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SYNTHESIS OF SIGNALS ADAPTIVE SPATIAL PROCESSING ROBUST ALGORITHMS ON THE BASE OF THE PROBABILISTIC APPROACH

The work is devoted to the problem of the analysis and synthesis of adaptive signal processing algorithms for the signals with not exact parameters knowledge. The instance of the traditional signal processing theory extension for the case of signals with not exact parameters knowledge is analyzed. The diagrams of algorithms quality received as a result of imitating modeling are given.

adaptive processing, algorithm quality, imitating modeling, signal, synthesis

Introduction

A priori uncertainty about the properties of the signals is typical for many practically important applications. But traditional algorithms don't provide the optimization of adaptive spatial processing of the signals (ASPS) and moreover they can turn out as absolutely inoperative. Because of it the problem of development of algorithms ASPS that can work effectively under the conditions of a priori uncertainty about the properties of the signal (vector of signal) is extremely actual problem. The task of such algorithms synthesis was formulated – as a rule – as a task of decreasing of sensitiveness of procedures that form the optimal vectors of weighting coefficients (VWC) to the errors of a priory data about the vector of useful signal. The decisions of such tasks are named as robust algorithms of ASPS. The decreased sensitiveness is understanded as more slow diminution of output relation "signal/(interference + noise)" (RSIN) as functions of the quantity that characterizes an error in preliminary data in comparison with similar functional dependence, which can be found in the case of algorithms that implement the corresponding optimal VWC.

Synthesis of algorithms

An analysis of the results of the works devoted to the problem of ASPS with roughly known parameters shows that there are two main approaches to analysis and synthesis of robust algorithms, namely probabilistic and deterministic. Let us analyze the first of them.

The probabilistic approach is based on the accounting a priori uncertainty about a signal by introduction in the model of a vector $\vec{S}(t)$ of random components interpreted as fluctuations of amplitudes and phases of signals $s_j(t)$ on the outputs of antenna elements (AE). [1] Hereinafter the probabilistic approach was extended to the problem of robust algorithms synthesis. In this case the output signals of AE or properly the output signal of the antenna array (AA) is deformed deliberately with additionally generated pseudorandom processes simulating errors in a priori data and use for calculation of the VWC based on the traditional algorithms.

Therefore, probabilistic approach leads to synthesis of robust algorithms with preliminary distortion of the signals to be analyzed. The principal feature of this approach is an assumption about random nature of the spatial structure of useful signal.

We should point out that both probabilistic and deterministic approaches lead to the solutions with heuristic and mostly qualitative character. We can say that these algorithms are "less sensitive', than procedures using optimal VWC. As a rule, they don't use any numerical measure of the decreasing of sensitiveness and the possibility of the algorithm application is based on the simulation technique.

On the base of the hypothesis about distribution of spatial and time structures of the signal the vector $\vec{S}(t)$ from the outputs of AE can be represented as $\vec{S}(t)=s(t)\vec{V}_s$. In case of such representation vector

$$\vec{V}_s=\left[a_1(\Theta_s)e^{j\varphi_{1s}}\ a_2(\Theta_s)e^{j\varphi_{2s}}\ldots a_N(\Theta_s)e^{j\varphi_{Ns}}\right]^T$$

determines the spatial structure of the signal and it contains all information that is needed to implement an algorithm of ASPS.

Following to [1], let's assume, that amplitudes and phases of signals on outputs of AE include the random parts, that is vector $\vec{S}(t)$ represents as $\vec{\bar{S}}(t)=s(t)\vec{\bar{V}}_s$, where

$$\vec{\bar{V}}_s=\left[(a_1(\Theta_s)+\bar{a}_1)e^{j(\varphi_{1s}+\bar{\varphi}_1)}\ (a_2(\Theta_s)+\bar{a}_2)e^{j(\varphi_{2s}+\bar{\varphi}_2)}\ldots(a_N(\Theta_s)+\bar{a}_N)e^{j(\varphi_{Ns}+\bar{\varphi}_N)}\right]^T;\quad \bar{a}_k=\bar{a}_k(t),$$

$$\bar{\varphi}_k=\varphi_k(t),\quad k=\overline{1,N},$$

are independent random processes to describe the amplitude and phase errors. Let us assume that amplitude and phase fluctuations are mutually independent and

$$E\{\bar{a}_k\bar{a}_l^*\}=E\{\bar{\varphi}_k\bar{\varphi}_l^*\}=0\forall k\neq l;$$

$$E\{\bar{\varphi}_k\}=E\{\bar{a}_k\}=0,\ k=\overline{1,N};$$

$$E\{\bar{a}_k^2\}=\sigma_{ka}^2,\quad E\{\bar{\varphi}_k^2\}=\sigma_{k\varphi}^2.$$

Because these errors are very small [1], let us write that

$$(a_k(\Theta)+\bar{a}_k)e^{j(\varphi_{ks}+\bar{\varphi}_k)}\approx(a_k(\Theta_s)+\Delta\bar{g}_k)e^{j\varphi_{ks}},$$

$$\Delta\bar{g}_k=\bar{a}_k+j\varphi_k. \qquad (1)$$

Using (1) let us represent vector $\vec{\bar{V}}_s$ as

$$\vec{\bar{V}}_s=\vec{V}_s+\Delta\vec{V}_s,$$

$$\Delta\vec{V}_s=\left[\Delta\bar{g}_1e^{j\varphi_{1s}}\ \Delta\bar{g}_2e^{j\varphi_{2s}}\ldots\Delta\bar{g}_Ne^{j\varphi_{Ns}}\right]^T. \qquad (2)$$

Using (2) we can form a correlative matrix (CM)

$$\mathbf{R}_{\bar{s}\bar{s}}=E\{\vec{\bar{S}}(t)\vec{\bar{S}}^H(t)\}=\mathbf{R}_{ss}+\mathbf{G}, \qquad (3)$$

where

$$\mathbf{R}_{ss}=P_s\vec{V}_s\vec{V}_s^H;\ \mathbf{G}=P_s\mathbf{G}';\ \mathbf{G}'=diag\{\sigma_1^2\sigma_2^2\ldots\sigma_N^2\};$$

$$\sigma_k^2=\sigma_{ka}^2+\sigma_{k\varphi}^2.$$

It is clear from (3) that under the existence of random fluctuations the real CM $\mathrm{R}_{\bar{s}\bar{s}}$ is differ from supposed matrix $\mathbf{R}_{cc}=\beta\mathbf{R}_{ss}=\vec{V}_s\vec{V}_s^H$ only by the values of diagonal elements. From an assumption that $\sigma_0^2=\sigma_1^2=\ldots\sigma_N^2$, it follows that $\mathbf{G}=\sigma_0^2\mathbf{I}$ and diagonal elements of the matrices $\mathbf{R}_{\bar{s}\bar{s}}$ and $\mathbf{R}_{cc}$ differ on the same value. Therefore, in this case the expressions for optimal VWC based on the hypothesis about partibility of spatial and time structures of the signal is true. Thus, in case of traditional criteria we have the following:

$$\vec{\bar{W}}=\beta\mathbf{R}_{\bar{x}\bar{x}}^{-1}\vec{V}_y;\ \mathbf{R}_{\bar{x}\bar{x}}=\mathbf{R}_{\bar{s}\bar{s}}+\mathbf{R}_{in};\ \vec{V}_c=\beta\vec{V}_s. \qquad (4)$$

It is follows from (4) that replacement of $\overline{\mathbf{R}}_{xx}$ by corresponded consistent estimate $\hat{\bar{\mathbf{R}}}_{xx}$, such as

$$\hat{\bar{\mathbf{R}}}_{xx}=\frac{1}{K}\sum_{k=1}^{K}\vec{\bar{X}}(k)\vec{\bar{X}}^H(k),$$

where $(\vec{\bar{X}}(k)=\vec{\bar{S}}(k)+\vec{I}(k)+\vec{N}(k))$ is a vector from the outputs of AEs, than algorithms of ASPS can be represented using the traditional representation.

This conclusion determines the practical approaches for creation of robust procedures. It is clear that under the fixed value of error of a priori data such as $\delta_0=\|\mathbf{R}_{cc}-\mathbf{R}_{ss}\|_B$ $(\delta_{01}=\|\vec{V}_c-\vec{V}_s\|,\|\vec{V}_c\|=\|\vec{V}_s\|)$ and using such designations as ($\vec{W}_1=\lim\limits_{(P_s/\sigma_n^2)\to\infty}\vec{W}$, $\vec{W}_2=\lim\limits_{(P_s/\sigma_n^2)\to\infty}\vec{\bar{W}}$), we can find the following inequality

$$\vec{W}_1^H\mathbf{R}_{ss}\vec{W}_1<\vec{W}_2^H\mathbf{R}_{ss}\vec{W}_2. \qquad (5)$$

Expression (5) is true because $\mathrm{cond}\,\mathrm{R}_{xx}<\mathrm{cond}\,\overline{\mathrm{R}}_{xx}$, that follows from application of Gershgorin theorem [2] to identically normalized matrices $\mathbf{R}_{xx}$, $\mathbf{R}_{\bar{x}\bar{x}}$. It can be shown that for arbitrary δ_0, P_s/σ_n^2 there exist the values of dispersions σ^2_{ka}, $\sigma^2_{k\varphi}$ (there is such matrix **G** in (3)) that $\vec{\bar{W}}^H \mathbf{R}_{ss}\vec{\bar{W}} \ge \vec{W}^H \mathbf{R}_{ss}\vec{W}$. Therefore, if both amplitudes and phases of the signals from the outputs of AEs are purposely distorted with help of mutually and spatial uncorrelated random processes with zero average of distributions (that means that vector $\vec{X}(t)$ is replaced by vector $\vec{\bar{X}}(t) = \vec{X}(t) + \Delta\vec{V}_s$), then the values of the components of vector $\Delta\vec{V}_s$ can be found for any signal-interference situation and in this case the algorithms

$$\vec{W}_p = \beta\hat{\mathbf{R}}^{-1}_{xx}\vec{V}_c, \hat{\mathbf{R}}_{xx} = \frac{1}{K}\sum_{k=1}^{K}\vec{\bar{X}}(k)\vec{\bar{X}}^H(k); \qquad (6)$$

$$\vec{W}_p(k+1) = \vec{W}_p(k) + \mu_k\left(\vec{V}_c - \hat{\mathbf{R}}_{xx}\vec{W}_p(k)\right); \qquad (7)$$

$$\vec{W}_p(k) = \vec{W}_p(k-1) + \mu_k\left(\vec{V}_c - \vec{\bar{X}}(k)\bar{y}(k)\right), \bar{y}(k) = = \vec{W}^H_p(k-1)\vec{\bar{X}}(k), \qquad (8)$$

will be “more effective” (they permit the greater value of output RSIN) than corresponding traditional procedures under the conditions of a priori uncertainties. Let us point out that such approach can be applied to all practical algorithms of ASPS. The desired effect that is analogous to influence of uncorrelated fluctuations of amplitudes and phases of the signals can be reached whether thanks to their imitation by the generator of random numbers or by addition of the matrix G to the selective CM $\hat{\mathbf{R}}_{xx}$. As an example we can use the figures 1, 2, 3, that depict the diagrams from [3] for illustration of the quality of traditional and robust (algorithm (6) with $\mathbf{G} = \sigma^2_0\mathbf{I}$) procedures ASPS under the conditions of a priori uncertainty about the signal properties. Here fig. 1 shows the dependences RSIN on the output of spatial filter (SF) as functions on the input relation signal/noise, fig. 2 shows the dependences of the output σ^2_0 as functions on absolute value of parameter σ^2_0, which were got with help of algorithm (6). At last, fig. 3 represents the curves, that characterize dependences of output RSIN as functions on the error value in the assignment of direction of arrival of the signal $\Delta\Theta = \Theta_c - \Theta_s$ where Θ_c is supposed direction of the signal arriving and Θ_s is real direction of the signal arriving for traditional algorithms with $\delta_0 = 3$.

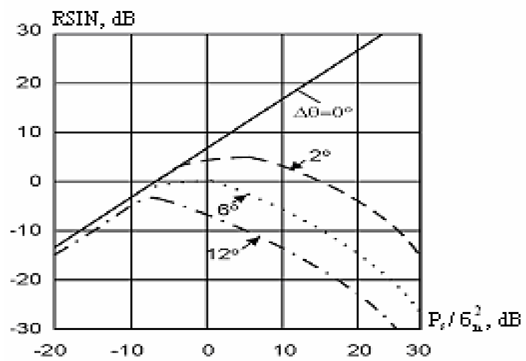

Fig. 1. Dependence of output RSIN on the input relation signal/noise

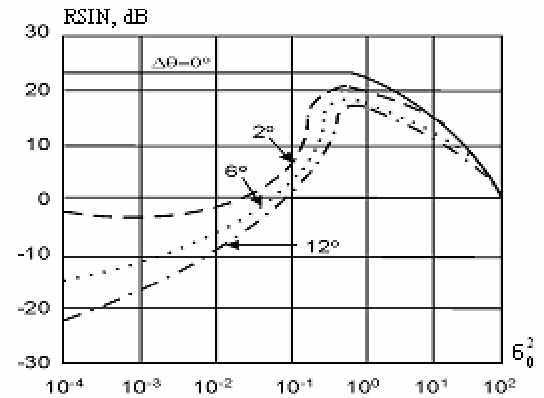

Fig. 2. Dependence of output RSIN on the value of parameter σ^2_0

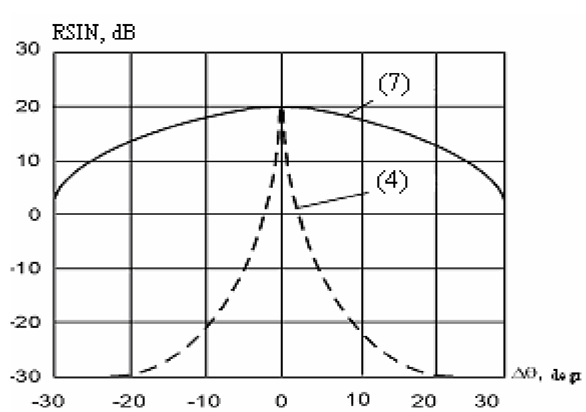

Fig. 3. Dependence of output RSIN on the value of error in the direction of signal arriving

The following assumptions about signal-noise situation and characteristics of AA were used to form these diagrams: linear equispaced antenna array with four elements; the distance between AEs is equal to $m_0/2$; isotropic and noninteracting AEs; the input relation interference/noise $10\lg P_1/\sigma_n^2 = 40$ dB (there is only single interference that is uncorrelated with useful signal); an angle of interference arriving $\Theta_1 = -50^\circ$; the supposed direction of the signal arriving $\Theta_c = 0^\circ$; real angle of signal arriving Θ_s is variable quantity; input relation signal/noise $10\lg P_s/\sigma_n^2 = 20$ dB; carrier frequencies of signal and interference are equal. It was supposed that carrier frequency of signal, structure and characteristics of antenna array are known exactly and some supposed angle of signal arriving Θ_y is known instead of real angle of signal arriving Θ_s, that means that the only source of differences between the vectors $\vec{V}_c$ and $\vec{V}_s$ is inaccurate information about signal arriving direction.

It is evident from the diagrams that in conditions of a priori uncertainty about the signal spatial structure the robust algorithms provide significantly higher meaning of the output signal / noise relation, than the appropriate traditional algorithms. But the question of how to carry out this appropriate selection of **G** remains open. For the elementary case $\mathbf{G} = \sigma_0^2\mathbf{I}$ in [3] it is offered to choose σ_0^2 according to the expression $\sigma_0^2 = N P_s/\sqrt{2}$. However it is received proceeding from a condition of the signal / noise relation maximization on the output of the adaptive array without allowance for noise. Moreover, for every concrete signal- noise situation there is the "best" σ_0^2 value.

Believing as a criterion of the choice of σ_{opt}^2 criterion of "the signal / noise relation maximum ", we shall receive

$$\sigma_{opt}^2 = Arg\max\left(\frac{\vec{W}^H(\sigma_0^2)\mathbf{R}_{ss}\vec{W}(\sigma_0^2)}{\vec{W}^H(\sigma_0^2)\mathbf{R}_{in}\vec{W}(\sigma_0^2)}\right),$$

$$\vec{W}(\sigma_0^2) = \beta\left(\mathbf{R}_{xx} + \sigma_0^2\mathbf{I}\right)^{-1}\vec{V}_c . \qquad (9)$$

For any non trivial combination of matrixes $\mathbf{R}_{ss}$ and $\mathbf{R}_{in}$ the value of σ_{opt}^2 is a single one and its definition is possible only at the precise knowledge of the appropriate correlation matrices (replacement of $\mathbf{R}_{in}$ by a correlation matrix $\mathbf{R}_{xx}$ in this case is inadmissible).

Conclusion

Hence, in conditions of a priori uncertainty about properties of the signal the optimization of the received in the frameworks of the probabilistic approach of robust procedures is essentially impossible. Moreover, due to the necessity to use $\mathbf{R}_{xx}$ the optimization of such algorithms is impossible even with not exact knowledge of the signal spatial structure.

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