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M. CHERKASKYY

National University "Lviv Polytechnic", Ukraine

INTERDEPENDENCE OF COMPLEXITY CHARACTERISTICS AND COMPUTER SYSTEMS DEPENDABILITY

Interdependence of complexity characteristics of SH-model, and computer system physical characteristics, which influence on its dependability, is examined. A SH-model, elementary transducer, technical and informative complexity characteristics are definite. The methods of optimization of complexity characteristics and of some dependability characteristics are pointed.

complexity characteristics, structural complexity, SH-model (software/hardware algorithm model), dependability, system of diagnostics

Introduction

Technology of the computer systems design on the basis of architectural methods draws on limited set of characteristics: time, hardware and capacitive complexities. Technology of the computer design on the basis of software/hardware model (SH-model) has the extended set: time, hardware and capacitive complexities (technical characteristics), program and structural complexities (informative characteristics). Practice of analysis of system integration decisions and of circuit technique decisions testifies to interdependence of the above characteristics. For example, time complexity of data transformation node depends not only on hardware complexity. It can be diminished due to the changes of quantitative values of informative characteristics. Unlike the architectural methods, methods of design on the basis of SH-model allow to use the values of program complexity and of structural complexity to estimate intellectual content of the accepted decisions.

Definition of SH-model

Definition: the SH-model is the seven:

$$B = \langle D, Q, q_0, q_f, G, P, M \rangle, \quad (1)$$

where

D – Final set of symbols of the external alphabet;

Q – Final set of conditions of SH-model;

q_0, q_f – Initial and final conditions,
 $q_0, q_f \in Q$;

G – Hardware configuration of the model;

P – Program, $P = \{y_i \mid i = 1, I\}$;

M – Memory.

$$G = (X, U), \quad (2)$$

where X – set of elementary converters;

U – connections.

The SH-model has no once and for all established structure. However, each concrete model of algorithm concerning hardware construction has precisely outlined structure which consists of two sets: sets of elementary converters and sets of interconnections:

$$\begin{aligned} E &= \{e_1, e_2, \dots, e_n\}; \\ U &= \{u_1, u_2, \dots, u_m\} \end{aligned} \quad (3)$$

Elementary converters. Hardware tools contains one or several elementary converters. Elementary converter x_i is a unit of hardware complexity.

$$\forall x_i = 1 \quad (4)$$

The elementary converter x_i transforms some set of initial data into set of output data d_i :

$$x_i : \{d_i\} \rightarrow \{d_i'\} \quad (5)$$

The time complexity l_i of one converter is accepted to equal one unit:

$$\forall i, l_i = 1 \quad (6)$$

An elementary converter is represented as a black box. A formal definition for the operation of transforming data by the elementary converter is possible with using any known model of algorithm, including the SH-model.

This means that the concept of the “SH-model” and “elementary converter” have hierarchical meaning. Thus the property of hierarchy is appropriate for the SH-model. I should note that abstract models of calculators don’t have such a property.

The usage of elementary converters as a component of a model of algorithm makes it necessary to take into consideration the system of connection

$$U = \{U_i \mid i=1, j\}$$

between them. Without this system the analyses of hardware implementation won’t be complete.

The configuration of communications influences other characteristics of computer tools, in particular, it will changes the value of hardware and program complexities. This leads to the next requirement: taking into account the configuration of connections or requires the introduction of new additional characteristic of SH-model – the structural complexity.

Properties of SH-model. SH-model has all the properties, that arise from the verbal intuitive definition of algorithm:

Discreteness. SH-model’s work is carried out by a set of steps, which are limited in time. Any step may include elementary transformation operations, transferring of data from one elementary converter to another, and also operation writing data into memory elements.

Elementariness. Operations of processing, transferring and writing of data in the limits of one ele-

mentary converter taken as memory elements of the SH-model are simple and local in time and space. The term “Elementariness” gains hierarchical sense.

Determinacy. Each algorithm’s step is completely determined by the function of the elementary converter and the program’s command. The direction of data transfer from one converter to another is accurately determined by the direction of intercircuit connections or set by program’s commands.

Direction. If a process of data transformation can’t be carried out according to the set configuration of the connections and the program, there should be an instruction, stipulating what to do in such a situation.

Mass character. This property has a two-level sense:

1- *Multi-functionality* – one and the same SH-model may be used for solving tasks belonging to some limited amount of classes, that is formalized;

2- *Mass character in a traditional explanation* – one and the same SH-model may be used for any number of problems that differ by a set of input data, while the rule of immediate transformation is constant.

Hierarchy. Each elementary converter may be represented by a SH-model of a lower hierarchical level. On the other hand, every SH-model may be used as an elementary convector of a higher hierarchical level.

Parameters of SH-model. The parameters of an algorithm – the rule for beginning, the rule for ending – are given by the program. Systems of intermediate and final results, the rule of direct processing are determined by the hardware or hardware-software tools. The system of the initial data is given by the memory, external in relation to the SH-model [8].

Complexity’s characteristics of SH-model

In the processes of SH-model synthesis, analysis and optimization it is suggested to use four comple-

xity characteristics: hardware, time, programming and structural.

Hardware complexity. the quantity of elementary converters and elements of temporary memory of an hierarchical level of the SH-model's hardware:

$$A = |X|, \quad (7)$$

where X is a set of the elements in the scheme.

This definition reflects the hierarchical structure of computer devices. If we view the SH-model as an operational device, we would view the elementary converters as one-digit cells or valves; for the level of register transfers an operational device, for the system level an SH-model is a computer etc.

At the same time, there exists another approach to evaluating the hardware complexity. The hardware complexity of microprocessors may be determined by the quantity of transistors, placed on a crystal. The constant increasing of the hardware complexity (which now reaches 100 000 000 transistors) creates favorable conditions for broadening of the functional abilities for improving practically all consuming characteristics of processors, particularly the accuracy of calculations, productivity increase (without increasing of the physical sizes of computers or broadening of functions).

This definition of the hardware complexity does not contradicts the term "equipment size", that is used in computing.

Time complexity. In the metric theory, under the term "Time complexity" the quantity of elementary operations, for example, steps of the Turing Machine, is understood. In the software/hardware theory time complexity is determined in a slightly different way.

Time complexity of the SH-model is determined by the quantity of elements of the scheme, placed along the maximally critical way of the signal's spreading.

$$L = |\max X_i|, \quad (8)$$

where $\max X_i$ is a tuple of SH-model's elements that belong to maximally critical way of the signal's spreading, including the repeated passing of elements in a loop.

A unit of the time complexity is an elementary converter of hierarchical level of a scheme. The transition from time complexity to determining the time of the scheme's initiation is given in the next formula:

$$T = \sum_{\tau_{e_i} \in \max |e_i|} \tau(e_i), \quad (9)$$

where $\tau(e_i)$ is a real time of signal's delay at the e_i element.

Hardware and time complexities define the properties of computer devices and are known as technical characteristics. SH-models are characterized by the programming and structural complexities in addition to the hardware and time ones.

Programming complexity. Let us analyze the time chart of some device's work on the level of register transfers in relation to the information it holds. The time chart is a two-dimensional table, on the axis of abscissas f which the time units are shown and on the axis of ordinates the control entries for the functional scheme devices.

On the level of the processor each Assembler command corresponds to a microprogram (time chart). The configuration of the control signal states is set for each microprogram. By using the logarithmic measure for the level of entropy of the time chart, we get:

$$P = -F \log_2 \frac{F}{n \cdot m}, \quad (10)$$

where

$$F = \sum_L f_l;$$

n – the quantity of control entries;

m – the quantity of the time units of the time chart;

f_l – the quantity of control signals of the l -th fragment of the time chart for the chosen hierarchy level of hardware structure;

L – the quantity of fragments of the time chart, configurations of which aren't repeated.

The structural complexity of the algorithm device is the entropy of the incidence matrix:

$$S = -E \log_2 \frac{E}{q \cdot r}, \quad (11)$$

where

$$E = \sum_L \varphi_l$$

φ_l – the quantity or the intercircuit connections of the n -th fragment of the incidence matrix scheme;

$q \times r$ – matrix size.

Capacitive complexity of a SH-model is equal to the quantity of the external memory cells that is needed for solving the problem given.

The inter-dependence between complexity's characteristics and interference immunity

For each complexity's characteristic it is possible to find empirical relationships with physical system characteristics. For example, the time complexity makes it possible to find the response time and the productivity of the device if the computer construction's element base is known. Hardware and memory space complexities can define the volume of the equipment. The value of program complexity and the structural complexity - in its smaller measure - makes it possible to oriented-estimate the system's development time. An increase in the structural complexity is, as a rule, accompanied by a rise in productivity. It is possible to reach by changing the values of the characteristics of complexity a rise in the system's productivity and decrease the time of its design due to an increase in the apparatus complexity. This op-

timization can improve also cost. In the comparison with the program and structural complexities apparatus complexity today is substantially cheaper.

Let us limit a study of the interdependence of the complexity's characteristics and reliability's characteristics by examples. Let us examine electromagnetic focusing, reciprocal effects between the elements of the systems, which generate interferences. Let us examine also some by the aspects of diagnostics. The physic- mechanical reasons for failures, which include aging the elements of system, destructive ambient effect on these elements, are not connected with the theory of complexity and in the report are not examined.

The known methods of eliminating the electromagnetic interferences consist in the minimization of capacitance interference between the connections. For this reason are used screens, decrease the length of line that transfer signals, the apparatus fulfillment of micro-instructions. These fighting methods with the interferences can be described in the terms of the theory of complexity. The decrease of the total length of connections practically coincides with the decrease of structural and an increase in the apparatus complexity.

It is expedient to carry out the analysis of structural complexity in the hierarchical sequence. At cell level of super-large integrated circuits the structural complexity can be reduced by redistributing the connections between the units and by elements. For example, in the devices such as associative memory this is achieved by the use of the distributed logical gates "And" and "Or", which substantially decreases the capacitance interference, but this way leads to the loss of speed. At the level of units the minimization of capacitance interference is achieved by the replacement the matrix organization of the memory with a linear organization. At the level of processors the apparatus fulfillment of microinstructions decreases the program and structural complexity. In this

case the risk of failure due to the electromagnetic effect substantially will decrease. The transfer circuits of control signals are especially critical for these effects. In all given examples an improvement in the interference immunity is achieved by an increase in the hardware complexity.

The examined schema-technical solutions are not isolated, they emphasize the general tendency: an increase in the interference immunity of computer devices, otherwise the decrease of structural complexity occurs due to an increase in the time complexity and increase in the hardware complexity.

Minimax optimization

The main necessities, which are advanced with the design of computer systems, are:

- the guarantee of a given level of productivity on certain set of tasks
- the accomplishment of project within the allocated characteristics of reliability.

If the achievement of processor's maximal productivity is determined, mainly, by the time complexity of the execution of the set of commands, then the duration of development essentially depends on the information characteristics of complexity – program and structural. These characteristics complement each other – so, the increase of one of them must influence the decrease of another. Experience shows that the labor expense for the development of processor in the larger measure depends on program complexity, than from the structural complexity. The minimization of program complexity in considerably larger degree affects the period of development, than the minimization of structural complexity. Furthermore, the minimization of the program complexity of system makes it possible: to increase the speed of the execution of commands, to decrease the probability of the appearance of errors with the design, and to facilitate understanding the processes, which flow in the computers.

For guaranteeing given values of the productivity of system and time of its design it is used the method of the minimax optimization of the characteristics of complexity. This method is interpreted as reaching the minimum values of some characteristics of complexity (time and program) with a certain permissible increase in others (hardware, memory space and structural complexities). Let us note, the cost weight of time and program complexities substantially exceeds the cost weight of the hardware, memory. Structural complexity among them occupies intermediate place. The possibility of an increase in the hardware and memory complexity is the basic method of obtaining the high values of the characteristics of contemporary computer systems.

The basic operations of minimax method for our application are the following:

- from entire collection of characteristics separate two - time and program;
- select the methods of the minimization of the time complexity, which do not contradict the minimization of program complexity;
- if there are several methods, which are neutral with respect to the program complexity, then we select a method, which leads to a minimum increase in the structural complexity;
- it is allowed to increase the given limits of hardware and capacitive complexity, if they make it possible to decrease the time complexity.

In case, when the decrease of time complexity coincides with the decrease of program complexity they are used the following methods: commands and micro-commands are carried out by apparatus method, accesses to RAM are minimized, transfers are substituted by switching, the quantity of types is minimized, instruction formats and data formats are minimized. The decrease of time complexity due to the pipelining and the parallelism practically does not influence program complexity. All enumerated

methods are accompanied by an increase in the structural, memory and especially hardware complexity.

The optimization of diagnostics system

As an object of design the model of diagnostics system by the characteristics of complexity does not differ from the universal SH- model. It is a component of it. Its requirements are the same - the minimization of the time of search and elimination of malfunction and the minimization of the time of development. According to the minimax method for the optimization of the diagnostics system it is necessary to minimize the values of time and program complexity due to an increase in the hardware, capacitive and partially structural complexity. The formulated requirements satisfy first of all the system of hardware diagnostics - diagnostics by a set of processes. For example, the using parallel computer circuits (hot reserving). It provides the reserve of system as a whole and on the separate units. The units of computer circuits with data processing without the loss of accuracy are effectively diagnosed also on the modules of simple numbers.

Diagnostics systems, which are developed for the already designed computer devices, require besides the hardware a temporary redundancy. In this case substantially increase the time of design, program complexity, the cost of project. The described methods of the optimization of system here are also applicable.

Conclusions

1. Between the characteristics of complexity and the characteristics of the reliability of computer system there is an empirical dependence.
2. The theory of the systems complexity can be useful for the minimization of electromagnetic interferences.
3. The theory of complexity can be fully used for the design of the diagnostics system.

References

1. Черкаський М.В. SH-модель алгоритму.// Комп'ютерна інженерія та інформаційні технології. Вісник НУ "Львівська політехніка" – Львів, 2001, № 433. – С. 127-134.
2. Cherkaskyy M. Theoretical Fundamentals Software/Hardware Algorithms. //Modern Problems of Radio Engineering, Telecommunications and Computer Science. Proceedings of the International Conference TCSET'2004. February 24-28, 2004, Lviv-Slavsko, Ukraine. – Lviv, Publishing House of Lviv Polytechnic, 2004. – P. 9-13.

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Рецензент: д-р техн. наук, проф. В.С. Харченко, Національний аерокосмічний університет ім. М.С. Жуковського «ХАІ», Харків.