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COMPROMISE KINETIC-FLUID MODEL OF ELECTRONS DYNAMICS IN ELECTRIC PROPULSION DEVICES WITH CLOSED ELECTRONS DRIFT AS AN ALTERNATIVE TO THE HYBRID PIC-FLUID METHOD

Electric propulsion devices with closed electron drift include Hall effect thrusters, plasma-ion thrusters with a radial magnetic field, and helicon thrusters, which are sources of plasma, ions, and electrons. Within the framework of the hybrid PIC-Fluid method for calculating a Hall effect thruster, which has been actively replicated in recent decades, the level of detail in the PIC unit does not correspond to the criterion of substance continuity, and the fragmentary set of equations in the Fluid unit does not contain several terms necessary for the calculation and indicates a profound misunderstanding of the origin and limits of applicability of the equations and the true nature of the processes. The calculation of ionization characteristics, the height of the potential barrier at the plasma boundary, the electrons and ions, and their energy fluxes to the surface of the thruster chamber is carried out using a Maxwell distribution, the conditions for the formation of which did not correspond to the realities in the rarefied plasma of electric propulsion devices. The closeness of the calculated integral characteristics to the measured ones is achieved using empirical coefficients with a difference of tens of times in different publications with a complete inability to predict the characteristics of samples of electric propulsion devices that have not yet been developed and tested. In this paper, a compromise method is proposed, the possibility of which is due to the closeness of the electron velocity distribution to isotropic due to the influence of a strong magnetic field and non-specular reflection of electrons from the potential barrier at the plasma boundary. The method operates with the angular moments of the distribution function without integrating the components of the kinetic equation by the velocity module. To calculate the densities of mass, momentum, energy, and their fluxes considering dissipative processes, it is sufficient to determine the angular moments of the second and third orders, the traces of which include the moments of the zero and first orders, respectively. Equations of angular moments are given as intermediates between the kinetic equation and the velocity distribution function moments equations. The expansion of the velocity distribution function in a series of angular moments is recorded. Calculations have been performed that show sufficient agreement with the known measurement results and a significant difference in the characteristics of the Langmuir layer and plasma at the boundary with it from those found using the Maxwell distribution. The use of the obtained results allows us to significantly increase the accuracy of predicting the thruster parameters and thereby reduce the volume of costly experiments to optimize their characteristics.

Keywords: *Hall effect thruster; velocity distribution function; moments of the distribution function; kinetic equation; Langmuir layer; Hybrid PIC-Fluid.*

1. Introduction

1.1. Motivation for the study

Currently, the most widely used types of electric propulsion thrusters (EPT) are Hall effect thrusters (HET) and plasma-ion thrusters (PIT). Both HET and PIT with radial magnetic field in ionization chamber (PITR) relate to devices with closed electron drift in crossed electric field with main axial projection and magnetic field with main radial projection. A promising direction in the electric propulsion are also helicon devices (HD): thrusters as well as plasma and electrons sources.

The main task of mathematical modeling of processes in electric propulsion devices is to predict their main parameters in order to reduce the costs of their design and testing. One of the key operational parameters here is power efficiency. Thus, it is important to identify the components of energy losses and use adequate methods to describe them.

In the case of HET, the main components of such losses are:

- ions velocity dispersion in the external beam of the thruster;
- electrons energy losses due to ionization;
- losses of ions, electrons and their energy in flows to the anode and dielectric walls of the channel.

1.2. Current state

In the last quarter of a century, the so-called Hybrid PIC-Fluid (HPF) method has become widely used, where

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the problem of ions dynamics is solved in the particle-in-cell (PIC) unit, and the problem of electrons dynamics – in the Fluid unit [1].

The problem of ion velocity dispersion in the PIC-unit was solved by dividing the longitudinal section of the thruster channel into several dozen cells in the axial and radial directions with an average number of macro-particles of the order of several dozen.

In the Fluid-unit, the overestimated values of the electron energy flux to the channel walls, found using the Maxwell distribution, were corrected using the concept of secondary electron-electron emission. The underestimated values of the axial electron current, found taking into account scattering in the plasma volume, were corrected using 'empirical coefficients' for so called 'anomalous mobility', freely varied by each new author in a wide range with the purpose of 'fitting' the results of calculations of two or three integral characteristics of the thruster to the measurement results. The distributions of local characteristics found in this way differed significantly from the real ones.

1.3. Purpose and approach

The objectives of this study are:

- to analyze the reasons why the above-mentioned problems using the PIC-Fluid method have not actually been solved;

- to present of the method for solving these problems using a compromise kinetic-gas dynamics model;

- to demonstrate and analyze preliminary calculation results.

2. Formulation of the problem

2.1. Problem in PIC-unit

The necessity of using PIC in describing ion dynamics is based, on the one hand, on the authors’ conviction that it is impossible to solve this problem by means of gas dynamics and, on the other hand, on the lack of understanding of the value of the error when recalculating solutions in the PIC unit to gas-dynamic parameters in the Fluid unit.

The measured values of ion velocity dispersion in HET are in the range of 5-10%. In order for the error in calculating the distribution by two coordinate projections and two velocity projections not to exceed 10%, it is necessary to have at least 10 macro-particles in each of these four directions – with a total number of 10000. Several dozen macro-particles, for example 100, in each cell means an error of the method of at least 30%. It means that the sought value of ion dispersion is deeply within the error limits of the method.

The conviction that it is impossible to solve this problem by means of gas dynamics can only be explained by the fact that the authors know only one approximation within its limits – the method of local thermodynamic equilibrium, which is not applicable to rarefied plasma in HET. The dispersion of any projection of the ion velocity in gas dynamics is represented by the corresponding component of their pressure tensor – in this case, axial-axial $P_i^{(xx)}$, for finding which the equation given in the paper [2] can be used:

$$\frac{\partial P_i^{(xx)}}{\partial t}+\frac{\partial}{\partial x}\left(V_{ix}P_i^{(xx)}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(V_{ir}P_i^{(xx)}r\right)+ +2P_i^{(xx)}\frac{\partial V_{ix}}{\partial x}=m_i V_{ix}^2\frac{\delta n_e}{\delta t}, \tag{1}$$

where V_{ix} – axial projection of ions mass flux velocity; m_i – ion mass; $\frac{\delta n_e}{\delta t}$ – volume ionization rate.

2.2. Problem with 'anomalous mobility'

The problem of 'anomalous mobility' is related to the fact that estimates of the axial electron flux, taking into account their scattering only on atoms and ions in the plasma volume, turn out to be significantly lower than those actually existing in HET and PITR.

The first attempt to explain and describe the real electrons axial mobility was to take into account the so-called wall conductivity by introducing into electrons motion equation, in addition to the frequencies of electron-atom ν_{ea} and electron-ion ν_{ei} scattering, also the frequency of collisions with the wall ν_w.

This attempt turned out to be insufficient, since it did not solve the problem of anomalous mobility not only in the channel, but also in the external beam of the thruster. As a result the expression appeared (in our notation):

$$\vec{V}_{e\Delta}=i_\perp\left(\frac{m_e\left(\nu_{ei}+\nu_{ea}+\nu_w\right)}{eB^2}+\frac{K_B}{B}\right)E_\perp^{(eff)}, \tag{2}$$

$$\vec{E}^{(eff)}=\vec{E}+\frac{\nabla P_e}{en_e}, \tag{3}$$

where B – magnetic induction; $i_\perp$ – unit vector, transverse to the magnetic line in the axial-radial plane; $E_\perp^{(eff)}$, $\vec{V}_{e\Delta}$ – effective electric field tension and the part of electrons mass flux velocity crossed to magnetic line;

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K_B – some empiric coefficient; n_e, P_e – electrons population and pressure.

The second term in expression (2) is introduced with reference to the anomalous "Bohm" diffusion coefficient [3] – the paper from 1949:

$$D_{Bohm} = \frac{k T_e}{16 e B}, \quad (4)$$

where according to (2) must be $K_B = 1/16$.

The authors of the paper [4] explain the presence of the second term in (2) by plasma turbulence, without being embarrassed by the absence of any characteristics of the plasma itself in this term.

There is an expression similar to (2), but it was not obtained "from scratch", but as a result of solving the equation of motion of electrons in a strong magnetic field:

$$\vec{V}_{e\Delta} = \left(i_\perp \frac{m_e\left(\nu_{ei} + \nu_{ea} + \nu_w\right)}{e B^2} + i_\varphi \frac{1}{B} i_B \right) E_\perp^{(eff)}. \quad (5)$$

It can be noted that the terms in brackets are vectors transverse not only to the magnetic line, but also to each other. The sum of their absolute values in (2) has no physical sense.

Multiple replication of an expression (2) demonstrates the “development” of the coefficient K_B from $1/16$ [1] through $1/100$, $1/107$ to $1/160$ [4] down (when describing processes in the thruster channel) to $1/4$ [1] and higher – in the external beam, a total in 40 times.

The right of the author of each new paper to "assign" the values ν_w and K_B allows "adjusting" the results of calculations of the thruster integral characteristics to the measured ones. A comparison of the calculated and measured distributions of the parameters in space (presented by far from all the papers) shows a significant discrepancy. In general, as in paper [5], in this case it turns out that calculations along the entire length of the thruster channel show the flow of ions exclusively outward, whereas in the reality of HET in the near-anode region (almost a half the length of the channel) there is a flow of ions to the anode, which reflected in the absence of electric potential sign change together with ions flow [6].

In this case, the main task in any modeling is actually lost – not to comment on the results of measurements of already existing thruster sizes, but to predict the parameters of those that have not yet been created.

The presence of the second term in (2) is connected, among other things, with the absolutely unfortunate name "wall mobility". The reflection of electrons occurs not from the wall, but from the potential barrier in the boundary bipolar shield. Such a shield exists at the plasma boundary with any neighborhood – both with the wall and with the surrounding vacuum. This shield is non-uniform and non-stationary due to plasma oscillations – the reflection of electrons is, on average, elastic, but not specular.

Also, there are no selected directions in scatterings both in the volume and in the boundary shield. These scatterings are not the factors of mobility. Their role is not to initiate the axial flow of electrons, but to do not allow the magnetic field to transform the axial flow into a completely closed azimuth one.

Thus, the so-called "wall mobility" is actually a boundary scattering that occurs both in the thruster channel and in the external beam. This effect itself cannot be directly represented in the gas dynamics equations written for a point in the volume, but as a boundary condition for the flow of electron momentum flux azimuth projection to the boundary, as was done in [8]:

$$\dot{p}_\varphi^{(S)} = \eta_p \frac{v_e}{4} m_e n_e V_{e\varphi}, \quad (6)$$

where v_e – average electron velocity module; $\eta_p = 0.1 - 0.2$ – electron momentum relaxation coefficient of during scattering in a shield.

In the equations of gas dynamics, the flow of the azimuth projection of the electron momentum towards the boundary without a mass flow must be represented by the corresponding radial-azimuth component $P_e^{(r\varphi)}$ of the pressure tensor:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(P_e^{(r\phi)} r^2\right) + e n_e V_{ex} B = \\ = -\left(\nu_{ea} + \nu_{ei}\right) m_e n_e V_{e\phi}. \quad (7)$$

Directly in the equation of motion, the named effect can be represented taking into account the results of paper [8], in which, neglecting the curvature of the channel, the almost constancy of the value $m_e n_e V_{e\varphi}$ along the magnetic line is shown. Taking into account the curvature of the channel, this means that the torque density $m_e n_e V_{e\varphi} r$ is constant. Using the techniques presented in [2], this allows us to write:

$$e n_e V_{ex}(B r) = \\ = -\left(\nu_{ea} + \nu_{ei} + \eta_p \frac{v_e}{2 \Delta r} \right)\left(m_e n_e V_{e\varphi} r\right), \quad (8)$$

where Δr – channel width.

Taking into account the above, expressions (6) – (8) should be used to describe processes both in the thruster channel and in the external beam without using the second term in (2).

2.3. Problem with electrons energy lost

The characteristics of the electron and their energy flux from the plasma, recorded using the Maxwell distribution, have the form:

$$\Gamma_e = n_e \sqrt{\frac{k T_e}{2\pi m_e}} \exp\left(-\frac{e\Delta\varphi}{k T_e}\right), \qquad (9)$$

$$q_s = \Gamma_{ep}\left(e\Delta\varphi + \varepsilon_{s0}\right), \qquad (10)$$

$$\frac{e\Delta\varphi}{k T_e} = \ln\left(\sqrt{\frac{m_i}{2\pi m_e}}\right), \qquad (11)$$

$$\varepsilon_{s0} = 2 k T_e, \qquad (13)$$

where Γ_e –electrons flux density from plasma; q_s – electrons energy flux density from plasma; $\Delta\varphi$ – potential drop inside bipolar shield; ε_{s0} – average residual energy of an electron in the end of shield.

The idea of secondary electron-electron emission arose due to fact that the values of Γ_e and q_s found using (9), (10) significantly exceed the actual values in HET.

This problem is solved in the paper [1] and many other ones using the expressions (in our notation):

$$\Gamma_{e\Sigma} = (1-\delta)\Gamma_e, \qquad (14)$$

$$q_{s\Sigma} = \Gamma_{ep}\left(e\Delta\varphi + \varepsilon_{s0} - \delta e\Delta\varphi\right), \qquad (15)$$

$$\frac{e\Delta\varphi}{k T_e} = \frac{1}{2} + \ln\left((1-\delta)\sqrt{\frac{m_i}{2\pi m_e}}\right), \qquad (16)$$

where Γ_e – primary electrons flux density from plasma; $\Gamma_{e\Sigma}$, $q_{s\Sigma}$ – summary electrons flux density and electrons energy flux density from plasma; δ – coefficient of secondary electron-electron emission.

The presence of the first term in the right part of (16), characteristic of all known HPF-style papers, is one of many examples of the practice of using "ready-made" expressions without understanding their origin and limits of applicability. Expression (16) comes from the formula for the float potential in the theory of a spherical Langmuir probe, with zero total current from the plasma to the probe:

$$\frac{e\varphi_f}{k T_e} = \frac{1}{2} + \ln\left(\sqrt{\frac{m_i}{2\pi m_e}}\right), \qquad (17)$$

where the first term

$$\frac{e\Delta\varphi_{pl}}{k T_e} = \frac{1}{2}, \qquad (18)$$

represents the potential difference between the plasma at a large distance from the probe and the boundary of the plasma with the bipolar layer near the probe.

The potential drop inside the layer itself is represented only by (11).

The potential difference $\Delta\varphi_{pl}$ arises as a self-consistent electric field generated by the plasma itself, equalizing the flows of electrons and ions to the probe due to the transfer of energy from electrons to ions, equal to $e\Delta\varphi_{pl}$, due to which the velocities of both components at the plasma boundary are equal to the ion-sound ('Bohm') velocity:

$$\frac{m_i V_B^2}{2} = e\Delta\varphi_{pl} = \frac{k T_e}{2}. \qquad (19)$$

The use of both records (16) and (19) within the HPF models means that one component of the total energy flux of electrons and ions to the surface is taken into account twice – both in electrons and in ions energy fluxes.

Moving along a magnetic line, only 2% to 5% of secondary electrons in HET are scattered on atoms, ions or electrons – the bulk of them reaches the opposite wall in the channel and do not have time to mix with the primary electrons in the plasma. One can note that expression (15) takes into account the difference in the energy distributions of secondary and primary electrons, which is not taken into account in many papers in the HPF-style. Almost all secondary electrons reach the opposite surface without being able to induce secondary emission in turn.

This means that one need to write instead of (15):

$$\Gamma_e = (1-\delta)\Gamma_{ep} + \delta_{opp}\Gamma_{ep\_opp} \approx \Gamma_e, \qquad (20)$$

where $\delta_{opp}\Gamma_{ep\_opp}$ – the secondary electrons flow from opposite wall.

The only result of secondary emission is a negligible correction to the electrons population in the plasma, on the order of a small fraction of the electrons reaching the surface.

In a flow along the entire length of the channel with a velocity limited by the magnetic field, each electron has

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time to collide with only few other electrons, which is absolutely not enough to form a distribution that is so-so closed to Maxwell one. Any attempt to model processes in electric propulsion devices with closed electron drift without solving the kinetic task is doomed to failure.

3. Compromise kinetic-fluid model

3.1. Categories and equations set

Being not Maxwell one, electrons velocity distribution function in HET and ionization chamber of PITR however is closed to isotropic one due to two factors: strong magnetic field and non-mirror reflection of electrons from Langmuir bound of plasma [2]. Thus, it is possible to built the equations system with detailing not by degrees of distribution deviation from Maxwell one (as in the method of local thermodynamic equilibrium), but by degrees of deviation from isotropic one.

The main object in kinetics is the particle velocity distribution function [9]. The main gas-dynamic parameters are the moments of the distribution function – tensors of different ranks [10]:

$$\mathbf{M}^{[n]}(t,\vec{r}) = m\int f(t,\vec{r},\vec{v})\vec{v}^{\lceil n\rceil}d^3\vec{v} = \rho_M\left\langle \vec{v}^{\lceil n\rceil}\right\rangle, \quad (21)$$

$$d^3\vec{v} = d\Omega v^2 dv, \quad (22)$$

where m – particle mass; ρ_M – mass density; $d\Omega = \sin\vartheta d\vartheta d\varphi$ – solid angle element in spherical coordinates; $\langle\quad\rangle$ – averaging symbol; $\vec{v}^{\lceil n\rceil}$ – factors of $\vec{v}$ [10]:

$$\vec{v}^{\lceil n\rceil} = \underbrace{\vec{v}\vec{v}..\vec{v}}_{n\,\text{times}}. \quad (23)$$

For instance: $\mathbf{M}^{[0]} = \rho_M$ – mass density; $\mathbf{M}^{[1]} = \vec{p}^{(V)}$ – mass flux density≡momentum density; $\mathbf{M}^{[2]} = \mathbf{\Pi}$ – momentum flux density (kinetic tensor), half of which trace $\frac{1}{2}\mathrm{Tr}\mathbf{\Pi} = \varepsilon^{(V)}$ is the energy density; $\mathbf{M}^{[3]} = \mathbf{Q}$ – the 3rd rank nameless tensor, half of which vector-trace $\frac{1}{2}\mathrm{Tr}\mathbf{Q} = \vec{q}$ is the energy flux density.

The basic equation in kinetics is Boltzmann kinetic equation, which writing to electrons is [9] and the equation of the velocity distribution function moment of the n^{th} order can be obtained by multiplying all the terms of the kinetic equation by $m_e\left\langle \vec{v}^{\lceil n\rceil}\right\rangle$:

$$\frac{\partial f_e(\vec{v})}{\partial t} + \nabla\cdot\left(f_e(\vec{v})\vec{v}\right) - \frac{e}{m_e}\nabla_v\cdot\left(f_e(\vec{v})\left(\vec{E}+\vec{v}\times\vec{B}\right)\right) = \frac{\delta f_e(\vec{v})}{\delta t}, \quad (24)$$

$$\frac{\partial \mathbf{M}_e^{[n]}}{\partial t} + \nabla\cdot\mathbf{M}_e^{[n+1]} + \frac{e}{m_e}\left\lfloor \mathbf{M}_e^{[n-1]}\vec{E} + \mathbf{M}_e^{[n]}\times\vec{B}\right\rfloor = \frac{\delta \mathbf{M}_e^{[n]}}{\delta t}, \quad (25)$$

where $\nabla_v = i_x\frac{\partial}{\partial v_x} + i_y\frac{\partial}{\partial v_y} + i_z\frac{\partial}{\partial v_z}$ – Hamiltonian operator in the space of velocity; $\frac{\delta f_e(\vec{v})}{\delta t}$, $\frac{\delta \mathbf{M}_e^{[n]}}{\delta t} = m_e\int\frac{\delta f_e(\vec{v})}{\delta t}\vec{v}^{\lceil n\rceil}d^3v$ – collision integral – change $f_e(\vec{v})$ and $\mathbf{M}_e^{[n]}$ per unit time as a result of collisions; $\lfloor\mathbf{A}\rfloor$ – operation of transforming an arbitrary tensor $\mathbf{A}$ into symmetric one.

The main technique of the compromise kinetic-fluid model is the transition from kinetics to gas dynamics in two steps:

$$\mathbf{M}_e^{[n]} = 4\pi m_e\int_0^\infty \mathbf{F}^{[n]}(v)v^{n+2}dv, \quad (26)$$

$$\mathbf{F}^{[n]}(v) = \frac{1}{4\pi}\int f_e(\vec{v})\, i_v^{\lceil n\rceil}d\Omega, \quad (27)$$

$$\frac{\partial \mathbf{F}^{[n]}}{\partial t} + v\nabla\cdot\mathbf{F}^{[n+1]} + n\frac{e}{m_e}\left\lfloor \frac{1}{v}\mathbf{F}^{[n-1]}\vec{E} + \mathbf{F}^{[n]}\times\vec{B}\right\rfloor - \frac{e}{m_e}\vec{E}\cdot\frac{1}{v^{n+2}}\frac{\partial}{\partial v}\left(v^{n+2}\mathbf{F}^{[n+1]}\right) = \frac{\delta \mathbf{F}^{[n]}}{\delta t}, \quad (28)$$

where $\mathbf{F}^{[n]}(v)$ – distribution function angular moment; i_v – the unit vector in the direction of velocity; $\frac{\delta \mathbf{F}^{[n]}}{\delta t}$ – change $\mathbf{F}^{[n]}(v)$ per unit time as a result of collisions:

$$\frac{\delta \mathbf{F}^{[n]}}{\delta t} = \frac{1}{4\pi}\int \frac{\delta f_e(\vec{v})}{\delta t} i_v^{\lceil n \rceil} d\Omega . \quad (29)$$

Tensor degree of velocity possesses the following properties:

$$\mathrm{Tr}\, i_v^{\lceil n \rceil} = \boldsymbol{\delta} \overset{(2)}{\bullet} i_v^{\lceil n \rceil} = i_v^{\lceil n-2 \rceil}, \quad (30)$$

$$\Delta_\Omega i_v^{\lceil n \rceil} + n(n+1)\, i_v^{\lceil n \rceil} = n(n-1) \left\lfloor \boldsymbol{\delta}\, i_v^{\lceil n-2 \rceil} \right\rfloor, \quad (31)$$

where $\boldsymbol{\delta}$ – unitary tensor of the second rank; $\overset{(n)}{\bullet}$ – symbol for the multiple dot product [11]; Δ_Ω – angular momentum operator:

$$\Delta_\Omega = \nabla_\Omega \cdot \nabla_\Omega , \quad (32)$$

$$\nabla_\Omega \circ = i_\vartheta \circ \frac{\partial}{\partial \vartheta} + \frac{1}{\sin\vartheta} i_\varphi \circ \frac{\partial}{\partial \varphi} , \quad (33)$$

where ∇_Ω – Hamiltonian angular operator; $"\circ" = "\ ", "\times", "\cdot"$ – a symbol of possible action of the operator (gradient, curl, divergence) [11].

The angular momentum $\mathbf{F}^{[n]}$ has similar properties with $i_v^{\lceil n \rceil}$:

$$\mathrm{Tr}\mathbf{F}^{[n]} = \boldsymbol{\delta} \overset{(2)}{\bullet} \mathbf{F}^{[n]} = \mathbf{F}^{[n-2]}, \quad (34)$$

$$\Delta_\Omega \mathbf{F}^{[n]} + n(n+1)\mathbf{F}^{[n]} = n(n-1) \left\lfloor \boldsymbol{\delta}\, \mathbf{F}^{[n-2]} \right\rfloor . \quad (35)$$

The action of multiple dot product [11] $\mathbf{C}^{[m+n-2k]} = \mathbf{A}^{[m]} \overset{(i)}{\bullet} \mathbf{B}^{[n]}$ means $i \le m$, $i \le n$:

$$C^{(m_1 \ldots m_{m-i} n_{i+1} \ldots n_n)} = \\ = \sum_{k_1 \ldots k_i} A^{(m_1 \ldots m_{m-i} k_1 \ldots k_i)} B^{(k_1 \ldots k_i n_{i+1} \ldots n_n)} . \quad (36)$$

Thus, the proposed compromise model combines the features of kinetics and gas dynamics and is intermediate between them in terms of the complexity of solving problems. The angular moments $\mathbf{F}^{[n]}$ of the distribution function, like the total moments $\mathbf{M}_e^{[n]}$, represent the densities and flux densities of mechanical quantities, but also related to the unit range of the velocity module.

The equation of angular moment (28) in the first three terms of the left-hand side is similar to the equation of total moment (25), but in the last term it retains the kinetic feature. When moving to the equation of total moment taking into account (26), we get:

$$\int_0^\infty \frac{1}{v^{n+2}} \frac{\partial}{\partial v}\left(v^{n+2}\mathbf{F}^{[n+1]}\right) v^{n+2} dv = v^{n+2}\mathbf{F}^{[n+1]}\Big|_0^\infty = 0 \quad (37)$$

The system of the angular moments equations as well as the system of full moments equations is essentially open – when writing the equation for the next unknown moment of the n^{th} order, a divergence of the moment of order arises in the second terms on the left sides of (25) and (28). In any description, the system of gas dynamics equations is closed approximately using assumptions of one or another level of accuracy.

In EPT conditions, if it is necessary to calculate viscosity and thermal conductivity in electron dynamics, it is sufficient to take into account the moments $\mathbf{M}_e^{[0]}$, $\mathbf{M}_e^{[1]}$, $\mathbf{M}_e^{[2]}$ and $\mathbf{M}_e^{[3]}$.

Similarly, in the kinetic-fluid model it is sufficient to take into account $\mathbf{F}^{[0]}$, $\mathbf{F}^{[1]}$, $\mathbf{F}^{[2]}$ and $\mathbf{F}^{[3]}$ with the necessary approximate expression for $\mathbf{F}^{[4]}$. Considering that $\mathbf{F}^{[2]}$ includes $\mathbf{F}^{[0]}$, and $\mathbf{F}^{[3]}$ includes $\mathbf{F}^{[1]}$ (34), this means that it is necessary to write only two equations of the form (28).

It is possible to introduce spherical functions $\mathbf{f}^{[n]}(v)$, each of which includes only the degree of anisotropy not taken into account in the previous ones:

$$\mathbf{f}^{[n]}(v) = \sum_{k=0}^{n/2} (-1)^k c_{n-2k}^{(n-k)-} \left\lfloor \boldsymbol{\delta}^{\lceil k \rceil} \mathbf{F}^{[n-2k]}(v) \right\rfloor , \quad (38)$$

$$\mathbf{F}^{[n]}(v) = \sum_{k=\nu}^{n/2} c_{n-2k}^{(n-k)+} \left\lfloor \boldsymbol{\delta}^{\lceil k \rceil} \mathbf{f}^{[n-2k]}(v) \right\rfloor , \quad (39)$$

$$\nu = \begin{cases} 0, & n \le N \\ (n-N+1)/2, & n > N \end{cases} , \quad (40)$$

where N – the maximum degree of anisotropy of the distribution function, which needs to be taken into account in a specific approximation; $c_k^{(m)+}$, $c_k^{(m)-}$ – coefficients of the distributions:

$$c_k^{(m)+} = \frac{2k+1}{2m+1} \frac{2^k (2m-k)!\, m!}{(m-k)!\,(2m)!} , \quad (41)$$

$$c_k^{(m)-} = \frac{(2m)!}{2^{2m-k}(m-k)!\, m!\, k!} . \quad (42)$$

The velocity distribution function can be represented as a series of spherical functions:

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$$f(\vec{v}) = \sum_{n=0}^{N} (2n+1) \mathbf{f}^{[n]}(v) \overset{(n)}{\bullet} i_v^{\lceil n \rceil} = \\ = \sum_{n=0}^{N} (-1)^{N-n} \sum_{i=0}^{1} C_{n,i}^{(N)} \mathbf{F}^{[2n+i]}(v) \overset{(2n+i)}{\bullet} i_v^{\lceil 2n+i \rceil}, \quad (43)$$

$$C_{n,i}^{(N)} = (2(N+n+i)+1) c_{2n+i}^{(N+n+i)-}. \quad (44)$$

The approximate expression for $\mathbf{F}^{[4]}$ taking into account (38) – (40) has the form:

$$\mathbf{F}^{[4]} = \left\lfloor \frac{30}{35} \boldsymbol{\delta} \mathbf{F}^{[2]} - \frac{3}{35} \boldsymbol{\delta}^{\lceil 3 \rceil} \overset{(2)}{\bullet} \mathbf{F}^{[2]} \right\rfloor. \quad (45)$$

3.2. Preliminary results

The differential equations for angular moments allow solve the problem of the distribution of HET parameters in the channel and external beam. The materials of works [2, 7, 8, 12, 13] were used to formulate the boundary conditions and integrals of electron-electron, electron-ion, electron-atom and ionization collisions in the angular moments equations.

Figure 1 compares the logarithms of the calculated and Maxwell distribution functions corresponding to the same values of concentration and temperature, i.e. such that the integrals are simultaneously the same for the calculated and Maxwell functions:

$$4\pi m_e \int_0^{\infty} f_0(v) v^2 dv = \rho_{Me}, \quad (46)$$

$$4\pi m_e \int_0^{\infty} f_0(v) v^4 dv = \frac{3}{2} \frac{\rho_{Me}}{m_e} kT_e, \quad (47)$$

where $f_0(v) = \mathbf{F}^{[0]}(v) = \mathbf{f}^{[0]}(v)$ – equal to each other angular momentum and spherical function of 0^{th} orders (scalars).

Such graphs are constructed based on measurements of plasma parameters using a Langmuir probe [9]. In particular, the electron temperature is determined by the slope of the curve (straight line for Maxwell distribution). Boundary scattering effect was described by expression (6), (7) both in the channel and in external beam without use doubtful idea of secondary emission.

The quantitative degree of difference between the calculated functions and the Maxwell ones is the greater, the higher the electron temperature. At the same time, these differences are qualitatively the same. The electron temperature $kT_e = 4e\varphi_i$ is characteristic of the middle part of the HET channel in the region of transition from subsonic plasma flow to supersonic one.

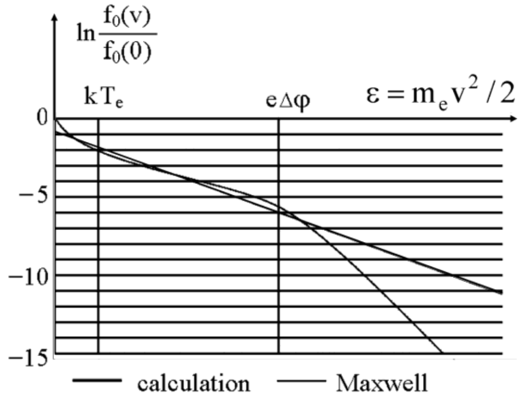

Fig. 1. Moment $f_0(v)$ distribution when $kT_e = 4e\varphi_i$

Four characteristic ranges of electron energy can be distinguished:

1) $0 \le \varepsilon < (\approx 2e\varphi_i)$;
2) $(\approx 2e\varphi_i) < \varepsilon < e\Delta\varphi - (\approx 2.5kT_e)$;
3) $e\Delta\varphi - (\approx 2.5kT_e) < \varepsilon < e\Delta\varphi + (\approx 0.5kT_e)$;
4) $\varepsilon > e\Delta\varphi + (\approx 0.5kT_e)$.

In regions 1 and 3 there is an excess, and in regions 2 and 4 there is a deficiency of electrons in comparison with the Maxwell distribution.

The excess of electrons in region 1 and the deficiency in region 2 are due to the active removal of electrons in ionization acts from the energy region near $\varepsilon \approx 3e\varphi_i$ with the maximum value of the ionization cross section to the region $\varepsilon < e\varphi_i$ with the zero value of this cross section.

The deficiency of electrons in region 4 is due to their active removal to the radial boundaries of the thruster channel.

The excess of electrons in region 3 is due to the correspondence of the calculated and Maxwell distribution functions: with a deficit of electrons in region 2, the simultaneous equality of integrals (45), (46) is possible only with their excess in at least two energy regions – "to the left" and "to the right" of region 2.

Figure 2 compares the logarithms of the calculated and Maxwell distribution functions corresponding to the same values of concentration and temperature $kT_e = 0.5e\varphi_i$, which is characteristic of the near-anode region of the channel and the external beam. Figure 3 shows the results of probe measurements of the electron energy distribution obtained in [14] for $kT_e = 4.5$ eV – the scale ε / kT_e is inserted by us. There

$$4\pi m_e f_0(v) v^2 dv = f_e(\varepsilon) d\varepsilon, \quad \varepsilon = \frac{m_e v^2}{2}, \quad (49)$$

$$f_e(\varepsilon) = \frac{4\pi}{\sqrt{m_e}} \sqrt{\varepsilon}\, f_0\left(\sqrt{\frac{2\varepsilon}{m_e}}\right). \quad (50)$$

The difference compared to Fig. 2 is that it does not introduce a multiplier $\sqrt{\varepsilon}$ into the distribution function – while the picture in the region of small energies is more informative than in Fig. 3. But this difference when $\varepsilon \geq 3kT_e$ becomes not so significant.

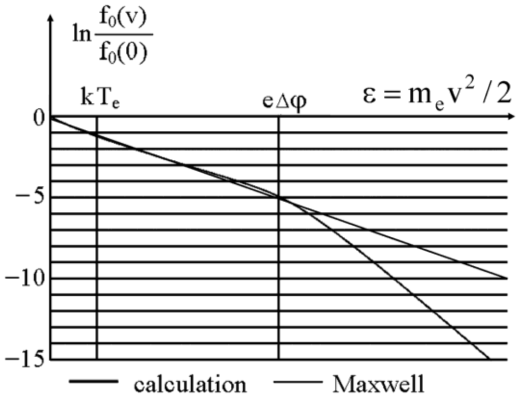

Fig. 2. Moment $f_0(v)$ distribution when $kT_e = 0.5e\varphi_i$

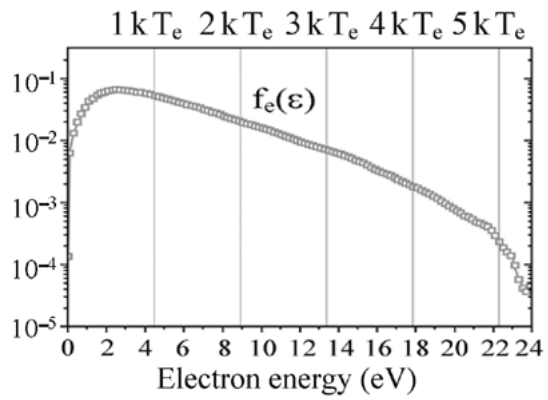

Fig. 3. Electrons energy distribution measured in far-field plume of PPS 1350 ML Hall thruster [14]

Calculation by linear approximation gives the same results as calculation by the method, which is actually the core of kinetics-fluid model, as evidenced by the graph break in the vicinity $\varepsilon = 5kT_e$, just like in Fig. 2.

It is characteristic that in [14] the results of measurements in the external jet of the thruster are presented, where there are no walls and, accordingly, no secondary electron-electron emission, which has been used for many years to "correct" the results of calculations using the Maxwell distribution that does not exist in HET.

The calculations were carried out in the range $0.5e\varphi_i \leq kT_e \leq 6e\varphi_i$. The value of the potential barrier in the boundary layer varied from $e\Delta\varphi = 4.9kT_e$ to $e\Delta\varphi = 5.1kT_e$ compared with the value for the Maxwell distribution $e\Delta\varphi = 5.27kT_e$, and the value of the average residual energy of an electron at the end of the layer varied from $\varepsilon_{s0} = 0.90kT_e$ to $\varepsilon_{s0} = 0.96kT_e$ compared with the value for the Maxwell distribution $\varepsilon_{s0} = 2kT_e$.

In the range with a relative error of no more than 1.1 %, we can assume:

$$e\Delta\varphi + \varepsilon_{s0} = 6kT_e. \quad (51)$$

The value of the total energy carried away by electrons from the plasma, calculated according to Maxwell, turns out to be approximately 20% higher than the actual value and with the addition once again of value $\Delta\varphi_{pl}$ (18) – by 30%.

Here, the only term in equation (28) sensitive to scale characteristics was the electron-electron scattering integral, proportional to the electron temperature. However, the contribution of this term to the electron velocity distribution was within 2.5% at $kT_e = 0.5e\varphi_i$ and 0.25% at $kT_e = e\varphi_i$. Thus, the obtained results are universal over a wide range of HET operating modes.

4. Conclusions

On the example of HET, it is shown that the values of the sought parameters of electric propulsion devices when using the PIC-Fluid method are well within the error limits in the PIC unit, and in the Fluid unit they are represented by a random set of records of a low level of detail in the absence of understanding of their origin and conditions of applicability. The closeness of the calculated integral characteristics to those measured for several already developed standard sizes of HET is achieved due to the variation in a wide range of the so-called 'empirical coefficients' in the event of a catastrophic mismatch of the distributed local characteristics. The main prognostic function of mathematical models – prediction of the characteristics of possible new models of the thruster – is completely lost.

In this paper we have introduced compromise kinetic-fluid model for description of electrons behavior in electric propulsion devices with closed electrons drift. It has been shown that:

1) electrons velocity distribution function in electric propulsion devices with closed electrons drift being not

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Maxwell because of small collisions frequency is however close to isotropic due to strong magnetic field and non-mirror reflection of electrons from potential barrier inside Langmuir sheath;

2) it is possible to build the equations system of velocity distribution function angular moments, which permits to detail electrons velocity distribution in the level anisotropy enough to have more precise values of particles, momentum and energy flow densities to walls of chamber;

3) approximate numeric solutions for HET were obtained, which have shown significant difference comparatively with those, obtained with the use of Maxwell's distribution;

4) the solutions obtained together with the results of previous works of authors [2, 7, 8, 12, 13] can be used in calculation of electric propulsion devices with closed electrons drift exploitation parameters with higher precision than with use of Maxwell's ones.

Contributions of authors: conceptualization, methodology – **Serhii Nesterenko**; formulation of tasks, analysis – **Serhii Nesterenko**, **Huang Zhihao**; development of model, software, verification – **Huang Zhihao**; analysis of results, visualization – **Serhii Nesterenko**, **Shahram Roshanpour**; writing – original draft preparation, writing – review and editing – **Shahram Roshanpour**.

Conflict of Interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

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This study was conducted without financial support.

Data Availability

The manuscript has no associated data.

Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

All the authors have read and agreed to the published version of this manuscript.

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КОМПРОМІСНА КІНЕТИЧНО-ГАЗОДИНАМІЧНА МОДЕЛЬ ДИНАМІКИ ЕЛЕКТРОНІВ В ЕЛЕКТРОРАКЕТНИХ ПРИСТРОЯХ ІЗ ЗАМКНУТИМ ДРЕЙФОМ ЕЛЕКТРОНІВ ЯК АЛЬТЕРНАТИВА ГІБРИДНОМУ МЕТОДУ PIC-FLUID

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До електроракетних пристроїв із замкнутим дрейфом електронів належать холлівські двигуни, плазмово-іонні двигуни з радіальним магнітним полем та геліконні двигуни, джерела плазми, іонів та електронів. У рамках активно тиражованого в останні десятиліття гібридного PIC-Fluid методу розрахунку холлівського двигуна рівень деталізації в блоці PIC не відповідає критерію суцільності середовища, а фрагментарний набір рівнянь у блоці Fluid не містить декількох доданків, необхідних для розрахунку і свідчить про глибоке нерозуміння походження та меж застосування рівнянь та істинної природи процесів. Обчислення характеристик іонізації, висоти потенційного бар'єру на межі плазми, потоків електронів та іонів та їхньої енергії на поверхню камери двигуна здійснюється з використанням максвеллівського розподілу, умови формування якого не відповідають реаліям у розрідженій плазмі електроракетних пристроїв. Близькість розрахованих інтегральних характеристик до виміряних досягається із застосуванням 'емпіричних' коефіцієнтів з відмінністю в десятки разів у різних публікаціях при повній неможливості передбачення характеристик ще не розроблених і не випробуваних зразків електроракетних пристроїв. У цій роботі запропоновано компромісний метод, можливість використання якого обумовлена близькістю розподілу електронів за швидкістю до ізотропного через вплив сильного магнітного поля та недзеркальним відбиттям електронів від потенційного бар'єру на межі плазми. Метод полягає в оперуванні кутовими моментами функції розподілу без інтегрування складових кінетичного рівняння модулем швидкості. Для розрахунку щільностей маси, імпульсу, енергії та їх потоків з урахуванням дисипативних процесів виявляється достатнім визначення кутових моментів другого і третього порядків, сліди яких включають відповідно моменти нульового і першого порядків. Наведено рівняння кутових моментів, проміжні між кінетичним рівнянням та рівняннями моментів функції розподілу за швидкістю. Записано розкладання функції розподілу за швидкістю до ряду за кутовими моментами. Проведено розрахунки, які показали достатню відповідність з відомими результатами вимірювань та значну відмінність характеристик ленгмюрівського прошарку та плазми на кордоні з ним від знайдених з використанням максвеллівського розподілу. Використання отриманих результатів дозволяє суттєво підвищити точність прогнозування параметрів двигунів та за рахунок цього знизити обсяги витратних експериментів для оптимізації їх характеристик.

Ключові слова: холлівський двигун; функція розподілу за швидкістю; моменти функції розподілу; кінетичне рівняння; ленгмюрівський прошарок; Hybrid PIC-Fluid.

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