more preferable than JPEG in the case when it is planned to use recognition algorithms. Hence, further development and investigation of the DAC algorithm are promising.

Fig. 27. Test image "Monarch"

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ДИСКРЕТНЕ АТОМАРНЕ СТИСНЕННЯ ЦИФРОВИХ ЗОБРАЖЕНЬ

І. В. Брисіна, В. О. Макарічев

Предметом вивчення статті є дискретне атомарне стиснення (ДАС) цифрових зображень, що є процесом стиснення з втратами якості та заснованим на використанні дискретного атомарного перетворення (ДАП). **Метою** є дослідження ефективності алгоритму ДАС. **Завдання**: розробити загальну схему компресії з використанням ДАП і порівняти результати алгоритмів ДАС та JPEG. У даній роботі ми використовуємо **методи** цифрової обробки зображень, теорії атомарних функцій та теорії наближень. Для того, щоб порівняти ефективність алгоритмів ДАС та JPEG, ми використовуємо набори класичних тестових зображень та класичних аерофотозображень. Ми аналізуємо коефіцієнт стиснення та втрати якості, використовуючи рівномірну (U) й середньоквадратичну (RMS) метрики, а також відношення сигнал/шум (PSNR). ДАС – це алгоритм, який можна налаштовувати, виходячи з конкретних потреб. У даній роботі ми використовуємо режими «Оптимальний» та «Допустимий» цього алгоритму. Отримано наступні **результати**: 1) ДАС є кращим ніж алгоритм JPEG з точки зору U-критерію; 2) немає значних відмінностей між ДАС та JPEG з точки зору критеріїв RMS і PSNR; 3) коефіцієнт стиснення алгоритму ДАС вище ніж коефіцієнт стиснення алгоритму JPEG. Тобто за допомогою ДАС можна отримати більшу економію пам'яті ніж з використанням JPEG при не гіршій якості результатів. Такі результати є наслідком таких фундаментальних властивостей атомарних функцій, як гарні апроксимаційні властивості, високий порядок гладкості та існування базису з локальним носієм у просторах атомарних функцій. Оскільки узагальнені Fup-функції мають такі самі зручні властивості, цілком природнім є те, що аналогічні результати стиснення можна отримати за допомогою узагальненого дискретного атомарного стиснення, яке базується на використанні цих функцій. Також у роботі наведено інтерпретацію отриманих результатів з точки зору теорії наближень та теорії функцій. **Висновки**: 1) за допомогою ДАС можна досягти кращих результатів ніж з JPEG; 2) якщо у подальшому планується використання алгоритмів розпізнавання, то краще використовувати стиснення за допомогою ДАС ніж JPEG; 3) подальший розвиток та дослідження ДАС є перспективними.

Ключові слова: атомарна функція; дискретне атомарне перетворення; стиснення зображень з втратами якості; алгоритм JPEG.

ДИСКРЕТНОЕ АТОМАРНОЕ СЖАТИЕ ЦИФРОВЫХ ИЗОБРАЖЕНИЙ

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Предметом изучения статьи является дискретное атомарное сжатие (ДАС) цифровых изображений, которое является процессом сжатия с потерями качества и основанным на применении дискретного атомарного преобразования (ДАП). **Целью** является исследование эффективности алгоритма ДАС. **Задачи исследования**: разработать общую схему компрессии с применением ДАП и сравнить результаты алгоритмов ДАС и JPEG. В данной работе мы используем **методы** цифровой обработки изображений, теории атомарных функций и теории приближений. Для того, чтобы сравнить эффективность алгоритмов ДАС и JPEG, мы используем наборы классических тестовых изображений и классических аэрофотоснимков. Мы анализируем коэффициент сжатия и потери качества, используя равномерную (U) и среднеквадратическую (RMS) метрики, а также отношение сигнал/шум (PSNR). ДАС – это алгоритм, который можно настроить в соответствии с конкретными потребностями. В данной работе мы используем режимы «Оптимальный» и «Допустимый» этого алгоритма. Получены такие **результаты**: 1) ДАС лучше алгоритма JPEG с точки зрения U-критерия; 2) с точки зрения критериев RMS и PSNR между ДАС и JPEG нет значительных отличий; 3) коэффициент сжатия алгоритма ДАС выше коэффициента сжатия алгоритма JPEG. Другими словами, с помощью ДАС можно получить большую экономию памяти, чем при помощи JPEG, при не худшем качестве результатов. Этот результат является следствием таких фундаментальных свойств атомарных функций, как хорошие аппроксимационные свойства, высокий порядок гладкости и существование базиса с локальным носителем в пространствах атомарных функций. Так как обобщённые Fup-функции имеют такие же удобные свойства, вполне естественно, что аналогичные результаты сжатия можно получить при помощи обобщённого дискретного атомарного преобразования, которое основано на их применении. Также в работе приведена интерпретация полученных результатов с точки зрения теории приближений и теории функций. **Выводы**: 1) с помощью ДАС можно достичь лучших результатов, чем с JPEG; 2) если в дальнейшем планируется применение алгоритмов распознавания, то предпочтительнее использовать сжатие с помощью ДАС; 3) дальнейшее развитие и исследование ДАС является перспективным.

Ключевые слова: атомарная функция; дискретное атомарное преобразование; сжатие изображений с потерями качества; алгоритм JPEG.

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