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MODAL ANALYSIS OF A 20-DEGREE-OF-FREEDOM MODEL OF A TRANSPORT AIRCRAFT WING

The subject *of the study is the modal analysis of natural frequencies and vibration modes of a regional transport aircraft wing. The proposed model utilizes predefined functions of aerodynamic forces and moments to compute the wing's resonance frequencies and analyze vibration stability under resonance conditions. The study presents finite element analysis examples based on a 20-degree-of-freedom wing model with variable bending and torsional parameters along the span. Numerical methods were used to determine the natural frequencies and corresponding mode shapes, enabling the identification of the most critical modes for further analysis of the structure's dynamic behavior. This study* ***aims*** *to develop a reduced-order model and a computational algorithm for wing vibration analysis with fewer degrees of freedom compared to the original high-fidelity model, and to validate the accuracy of the dynamic characteristics for subsequent evaluation of vibration stability and flutter boundaries. The* ***objectives*** *include: developing a model that accounts for spanwise variations in bending and torsion; considering the partial-span location of the aileron; and incorporating the positions of the wing's stiffness axis and mass center axis. The* ***research methodology*** *is based on numerical schemes for mathematical modeling and dynamic system analysis, with an emphasis on finite element analysis to determine the wing's frequency characteristics. The analysis accounts for spanwise variations in stiffness and torsional moments within the mechanical control linkage.* ***Results and conclusions****: The study substantiates and applies key assumptions for constructing a computational model of a regional transport aircraft wing. Based on these assumptions, a finite element model with 20 degrees of freedom was developed. Unlike existing models, the proposed model incorporates not only the bending and torsional angles of the wing and aileron but also the spanwise variation of all structural parameters.* ***Scientific novelty*** *of the obtained results: for the first time, a novel method has been developed for constructing a reduced-order model of an aircraft wing with a significantly lower number of degrees of freedom, while maintaining a high level of agreement with the dynamic characteristics of the full-scale wing structure.*

Keywords: *natural frequencies; wing of a regional transport aircraft; aileron; freeplay; computational model; flutter; degree of freedom.*

1. Introduction

It is well known that oscillatory characteristics significantly influence the operational efficiency of an aircraft, as well as the reliability and flight safety. The spectrum of natural frequencies and mode shapes of an aircraft structure is determined both computationally and through experimental studies. The results of determining natural frequencies and mode shapes form the basis for analyzing the dynamic characteristics of the aircraft. If the wing twists and bends in certain manner the unsteady aerodynamic loads start feeding elastic motion of wing causing amplitudes to grow, eventually leading to structural failure or Limit Cycle Oscillation (LCO) [1]. LCO can accelerate the accumulation of structural fatigue damage and compromise controllability during flight. One can observe that vertical and horizontal tail plane control surfaces (Elevator and Rudder) represent the major cause of airframe vibrations (72%). These vibrations may have multiple consequences on the aircraft life cycle from the design to operations [2].

The study of natural frequencies and mode shapes has received considerable attention in modern scientific and educational-methodological literature. In particular, reference [3] presents a modal analysis of an aircraft wing, including theoretical and numerical calculations treating the wing as a cantilever beam. Natural frequencies and their associated vibration modes were obtained. As a result of the study, it was concluded that the aircraft wing can be considered a cantilever beam, neglecting all acting forces on the aircraft except gravity. In reference [4], the modal analysis of glider and transport aircraft wings is performed using the dynamic stiffness method. The wing is modeled as a system of dynamic stiffness beam elements undergoing bending-torsion deformation, assembled into a global dynamic stiffness matrix of the entire wing. The reference [5] presents an identification algorithm designed for the modal analysis of aircraft structures during flight tests, with a particular focus on processing short-duration tests with multiple inputs. Another important model worth mentioning is the pseudo two-degree-of-freedom model with pitch and plunge.

Similar to a cantilever plate, there are two-dimensional flexible aerodynamic profiles that are modeled as finite beam elements with chordwise bending, capable of both bending and twisting [6, 7]. References [8, 9] present an analysis of the nonlinear response of a typical airfoil section with control surface freeplay, subjected to periodic impulsive excitations in a low-subsonic flow, along with an associated wind tunnel test program.

The current study is devoted to the modal analysis of the natural frequency characteristics and vibration modes of an aircraft wing, taking into account further investigation of freeplay in the mechanical control linkage. The problem essentially reduces to determining the natural frequencies of a wing model subjected to external aerodynamic loading. It should be noted that this work aims to develop a reduced-order wing model for the investigation of free vibrations with 20 degrees of freedom, based on a reference model of a real aircraft wing with 48 degrees of freedom, and to validate the dynamic behavior and mode shapes at various frequencies.

2. Computational model

Consider the wing of a regional transport aircraft (Fig. 1.1), which features a high aspect ratio, a low sweep angle, and a tapered planform. The aileron, positioned near the wingtip in the outboard section, spans approximately one-quarter of the total wingspan.

It is assumed that the wing consoles can be modeled as a flat plate made of a homogeneous isotropic material (Fig. 1.2).

To satisfy the given position of the center of mass and the stiffness axis along the chord, box-section beams with a linearly varying cross-section along the wingspan are placed at the leading and trailing edges of the wing (Fig. 1.3).

Box profiles provide stiffness of the wing, both in bending and in torsion. It is assumed that the deflections of the aileron along the wingspan are minimally affected by the placement of attachment points and that the aero-

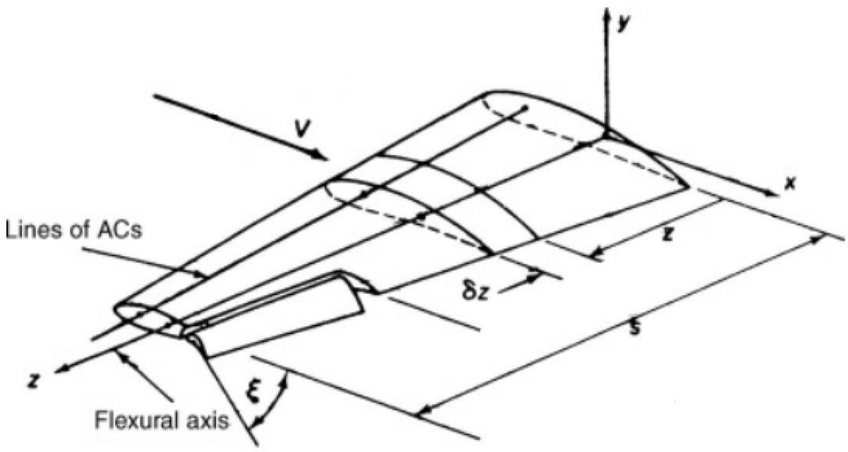

Figure 1.1. Typical wing of a regional transport aircraft

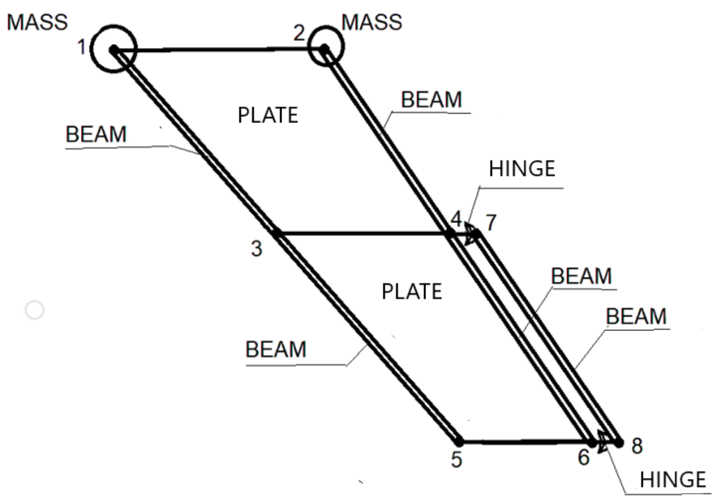

Figure 1.2. Computational model of the wing

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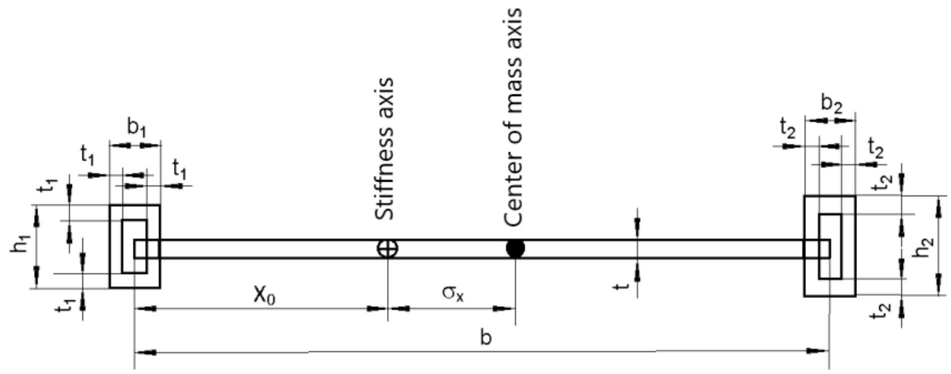

Figure 1.3. Cross-section of the computational model

dynamic forces acting on the aileron can be defined using either experimental data or computational dependencies. The aileron is modeled as a beam with a variable cross-section**,** with its ends hinged to the trailing edge of the wing.

2.1. Computational scheme

The plate modeling the wing has a trapezoidal shape, identical to the wing's planform. The dihedral angle between the wing consoles and the wing incidence angle relative to the longitudinal axis of the fuselage are not considered, meaning the wing is assumed to be positioned in a horizontal plane. The thickness of the plate is assumed to be constant throughout its entire surface. The selection of model parameters is based on the approximate characteristics of a regional transport aircraft wing, which are provided below.

Semi-span: L = 16.0 m.

Root chord length: b_{root} = 4.85 m.

Tip chord length: b_{tip} = 1.45 m.

Position of the stiffness axis (from the leading edge): X_0 /b = 0.37.

Aileron spanwise position:
Z_{start} = 11.5 m, Z_{end} = 14.3 m.

Aileron chord length:
$b_{root\ ail}$ = 0.7 m*;* $b_{tip\ ail}$ = 0.62 m.

Axial compensation: b_{comp} = 0.27 b_{ail}.

The aileron's stiffness axis coincides with the rotation axis.

The mass-inertial and stiffness characteristics of the wing at several cross-sections perpendicular to the stiffness axis are as listed in Table 1.1.

Below, in square brackets, the parameter notation adopted in Antonov Design Bureau is provided.

Linear mass – m [kg × s² / cm²].

Center of mass position relative to the stiffness axis – σ_x [cm],

where $\sigma_x > 0$ indicates that the center of mass is located behind the stiffness axis.

Total (intrinsic and transferred) linear moment of inertia of the cross-section – I_{mz} [kg × s²].

Torsional stiffness – GJ_{kr} [kg × cm²].

Bending stiffness in the vertical plane – EI_Z [kg × cm²].

Bending stiffness in the horizontal plane – EI_Y [kg × cm²].

The mass-inertial characteristics of the fuselage and empennage are as follows:

G = 13.403 kg,

X = 107 cm (measured from the stiffness axis at the root section, aft along the flight direction),

Y = 73 cm (measured from the stiffness axis at the root section, upward),

Table 1.1

Wing cross-section parameters

Z/L	m	σ_x	I_{mz}	GJ_{kr}	EI_Z	EI_Y
0.00	2.0×10^{-3}	-20.0	20.0	8.0×10^{10}	15.0×10^{10}	60.0×10^{10}
0.25	1.9×10^{-3}	40.0	25.0	5.0×10^{10}	10.0×10^{10}	40.0×10^{10}
0.50	1.2×10^{-3}	30.0	10.0	1.3×10^{10}	3.0×10^{10}	10.0×10^{10}
0.75	7.0×10^{-4}	8.0	2.0	0.25×10^{10}	0.65×10^{10}	3.0×10^{10}
1.00	2.1×10^{-4}	-5.0	0.3	0.1×10^{10}	0.15×10^{10}	1.0×10^{10}

$I_{mx} = 6.1 \times 10^5$ kg × cm × s²,
$I_{my} = 5.7 \times 10^6$ kg × cm × s²,
$I_{mz} = 6.0 \times 10^6$ kg × cm × s².

In five cross-sections along the wingspan, eight parameters are defined: chord length, stiffness center coordinate, center of mass coordinate, linear mass, mass moment of inertia in vertical bending, torsional moment of inertia, and moments of inertia in vertical and horizontal bending**.** Expressions for these parameters will be formulated based on the characteristics of the cross-section model. The origin of the cross-section coordinate system is placed at the leading edge**.** Cross-sectional dimensions are determined by selection.

$$EI_z = E_0 I_{0z} + E_1 I_{1z} + E_2 I_{2z}, \qquad (1.1)$$

where E_0, E_1 and E_2 – elastic moduli of the materials of the plate, the front beam, and the rear beam;

$$EI_y = E_0\left[I_{0y} + F_0\left(\frac{b}{2} - x_0\right)^2\right] + E_1\left(I_{1y} + F_1 x_0^2\right) + + E_2\left[I_{2y} + F_2\left(b - x_0\right)^2\right], \qquad (1.2)$$

where $I_{0y} = \frac{b^3 t}{12}$, $I_{1z} = \frac{b_1^3 h_1 - (b_1 - 2t_1)^3 (h_1 - 2t_1)}{12}$,

$$I_{2z} = \frac{b_2^3 h_2 - (b_2 - 2t_2)^3 (h_2 - 2t_2)}{12}$$

all dimensions are given in Fig. 1.3.

The parameters that need to be selected for our model are the elastic moduli and densities of the materials of the plate and the two box-section beams; the thickness of the plate; the height, width, and wall thickness of the box section. In total, there are 13 parameters.

2.2. Finite element model

The wing model is built using the finite element software ANSYS. The aileron is attached to the outer part of the wing console, so the console model consists of two SHELL181 plate elements along the wingspan. Accordingly, beam elements BEAM188 are placed along the leading and trailing edges of the wing. The aileron is connected to the outer element of the console and is also modeled using the BEAM188 beam element. The moment of inertia of the beam around the longitudinal axis corresponds to the moment of inertia of the aileron around its axis of rotation. The hinges connecting the aileron to the trailing edge of the wing are modeled using COMBIN40 connection elements, which allow setting freeplay in the joint.

The side of the plate that approximates the root chord is attached to the fuselage. The mass and moments of inertia of the fuselage are approximated using two structural mass elements MASS21.

The beam modeling the aileron is connected to the trailing edge of the wing by two COMBIN40 hinges, which include a clearance.

All finite elements in the constructed model have 6 degrees of freedom at each node, and their shape functions ensure continuous strain distribution across the wing. The total number of degrees of freedom in the unconstrained model is 48. Based on the physical meaning of the studied vibrations, the fuselage displacements in the horizontal plane should be excluded. In the case of symmetric flutter, rotations around the longitudinal axis (X-axis) should be constrained, while in the case of antisymmetric flutter, movement along the vertical axis (Y-axis) should be restricted. Thus, 4 degrees of freedom are constrained at nodes 1 and 2 (Fig. 1.2).

In the case of bending and torsional vibrations of the beam and plate, in addition to transverse deformations, small deformations also occur in the longitudinal direction. In linear vibration problems, these can be neglected. Similarly, the displacements of all nodes in the plane of the wing (XZ plane) can be neglected, removing 3 degrees of freedom in the remaining nodes of the wing model. Whether this affects the calculation of vibration frequencies and modes should be verified through numerical experiments.

The COMBIN40 connection elements leave only one degree of freedom for the aileron nodes relative to the trailing edge of the wing. Thus, after constraining all unnecessary degrees of freedom, the final finite element model has 20 degrees of freedom.

3. Modeling of free vibrations

Firstly, will be investigating the natural frequencies and mode shapes of symmetric vibrations in the model, where only the fuselage nodes are constrained from moving in the horizontal plane. The equation for modal analysis is as follows:

$$[M]\{\ddot{U}\} + [K]\{U\} = 0, \qquad (1.3)$$

where $[M]$ – is the mass matrix,

$[K]$ – is the stiffness matrix,

$\{U\}$ – is the vector of generalized nodal displacements.

Modal analysis determines the vibration characteristics (natural frequencies and mode shapes) of a structure or machine component. It can also serve as a starting point for more detailed dynamic analyses, such as transient dynamic analysis, harmonic analysis, or spectral analysis. Natural frequencies and mode shapes are crucial

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parameters when designing structures for dynamic loading conditions. If damping is present in the structure or machine component, the system becomes a damped modal analysis. In a damped modal system, the natural frequencies and mode shapes become complex.

The ANSYS calculation results are presented in Table 2.1 in the second column. The first two frequencies are zero, corresponding to the rigid body motions of the system - angular motion in pitch and linear motion in the vertical direction and therefore can be omitted. The other frequencies correspond to the bending-torsional vibrations of the wing.

Next, the frequencies and mode shapes were determined with all degrees of freedom constrained in the plane of the wing. The ANSYS calculation results are presented in Table 2.1 in the 3-rd column. We observe that the frequencies of the baseline and reduced models do not differ, meaning that the system with 20 degrees of freedom accurately captures all wing deformations of interest. Figure 2.1 shows the mode shapes.

In the bending-torsional mode shapes, a discontinuity is visible along the chord connecting the two plates. This occurs because, when generating the illustrations, ANSYS connects nodal points with straight lines rather than using cubic parabolas for shape functions.

4. Discussion on research results

Undoubtedly, in the modern world, the computer experiment plays a significant role in the research of complex phenomena, in particular, natural frequencies and vibration modes of an aircraft wing. In the present study we obtained the following results.

Mode 3 – combined bending and torsion mode. Bending behavior: the wing deflects downward along the Z-axis, which is typical for bending vibrations. Torsional component: at the wingtip and aileron region, there is visible twisting of the airfoil profile. The chordwise color gradient indicates rotational deformation of the cross-section, confirming the presence of torsional motion coupled with bending.

Mode 4 – pure bending mode. This mode is characterized by a distinct bending behavior, with the following features: smooth downward deflection along the entire span of the wing, with maximum displacement at the tip. No significant chordwise twist is observed (unlike Mode 3), indicating minimal or no torsional component. The deformation occurs primarily in the vertical plane (Y-axis direction).

Mode 5 – bending mode. This is a high-frequency bending mode, where the oscillation is mainly concentrated in the aileron region. There is no significant torsional contribution, and the deformation remains predominantly in the vertical plane.

Mode 6 – torsional mode. This mode represents a pure torsional deformation pattern: twisting occurs along the longitudinal axis, with a continuous change in angle along the span.

Mode 7 – bending mode. This mode again exhibits a bending character, with deformation along the structural span.

Mode 8 – bending mode. This mode is a classical high-frequency bending mode, characterized by pronounced vertical deflection and multiple curvature regions along the wing span.

Table 2.1

Natural frequencies of wing vibrations

Oscillation mode	Base model (48 DOF) Frequency, Hz	Reduced model (20 DOF) Frequency, Hz
1	7.82E-08	7.82E-08
2	2.44E-06	2.54E-06
3	5.27	5.27
4	7.63	7.63
5	33.885	33.885
6	116.29	116.29
7	124.84	124.84
8	157.53	157.53
9	270.29	270.29
10	284.78	284.78

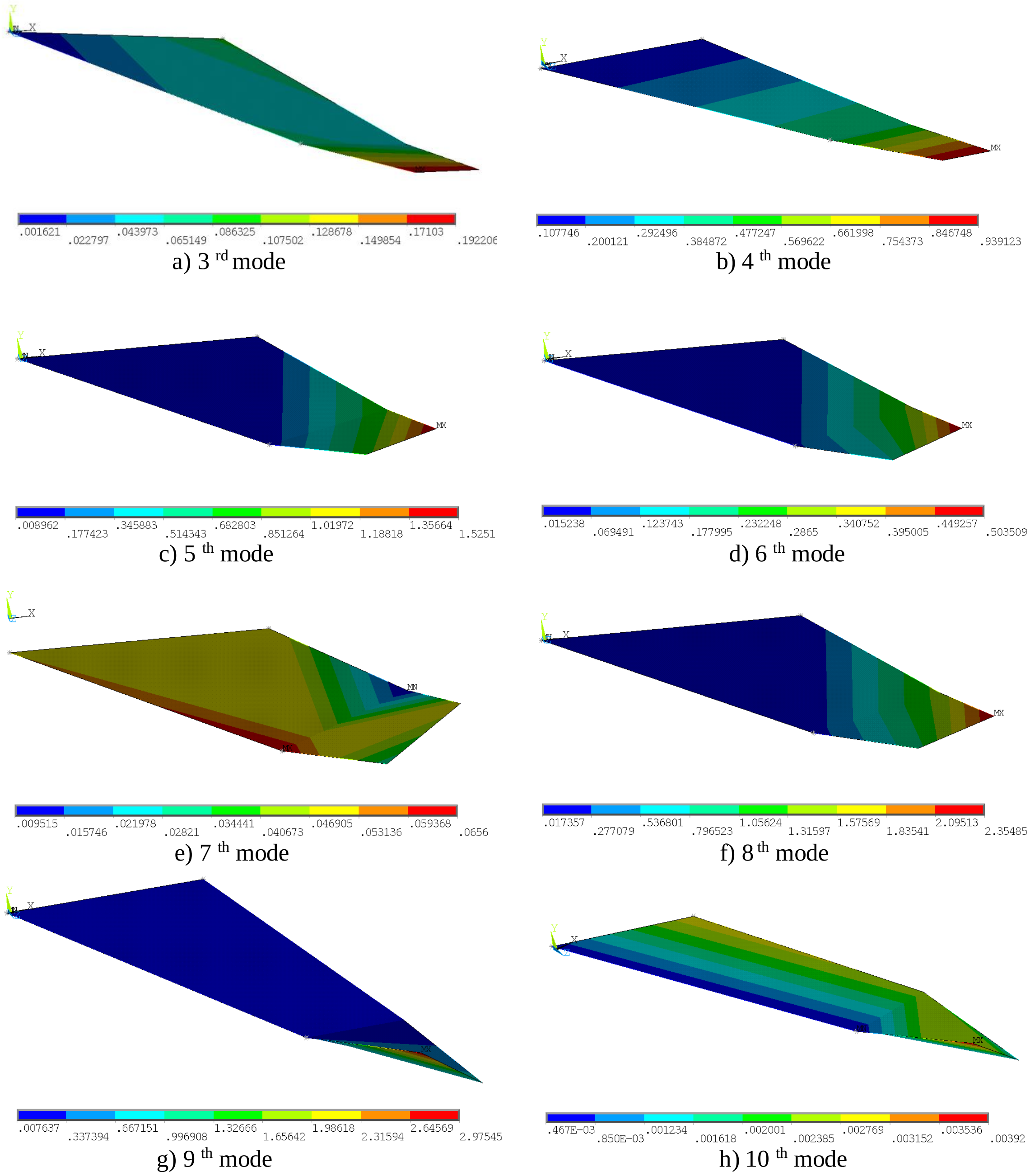

Figure 2.1. Wing mode shapes. Reduced model (20 DOF)

Mode 9 – torsional mode. This mode once again demonstrates a predominantly torsional deformation of the wing structure.

Mode 10 – bending mode. The tenth mode corresponds to a bending vibration mode, with clear flexure along the longitudinal axis of the structure.

As per [1], the focus is placed on solving the equations that define the flutter problem and the determination of eigenvalues, with final solutions expressed as combinations of speed and frequency, determined through a characteristic determinant equal to zero. It is shown that the flutter frequency ratios implicitly depend on the following five parameters: the position of the axis, the ratio of purely bending to purely torsional (twisting) frequencies, the dimensionless static unbalance, the dimensionless radius of gyration, and the relative density. The main objective of this section is to identify the characteristic values of the key parameters and to demonstrate the influence of varying each parameter individually on the characteristics of bending-torsional flutter. The subsequent stage of this research involves investigating flutter susceptibility within a defined flight envelope. As indicated in [10, 11], flutter can develop due to phase shifts, particularly when torsional oscillations precede

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bending ones. The positioning of the elastic axis ahead of the aerodynamic center of the wing cross-section plays a critical role in damping torsional vibrations. Understanding this relationship is essential for determining critical flutter speeds and improving the aeroelastic stability of the wing structure. Further analysis will include parametric studies based on reduced-order models, aiming to correlate modal characteristics with phase relationships and structural stiffness distribution.

Conclusions

This study presents a comprehensive modal analysis of a regional transport aircraft wing with a focus on developing a reduced-order computational model that accurately reflects the dynamic characteristics of the full-scale structure. The model incorporates spanwise variations in stiffness and inertial properties, aileron positioning. A finite element model with 20 degrees of freedom was constructed using modern numerical methods, capturing the essential bending and torsional behaviors of the wing and aileron system. The novelty of the research lies in the development of a reduced-order model that significantly decreases the computational complexity while maintaining a high level of accuracy in the estimation of natural frequencies and mode shapes. The implementation of variable cross-sectional properties and detailed modeling of the control surface enables a more realistic assessment of the wing's dynamic response. The results confirm that the proposed model can reliably predict critical modes of vibration, forming a solid foundation for further aeroelastic stability analysis, including the study of flutter phenomena and limit cycle oscillations. The methodological framework and computational scheme developed in this research can be applied in early design stages for evaluating structural dynamics and ensuring flight safety of transport aircraft.

Conflict of interest

The author declares that he has no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

Financing

The study was performed without financial support.

Data availability

The data that support the findings of this study are not publicly available due to reasons of confidentiality and internal policy.

Use of artificial intelligence

The author confirms that he did not use artificial intelligence technologies when creating the current work.

References

1. Bisplinghoff, R. L., Ashley, H., & Halfman, R. L. *Aeroelasticity*. 2nd ed. New York, Dover Publications, 1996. 860 p. Available at: https://books.google.com.ua/books/about/Aeroelasticity.html?id=jtqDQ2nTvvcC&redir_esc=y (accessed 10.10.2024).

2. Airbus S.A.S. Troubleshooting Airframe Vibrations. *Safety First*, 2017, vol, 24, pp. 26-32. Available at: https://www.theairlinepilots.com/forumarchive/flightsafety/airframe-vibrations.pdf (accessed 10.10.2024).

3. Demirtas, A. M., & Bayraktar, M. Free vibration analysis of an aircraft wing by considering as a cantilever beam. *Selcuk University Journal of Engineering Sciences*, 2019, vol. 7, iss. 1, pp. 12-21. DOI: 10.15317/Scitech.2019.178.

4. Banerjee, J. R. Modal analysis of sailplane and transport aircraft wings using the dynamic stiffness method. *Journal of Physics: Conference Series*, 2016, vol. 721, article no. 012005. DOI: 10.1088/1742-6596/721/1/012005.

5. Vayssettes, J., & Mercère, G. New developments for experimental modal analysis of aircraft structures. *MATEC Web of Conferences*, 2015, vol. 20, article no. 01001. DOI: 10.1051/matecconf/20152001001.

6. Kim, S. H., & Lee, I. Aeroelastic analysis of a flexible airfoil with a free play nonlinearity. *Journal of Sound and Vibration*, 1996, vol. 193, iss. 4, pp. 823-846. DOI: 10.1006/jsvi.1996.0317.

7. Hefeng, D., Chenxi, W., Shaobin, L., & Zhen, S. X. Numerical research on segmented flexible airfoils considering fluid-structure interaction. *Procedia Engineering*, 2015, vol. 99, pp. 57-66. DOI: 10.1016/j.proeng.2014.12.508.

8. Tang, D., Kholodar, D., & Dowell, E. H., Nonlinear response of airfoil section with control surface freeplay to gust loads. *AIAA Journal*, 2000, vol. 38, no. 9, pp.1543-1557. DOI: 10.2514/2.1176.

9. Tang, D., & Dowell, E. H. Aeroelastic airfoil with free play at angle of attack with gust excitation. *AIAA Journal*, 2010, vol. 48, no. 2, pp. 427-442. DOI: 10.2514/1.44538.

10. Süelözgen, Ö. Advanced aeroelastic robust stability analysis with structural uncertainties. *CEAS Aeronautical Journal*, 2021, vol. 12, iss. 3, pp. 43-53. DOI: 10.1007/s13272-020-00473-8.

11. Lowe, B. M., & Zingg, D. W. Efficient flutter prediction using reduced-order modeling. *AIAA Journal*, 2021, vol. 59, iss. 7, pp. 2670-2683. DOI: 10.2514/1.J060006.

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МОДАЛЬНИЙ АНАЛІЗ МОДЕЛІ КРИЛА ТРАНСПОРТНОГО ЛІТАКА З 20 СТУПЕНЯМИ ВІЛЬНОСТІ

О. В. Стародуб

Предметом дослідження є модальний аналіз власних частот та форм коливань крила регіонального транспортного літака. Запропонована модель передбачає використання заздалегідь визначених функцій аеродинамічних сил і моментів для обчислення резонансних частот крила. Наведено приклади, що включають скінченноелементний аналіз моделі крила з 20 ступенями вільності при змінних згинальних і крутильних параметрах уздовж розмаху. Застосовано чисельні методи для визначення власних частот та відповідних форм коливань, що дозволило ідентифікувати найбільш критичні моди для подальшого аналізу динамічної поведінки конструкції. **Метою** дослідження є розробка редукованої моделі та обчислювального алгоритму для аналізу коливань крила зі значно меншою кількістю ступенів вільності, порівняно з вихідною високоточною моделлю, а також верифікація точності динамічних характеристик для подальшої оцінки стійкості коливань і меж флатера. Основні завдання дослідження включають: побудову моделі, що враховує варіацію згинальної жорсткості та крутильного моменту уздовж розмаху; врахування часткового розміщення елерона по розмаху; включення положення осі жорсткості та осі центра мас крила. **Методи дослідження** базуються на чисельних схемах математичного моделювання та аналізу динамічних систем, зокрема на скінченноелементному аналізі для визначення частотних характеристик крила. Аналіз враховує змінну жорсткість та крутильні моменти у механічній проводці керування. **Результати та висновки**: у роботі обґрунтовано та прийнято ключові припущення для побудови обчислювальної моделі крила регіонального транспортного літака. На основі цих припущень розроблено скінченноелементну модель з 20 ступенями вільності. На відміну від існуючих моделей, запропонована модель враховує не лише згинальні та крутильні кути крила й елерона, але також змінність параметрів уздовж розмаху. **Наукова новизна** отриманих результатів полягає у розробці нового методу побудови моделі зі зниженою розмірністю, яка забезпечує високий ступінь відповідності динамічним характеристикам повномасштабної конструкції крила при істотно меншій кількості ступенів вільності.

Ключові слова: власні частоти; крило регіонального транспортного літака; елерон; люфт (вільний хід); розрахункова модель; флаттер; ступінь вільності.

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