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INVERSE PROBLEMS OF THERMOELASTICITY FOR RECTANGULAR PLATES

New inverse thermoelasticity problems for frictionally interacting layers have been formulated, in which unknown thermal loading (temperature of boundary surface and intensity of frictional heat flux) has been determined using additionally given vertical displacements of one of the outer boundary surfaces. The functional spaces, for which the problems are well-posed, have been found. The method for solving the problems has been suggested and numerically verified with the use of the solution of the direct problem. This paper deals with the determination of heating temperatures and temperature distributions on the upper surface of a thin rectangular plate, (defined as $-a/2 \le x \le a$, $-b/2 \le y \le b/2$). The expressions of the heating temperatures and temperature distributions have been obtained in series form, involving Bessel's functions with the help of the integral transform technique. Thermoelastic deformations have been discussed and illustrated numerically with the help of temperature and determined.

Key words: *inverse problem, inverse transient function, thermoelastic deformation, rectangular plate*

INTRODUCTION

The inverse thermoelastic problem consists in the determination of the temperature heat flux on the boundary surfaces of the solid body when the conditions of the displacement and stresses are known at some points of the solid under consideration.

The problem at issue was studied by many scientists: Grysa & Cialkowski (1980), Grysa & Kozlowski (1982) investigated one-dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. The problems of normal deflection of an axisymmetrically heated circular plate in the case of fixed and simply supported edges were considered by Boley & Weiner (1960). Further, Roychoudhuri (1973) succeeded in determining the normal deflection of a thin clamped circular plate due to ramp-type heating of a concentric circular region of the upper face. Ishihara & Noda (1997) considered the theoretical analysis of thermoelastoplastic deformation of a circular plate due to partially distributed heat supply.

In this paper, we have based our calculations on Roychoudhari (1973) and analyzed the thermoelastic deformation on the upper surface in a thin circular plate exposed to heating. The results, obtained in series form involving Bessel's functions, are illustrated numerically.

ANALYSIS

Consider a thin circular plate of radius a and thickness b defined as $0 \le r \le a, -b/2 \le z \le b/2$. The plate is initially of zero temperature.

The thermal stressed state of electronic plates under known initial-boundary thermal and mechanical conditions can be investigated with the use of well-known methods, such as [1-3].

However, for electronic plates, the information on thermal loading is often available only on some part of a surface due to the limited access that makes an arelevant thermoelasticity problem ill-posed. To determine a thermal mode and thermal stressed state of the block's electronic plates in this case, it is possible to use additional information and to converse the initial thermoelasticity problem to the inverse one [4,5].The problem of determining the mathematical and mechanical characteristics of plate material is also important. The mathematical and mechanical characteristics of the plate material are often considered as to be constant values in calculations. In reality the mathematical and mechanical characteristics depend on many factors, such as normal loading, temperature of plates, Young's module, Poisson's coefficient, etc., which are also changeable in the case of non-stationary process. In this paper is shown that the problem of finding the pattern law of the thermal stress and temperatures under known boundary and initial conditions can be solved by using additionally measured horizontal and vertical displacements of one of the outer boundary surfaces of the electronic plates, that is by solving of the inverse problem. Using the found time change of the thermal stress…… and known sliding velocity and contact pressure, it is possible to determine the time change heat flux intensity, which is required for adequate calculation of the electronic plates and for the choice of rational operating modes.

CHALLENGE

Let us consider a two-dimensional model of the electronic plates. We assume that the lower surface of the first plate is elastically fixed, while the second plate is moving with variable speed over the first one and is pressed to it. As a result of plate motion on the contact surface, heat is formed. The intensity of the heat flux equals the specific power of mechanical forces. The mechanical contact of plates is assumed to be ideal and the thermal (heat) flux is considered to be non-ideal.

Within the assumed framework, we need to solve the following problems:

– (1) unknown thermal loading on the fixed surface of the electronic plates at given temperature and vertical and horizontal displacements of the other boundary surface;
– (2) time change of the temperature's range (intensity of heat flux) using additionally known horizontal and vertical displacements of the outer loaded boundary surface of the electronic plates under given initial and boundary thermal conditions;

– the inverse thermoelasticity problems involve two parameters – thermal loading (temperature of the boundary surface) and heat flux intensity – that are determined by displacements of the boundary surface.

METHODS & DISCUSSION

In this paper we have relied on equations of the correspondent problem of thermoelasticity for the electronic plates consideration [3]. We have based our calculations on horizontal and vertical displacements of the second plate in the form of dependence of temperature of each plate upon the distribution. The temperatures of plates have been found with the help of the Laplace integral transformation of dependences on loading thermal factors and take into account that the displacements of the outer boundary surface of the second plate are known, we will obtain the convolution type of Volterra integral equations of the first kind

$$\int_0^{\tau} K_i(\tau-\xi) Q_i(\xi) d\xi = F_i(\tau), \qquad 0 \le \xi \le \tau, \qquad i=1,2 \qquad (1)$$

To determine the unknown function with the time moment In equations (1) F denotes known functions presented by the prescribed displacements and functions, which are given in the boundary and initial conditions, are known kernels.

The investigation of the kernel has shown that it is possible to find the unique continuous stable solution to problem (1) using the Laplace integral transformation, if input functions (displacements, temperature of the surface, intensity of the heat flux) are twice continuously differentiable from space. This means that in the indicated functional spaces problem (1) is well-posed. Integral equation (1), which corresponds to problem (20, can be reduced to the Volterra integral equation of the second kind with the kernel of integrable singularity. The unique continuous stable solution of the obtained equation can be found by the method of averaged functional corrections [6]. In this paper the inverse problem is well-posed if input functions (displacements, temperatures of surfaces) are continuously differentiable space. The obtained solutions of the formulated problems allow us to investigate the change of functions in time during the whole period of plates as well as dependence of the plates on the determinined parameters of the process (such as sliding velocity, contact pressure, temperature of contact surface, Young's module and Pouisson's coefficient).

NUMERICAL VERIFICATION

For the steklotekstolit's STEF plate the suggested method for solving the formulated problems has been numerically verified. For this purpose, we haveprescribed boundary thermal conditions and determined the time change of horizontal and vertical displacements of the outer boundary surface of the plate as a solution of the appropriate direct contact thermoelasticity problem. The found displacements are approximated by cubic spline with accuracy and have been used as given in inverse problems. Using the suggested approach, we have determined the solutions of inverse problems – the boundary surface temperature (problem (1)) and the coefficient of heat flux (problem (2)). The comparison of the found solutions with the functions given in direct problems has shown that the maximal relative deviation of the solutions from the appropriate known functions does not exceed in problem (10 and in problem (2). This provides evidence of satisfactory accuracy of obtained inverse problems solutions. It should be noted that smaller of the error of horizontal and vertical displacements approximation leads to the decrease of relative deviation that numerically confirms the stability of obtained solutions.

CONCLUSIONS

Within the framework of the two-dimensional model of the temperature stress of electronic plates, new inverse thermoelasticity problems have been formulated, in which unknown thermal loading (temperature of the outer boundary surface, heat flux) is determined by the additionally known time change of horizontal and vertical displacements of one of the outer boundary surfaces.

The method for solving the formulated problems has been suggested based on their reduction to the Volterra integral equations with further use of the Laplace integral transformation and the method of averaged functional corrections.

On the basis of the analysis of the formulated problems, the functional spaces in which the problems are well-posed have been determined.

For the electronic plates, the numerical verification of the suggested method for solving the formulated problems has been performed, which has confirmed its efficiency.

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ОБРАТНЫЕ ЗАДАЧИ ТЕРМОУПРУГОСТИ ДЛЯ ПРЯМОУГОЛЬНЫХ ПЛАСТИН

В. О. Повгородний

Новые обратные задачи термоупругости для фрикционно взаимодействующих слоев были сформулированы. В этих задачах неизвестная тепловая нагрузка (температура граничной поверхности и интенсивность фрикционного теплового потока) была определена с использованием данных вертикального смещения одной из внешних граничных поверхностей. Функциональные пространства, для которых обратные задачи корректны, были найдены. Был предложен способ решения обратных задач и проверен с использованием многократного решения прямой задачи. Эта статья посвящена определению температур нагрева и распределения температур на верхней поверхности тонкой прямоугольной пластины (определяемой как -a / 2 ≤ x ≤ a, -b / 2 ≤ y ≤ b / 2). Выражения температур нагрева и распределения температур были получены в виде ряда, включая функции Бесселя с помощью интегрального преобразования. Термоупругие деформации были обсуждены и проиллюстрированы численно с помощью численных методов определения температур.

Ключевые слова: обратная задача, обратная переходная функция, термоупругая деформация, прямоугольная пластина

ОБЕРНЕНІ ЗАДАЧІ ТЕРМОПРУЖНОСТІ ДЛЯ ПРЯМОКУТНИХ ПЛАСТИН

В. О. Повгородній

Нові обернені задачі термопружності для фрикційно взаємодіючих шарів були сформульовані. В цих задачах невідоме теплове навантаження (температура граничної поверхні та інтенсивність фрикційного теплового потоку) було визначене з використанням даних вертикального зміщення однієї з зовнішніх граничних поверхонь. Функціональні простори, для котрих обернені задачі коректні, були знайдені. Був запропонований засіб використання обернених задач та перевірений з використанням багатократного вирішення прямої задачі. Ця стаття присвячена визначенню температур нагріву та розподіленню температур на верхній поверхні тонкої прямокутної пластини (яка визначається як -a / 2 ≤ x ≤ a, -b / 2 ≤ y ≤ b / 2. Вираження температур нагріву та розподілення температур були одержані у вигляді ряду, враховуючи функції Беселя за допомогою інтегрального перетворення. Термопружні деформації були розглянуті та проілюстровані чисельно за допомогою чисельних методів визначення температур.

Ключові слова: обернена задача, обернена перехідна функція, термопружня деформація, прямокутна пластина

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