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MAHDI MORTAZAVI<sup>2</sup>**<sup>1</sup> *National Aerospace University (KhAI), Kharkov, Ukraine*<sup>2</sup> *Amirkabir University of Technology, Tehran, Iran***ASSESSMENT INFLUENCE OF INCOMPLETE SIMILARITY OF DYNAMICALLY  
SCALED MODELS ON FLIGHT-TEST RESULTS**

*In this paper, the effect of incomplete similarity of inertial properties of dynamically scaled models of the airplane is investigated. The approach of flying quality analysis is presented to examine the possibility of using the approximate similar model to obtain reliable data from the flight test. Relationships are proposed for the longitudinal and lateral-directional flying qualities between the full-scale airplane and its model. Sensitivity analysis of flight characteristics for changes in the moment of inertia is performed. Results of analyses on Boeing-747 showed that the 7% changing the moment of inertia in the lateral mode and 13% in longitudinal mode can change the level of flying quality based on MIL-8785C standard. The proposed method improved the previous methods and can give the definite range for allowable deviations of moments of inertia errors for acceptance of the model as a research tool.*

**Keywords:** *approximate similarity, inertial properties, dynamically similar/scaled model, simulation, flight test, flying quality, integral performance index.*

**Introduction**

Several methods in aircraft design and development process are used for aircraft behavior prediction such as theoretical methods and simulation, computational fluid dynamics (CFD) and wind tunnel testing. However, all of these methods have errors in comparison with real flight conditions. One of the most effective techniques for aeronautical researches is free-flying dynamically similar/scaled model (FDSM) [1].

FDSMs have been widely used during the creation of new flying vehicles, for testing aerodynamic concepts, radical configurations, control systems development and exploring high-risk flight envelopes [2-4]. Moreover, FDSM can be used for fast evaluation of concepts in early design stages [5]. FDSM is not only geometrically but also dynamically scaled replicas. Dynamical scaling includes dimensional and scaling for weight, inertia, actuator dynamics and control system response. The similitude requirements and scaling relationship applied to FDSM testing were presented by Wolowicz et al [6].

Determination of mass-inertial parameters of FDSM and tuning is one of the main tasks in quality control of FDSM [7, 8]. In the FDSM design and development process, it is attempted to ensure the similarity criterion of mass-inertial parameters. However, errors are usually created during design and construction of FDSM. Thus, it is necessary to investigate non similar-

ties and tune them [9, 10]. However, in some cases, tuning of all parameters, especially moments of inertia is not feasible. If the similarity of mass-inertial characteristics is not ensured, the constructed model can be useless and will lead to considerable financial losses. One of the sources of non-similarities can be due to mass-inertial parameter's errors. The possibility of using this model and determination metrics for obtaining reliable data from flight research is the goal of the study.

In this regard, Betin presented the method of comparing the simulation results of the actual and ideal model to evaluate a level of similarity of FDSM with full-scale airplane [11, 12]. Comparison was conducted by an integral performance index of the automatic flight control system (AFCS). Betin proposed the allowable difference of 5% between the ideal and actual model responses by the integral performance index.

Furthermore, Sarlak tried to solve the problem of approximate similarity by designing a control system for compensating the non-similarities [13]. Moreover, Whorton discussed the metrics for similarity of two closed-loop dynamics systems [14]. The closed-loop systems could provide the desired output with proper control law. However, in some flight regimes such as post-stall and spin the control surfaces are not active completely. Thus, the examination of similarity in the open-loop system is essential. Although, in applications such as new-generation fighters where the open-loop plant is unstable, the similarity of the closed-loop system dynamics is relevant. Nevertheless, previous meth-

ods did not present the sensible criteria for acceptance of the model with deviations in mass-inertial characteristics.

Therefore, the goal of this paper is the presentation of new approach for assessment of incomplete similarity of FDSM. This approach is proposed based on flying quality analysis of the airplane. In this method, the deviation of mass-inertial parameters of the airplane is accepted to the extent that the level of flying quality does not change.

### 1. Problem statement

According to the similarity requirements of the rigid airplane, the actual values of main parameters of FDSM, i.e. mass, center of mass, axial and product of inertia have to be tuned to the required values.

$$(m_M^A, X_{CG}^A, Y_{CG}^A, Z_{CG}^A, I_X^A, I_Y^A, I_Z^A, I_{XY}^A, I_{YZ}^A, I_{ZX}^A) \rightarrow (m_M^R, X_{CG}^R, Y_{CG}^R, Z_{CG}^R, I_X^R, I_Y^R, I_Z^R, I_{XY}^R, I_{YZ}^R, I_{ZX}^R), \quad (1)$$

where  $m$  – mass of the model;

$X, Y, Z$  – coordinates of CG;

$I$  – axial and product moments of inertia;

CG – subscript refers to center of gravity;

M – subscript refers to model;

A – superscript refers to actual model;

R – superscript refers to real model.

In addition to the linear scale factor, the mass-inertial parameters of FDSM depend on the flight altitude of model and full-scale airplane. Knowing the value of density ratio required mass-inertial values of FDSM are calculated using the scale factors of the model in the table 1 [9].

Table 1

The scale factors of FDSM

Parameters	Scale factors*
Linear dimension	$k_L = L_{AC}/L_M$
Density	$k_\rho = \rho_{AC}/\rho_M$
Mass	$k_m = m_{AC}/m_M = k_\rho k_L^3$
Inertia	$k_I = I_{AC}/I_M = k_\rho k_L^5$
Time	$k_t = t_{AC}/t_M = \sqrt{k_L}$
Linear velocity	$k_v = V_{AC}/V_M = \sqrt{k_L}$
Time	$k_t = t_{AC}/t_M = \sqrt{k_L}$
Control deflections	1
Angle of attack	1

\* Subscript AC and M refers to full-scale aircraft and model, respectively.

In the design and development of FDSM, in some cases the complete similarity of mass-inertial properties is not satisfied. Then the question is to what degree and in what ways FDSM and the full-scale airplane have to

be similar in order to obtain reliable results from FDSM flight test. To address these questions, Betin proposed the method for evaluation of ideal and actual FDSM by comparing the integral of squared errors of transient responses of motion parameters to step input. In this regard,  $I_0$  - integral performance index is defined as follows [9].

$$I_0 = \int_0^\infty [x(t) - x_\infty]^2 dt, \quad (2)$$

where  $x(t)$  is the simulated parameter and  $x_\infty$  is the steady-state response. If  $I_0$  for real model and ideal responses have a difference of less than 5%, two models are considered similar. However, this method does not guarantee the similarity and accepts the wide range of moment of inertia changes. For example, the effect of changes of  $I_{yy}$  on response of angle of attack to step input is simulated in Fig. 1 for a business jet airplane. Here the  $I_{yy}$  is changed up to 20%, and the differences in responses are shown. The difference between  $I_0$  for airplane with basic MI and MI increased by 20% is 2%. The 20% of deviation of MI seems to be too much for dynamic similarity problems, but this method accepts it. The idea of comparison of responses of the ideal and real object is rational; however, there is a need to define the metric of acceptance or rejection of approximate similarities. The main goal of FDSM design is the prediction of real airplane behavior. The mistakes in predictions can lead to considerable financial loss in airplane design and development program. Thus, the correct prediction of airplane specific behaviors can be considered as a metric for acceptance of a model. The behavior of airplane is defined by stability and controllability characteristics. The requirement on dynamic stability is typically expressed in terms of damping and frequency of a natural mode. This study uses this concept to assess the degree in which two dynamically scaled models can be said to be similar.

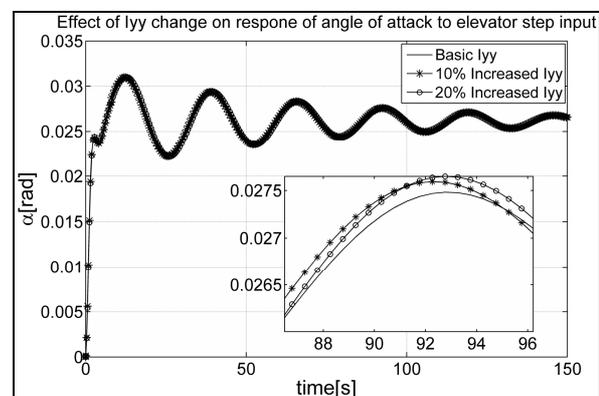


Fig. 1. Effect of moment of inertia change on response of angle of attack

## 2. Method

### 2.1. Flying qualities analysis

The airplane stability has a great importance for the flying qualities related to the passenger comfort, or for the fighting capabilities for a military aircraft. The flying quality levels are defined in Military Specification. The requirement on dynamic stability is typically expressed in terms of damping and frequency of a natural mode. Thus the United States (US) Air Force requires damping and frequency of the oscillation for various flight phases A, B and C at specific range values, according to the defined quality "Level 1", "Level 2" and "Level 3" [13]. In addition, in MIL-F-8785C piloted flying qualities are broken down by aircraft class. The manned aircraft classifications have been traditionally categorized into four different classes [15]:

- Class I: small, light-weight, medium maneuverability airplanes.
- Class II: medium-weight, low to medium maneuverability airplanes.
- Class III: large, heavy-weight, low maneuverability airplanes.
- Class IV: high-maneuverability airplanes.

The Flight Phase Categories are defined as follows:

- Category A: Nonterminal flight phases that require rapid maneuvering, precise tracking or precise flight path control.
- Category B: Nonterminal flight phases that are accomplished with gradual maneuvers (climb, cruise, descent)
- Category C: Terminal flight phases such as takeoff or landing.

In the next sections, the methodology of Flying Quality evaluation in the longitudinal and lateral-directional modes will be presented.

In the present analysis, the dimensional small disturbance equations system is used for the longitudinal and lateral directional motion, given through the linearization of the general equations. It was assumed that the motion of the airplane consists of small deviations from a reference steady flight condition. The longitudinal equations of motion are [16, 17]:

$$\begin{aligned} \dot{u} &= -g\theta\cos\theta_1 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e; \\ U_1 \dot{\alpha} - U_1 \dot{\theta} &= -g\theta\sin\theta_1 + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + \\ &+ Z_q \dot{\theta} + Z_{\delta_e} \delta_e; \\ \ddot{\theta} &= M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + \\ &+ M_q \dot{\theta} + M_{\delta_e} \delta_e, \end{aligned} \quad (3)$$

where  $U$  – Component of velocity along  $X$ ;

$u$  – Perturbed value of  $U$ ;

$g$  – Acceleration of gravity;

$\theta$  – Perturbed value of pitch attitude angle;

$\theta_1$  – Steady state pitch attitude angle;

$X_u$  – Forward acceleration per unit change of speed;

$X_{T_u}$  – Forward acceleration per unit change of speed (due to thrust);

$X_\alpha$  – Forward acceleration per unit angle of attack;

$X_{\delta_e}$  – Forward acceleration per unit elevator angle;

$U_1$  – steady state of velocity;

$\alpha$  – angle of attack;

$Z_u$  – Vertical acceleration per unit change of speed;

$Z_\alpha$  – vertical acceleration per unit angle of attack;

$Z_{\dot{\alpha}}$  – vertical acceleration per unit rate of change of angle of attack;

$Z_q$  – vertical acceleration per unit pitch rate;

$Z_{\delta_e}$  – vertical acceleration per unit elevator angle;

$M_u$  – pitch angular acceleration per unit change in speed;

$M_{T_u}$  – pitch angular acceleration per unit change in speed (due to thrust);

$M_\alpha$  – pitch angular acceleration per unit angle of attack;

$M_{T_\alpha}$  – pitch angular acceleration per unit angle of attack (due to thrust);

$M_{\dot{\alpha}}$  – pitch angular acceleration per unit rate of change of angle of attack;

$M_q$  – pitch angular acceleration per unit pitch rate;

$M_{\delta_e}$  – pitch angular acceleration per unit elevator angle.

The lateral-directional equations of motion are [16]:

$$\begin{aligned} U_1 \dot{\beta} + U_1 \dot{\psi} &= g\phi\cos\theta_1 + Y_\beta \beta + Y_p \dot{\phi} + Y_r \dot{\psi} + \\ &+ Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r; \\ \ddot{\phi} - \frac{I_{xz}}{I_{xx}} \ddot{\psi} &= L_\beta \beta + L_p \dot{\phi} + L_r \dot{\psi} + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r; \quad (4) \\ \ddot{\psi} - \frac{I_{xz}}{I_{zz}} \ddot{\phi} &= N_\beta \beta + N_{T_\beta} \beta + N_p \dot{\phi} + N_r \dot{\psi} + \\ &+ N_{\delta_a} \delta_a + N_{\delta_r} \delta_r, \end{aligned}$$

where  $\beta$  – angle of sideslip;

$\psi$  – Airplane heading angle;

$\phi$  – Airplane bank angle;

$Y_\beta$  – Lateral acceleration per unit sideslip angle;

$Y_p$  – Lateral acceleration per unit roll rate;

$Y_r$  – Lateral acceleration per unit yaw rate;

$Y_{\delta_a}$  – Lateral acceleration per unit aileron angle;

- $Y_{\delta_r}$  – Lateral acceleration per unit rudder angle;
- $I_{xz}$  – Product of inertia about XZ plane;
- $I_{xx}$  – Moment of inertia about X-axis;
- $L_{\beta}$  – Roll angular acceleration per unit sideslip angle;
- $L_p$  – Roll angular acceleration per unit roll rate;
- $L_r$  – Roll angular acceleration per unit yaw rate;
- $L_{\delta_a}$  – Roll angular acceleration per unit aileron angle;
- $L_{\delta_r}$  – Roll angular acceleration per unit rudder angle;
- $I_{zz}$  – moment of inertia about Z-axis;
- $N_{\beta}$  – Yaw angular acceleration per unit sideslip angle;
- $N_{T_{\beta}}$  – Yaw angular acceleration per unit sideslip angle (due to thrust);
- $N_r$  – Yaw angular acceleration per unit yaw rate;
- $N_{\delta_a}$  – Yaw angular acceleration per unit aileron angle;
- $N_{\delta_r}$  – Yaw angular acceleration per unit rudder angle.

These equations can be written in state-space form or analyzed by Laplace-transform technique. Using both methods the same characteristics equations for longitudinal and lateral-directional modes will be obtained. Longitudinal characteristics equation is in form of fourth order polynomial as equation 5.

$$As^4 + Bs^3 + Cs^2 + Ds + E = 0, \quad (5)$$

where the coefficients A, B, C, D and E are a function of dimensional derivatives of the airplane. The roots of this characteristics equation determine the dynamic stability of the airplane. These roots are affected by flight conditions, by airplane mass, mass distribution, geometry and aerodynamics characteristics. This equation factorises into two pairs of complex roots each pair of which describes a longitudinal stability mode. The stability of each mode is determined by the damping ratio  $\zeta$  and the undamped natural frequency by  $\omega$  [17]. The lower frequency mode is called phugoid (Ph) and higher frequency mode is called a short-period (SP) pitching oscillation. The values of  $\omega$  and  $\zeta$  for longitudinal modes are obtained as follows [16]

$$\omega_{SP} \approx \sqrt{\frac{Z_{\alpha}M_q}{U_1} - M_{\alpha}}, \quad (6)$$

$$\zeta_{SP} \approx \frac{-(M_q + \frac{Z_{\alpha}}{U_1} + M_{\alpha})}{2\omega_{SP}}, \quad (7)$$

$$\omega_{Ph} \approx \sqrt{\frac{-gZ_u}{U_1}}, \quad (8)$$

$$\zeta_{Ph} \approx \frac{-X_u}{2\omega_{Ph}}. \quad (9)$$

Lateral-directional characteristic equation is in form a polynomial:

$$A_1s^5 + B_1s^4 + C_1s^3 + D_1s^2 + E_1s = 0, \quad (10)$$

where the coefficients  $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$  and  $E_1$  are a function of dimensional derivatives of the airplane. The lateral-directional characteristic polynomial factorises into three real roots and a complex pair of roots. The roots, or poles, of the lateral-directional characteristic polynomial provide a complete description of the lateral-directional stability characteristics of the airplane. The zero root indicates neutral stability in yaw; the first non-zero real root describes the spiral (S) mode; the second real root describes the roll (R) subsidence mode, and the complex pair of roots describe the oscillatory Dutch-roll (DR) mode. The values of  $\omega$  and  $\zeta$  for lateral-directional modes are obtained as follows [16]

$$\omega_{DR} \approx \sqrt{N_{\beta} + \frac{1}{U_1}(Y_{\beta}N_r - N_{\beta}Y_r)}, \quad (11)$$

$$\zeta_{DR} \approx \frac{-(N_r + \frac{Y_{\beta}}{U_1})}{2\omega_{DR}}, \quad (12)$$

$$T_S \approx -\frac{(L_{\beta} + N_{\beta}\bar{A}_1)}{(L_{\beta}N_r - N_{\beta}L_r)}, \quad (13)$$

$$T_R \approx -\frac{1}{L_p}. \quad (14)$$

## 2.2. Relationship between model and full-scale airplane flying qualities characteristics

Assuming Froude similarity criterion and self-similarity of Reynolds and Mach number, results the aerodynamic equivalence. Some literatures used the concept of dynamic similarity for flying quality's evaluation of unmanned vehicles and scaled models [14, 18-19]. Therefore, the dependence of natural frequency and damping ratio of scaled model and full-scale airplane are as follows:

$$\omega_i^F = \omega_i^M / \sqrt{k_L};$$

$$\zeta_i^F = \zeta_i^M; \quad (15)$$

$i = SP, Ph, DR, S \ \&R.$

These relations can be obtained using the Eq. (6)-(9) and (11), (12) by substituting the relevant scale factors from table 1. For example, for phugoid mode it can be written:

$$\omega_{Ph}^F \approx \sqrt{\frac{-gZ_u^F}{U_1^F}} = \sqrt{\frac{-g}{U_1^F} \times \frac{-\bar{q}_1^F S^F (C_{L_u}^F + 2C_{L_1}^F)}{m^F U_1^F}} = \sqrt{\frac{-g}{\sqrt{k_L} U_1^M} \times \frac{-(k_L \bar{q}_1^M)(k_L^2 S^M)(C_{L_u}^M + 2C_{L_1}^M)}{(k_L^3 m^M)(\sqrt{k_L} U_1^M)}} = \frac{1}{\sqrt{k_L}} \sqrt{\frac{-gZ_u^M}{U_1^M}} = \frac{\omega_{Ph}^M}{\sqrt{k_L}}$$

$$\xi_{Ph}^F \approx \frac{-X_u^F}{2\omega_{Ph}^F} = \frac{\sqrt{k_L}}{\omega_{Ph}^M} \times \frac{-\bar{q}_1^F S^F (C_{D_u}^F + 2C_{D_1}^F)}{m^F U_1^F} = \frac{\sqrt{k_L}}{\omega_{Ph}^M} \times \frac{-(k_L \bar{q}_1^M)(k_L^2 S^M)(C_{D_u}^M + 2C_{D_1}^M)}{(k_L^3 m^M)(\sqrt{k_L} U_1^M)} = \frac{-X_u^M}{2\omega_{Ph}^M} = \xi_{Ph}^M$$

where  $q_1$  – Dynamic pressure;

$S$  – Wing area;

$C_{L_1}$  – Lift coefficient;

$C_{L_u}$  – Variation of airplane lift coefficient with dimensionless speed;

$C_{D_1}$  – Drag coefficient;

$C_{D_u}$  – Variation of airplane drag coefficient with dimensionless speed;

Superscript F refers to full-scale aircraft, and M refers to scaled model. In a similar way, the Eq. (15) can be proven for other modes of motion.

### 3. Results and discussion

For implementation of methodology an example is investigated. The B-747 transport aircraft of class III is selected for flying quality analysis. The main data of this aircraft are presented in table 2. The flying qualities requirements of the aircraft for flight phase of category A are presented in table 3 for longitudinal and lateral-directional modes of motion. Assuming a 1/20 scaled model for it, the main parameters of FDSM are determined and presented in table 4 using scale factors of table 2. Using the equations 6-10, natural frequency and the damping ratio of longitudinal and lateral-directional modes are calculated for full-scale aircraft. Then, using the equation 15, natural frequency and damping ratio of various modes are calculated for scaled model and presented in table 4. Results show that the B-747 has the flying qualities of level 1 in longitudinal and Dutch-roll

modes, and level 2 of quality of flight in spiral and roll modes.

Table 2

Full-scale airplane and its FDSM data

Parameters	Full-scale	Scaled model
Mass (kg)	288770	133.7
$I_{xx}$ (kg.m <sup>2</sup> )	24675890	28.6
$I_{yy}$ (kg.m <sup>2</sup> )	44877570	51.9
$I_{zz}$ (kg.m <sup>2</sup> )	67384150	78
$I_{xz}$ (kg.m <sup>2</sup> )	1315140	1.5
Altitude (m)	12190	1000
Speed (km/h)	956	213.9

Table 3

Flying quality requirements of airplane, class III, category A

MODE		Flying Qualities Requirements		
		Level 1	Level 2	Level 3
Short Period	$\omega_{SP}$	---	---	---
	$\xi_{SP}$	> 0.35, < 1.3	>0.25, < 2.00	> 0.15
Phugoid	$\omega_{Ph}$	---	---	---
	$\xi_{Ph}$	> 0.04	> 0	Unstable
Dutch-Roll	$\omega_{DR}$	> 0.5	> 0.5	> 0.4
	$\xi_{DR}$	> 0.19	> 0.02	> 0
	$\omega_{DR} \cdot \xi_{DR}$	> 0.35	> 0.05	---
Roll	$T_R$	< 1.4	< 3.0	---
Spiral	$T_S$	>17.3	> 11.5	> 7.2

Table 4

Flying quality (FQ) Characteristics of model and full-scale airplane

MODE		Scaled model	Full-scale airplane	Level of FQ
Short Period	$\omega_{SP}$	5.96	1.333	Level 1
	$\xi_{SP}$	0.353	0.384	
Phugoid	$\omega_{Ph}$	0.206	0.046	Level 1
	$\xi_{Ph}$	0.239	0.239	
Dutch-Roll	$\omega_{DR}$	3.95	0.885	Level 2
	$\xi_{DR}$	0.122	0.122	
	$\omega_{DR} \cdot \xi_{DR}$	0.483	0.108	
Roll	$T_R$	0.44	1.979	Level 2
Spiral	$T_S$	2.76	12.358	Level 2

The sensitivity analysis is performed on changes of moments of inertia in longitudinal and lateral-directional modes. Moment of inertia about y-axis has the effect on short-period mode characteristics. Fig. 2 shows that the 13% increase in  $I_{yy}$  reduces damping ratio of the short period to the value of level 2. Moreover, the responses of angle of attack to unit step input of the elevator are presented in Fig. 3 for basic and increased  $I_{yy}$ . The integral performance index ( $I_0$ ) difference is about 1%, what indicates that this metric for comparison is not effective enough. Analysis of phugoid modes showed that the  $I_{yy}$  has no effect on this mode.

Flying quality of Dutch-roll is level 2 and changing to level 3 needs more than 200% of  $I_{zz}$  increase. Therefore, the low variation of MI has the negligible effect on characteristics of this mode. The greatest effect of change in MI is observed on Spiral mode. As it has shown in Fig. 4, the 7% decrease of  $I_{zz}$ , reduces the flying quality level to 3. Finally, the 30% decrease of the  $I_{xx}$ , improved the flying quality level of Roll mode to level 1 as it has shown in Fig. 5.

It should be noted that this approach for various airplanes gives different allowable ranges of MI. Moreover, some additional flying qualities' requirements can be investigated based on MIL-F-8785C. After determination the allowable range of MI for airplane, regarding the relationship between flying quality of airplane and scaled model, the allowable error of MI for FDSM can be obtained as follows. The scale factor of MI from table 1 can be used to calculate the MI of the scaled model. Besides, it can be used to determine the range of allowable variation of MI.

$$k_I = I_{AC}/I_M = k_p k_L^5 \Rightarrow \Delta I_{AC}/\Delta I_M = k_p k_L^5 \Rightarrow \Rightarrow \Delta I_M = \frac{\Delta I_{AC}}{k_p k_L^5} \quad (18)$$

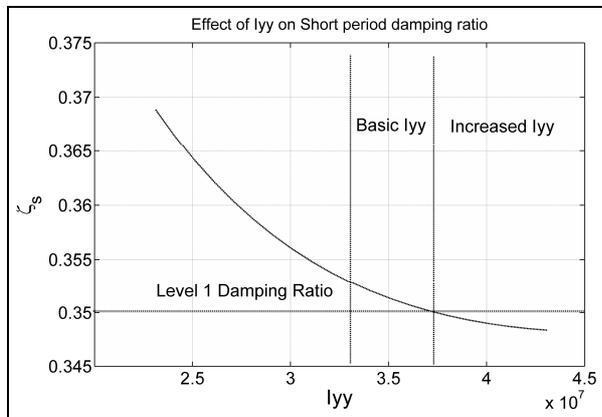


Fig. 2. Effect of moment of inertia change on short period damping ratio

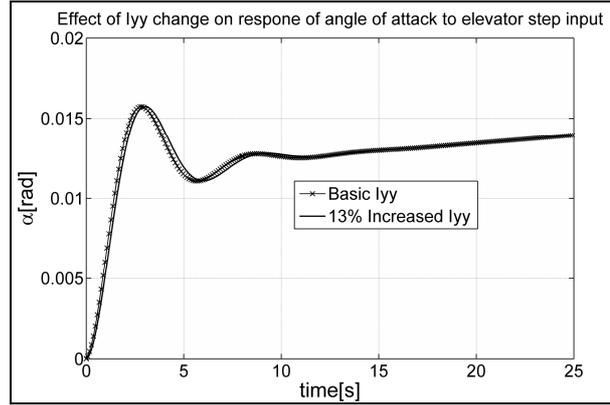


Fig. 3. Effect of moment of inertia change on response of angle of attack

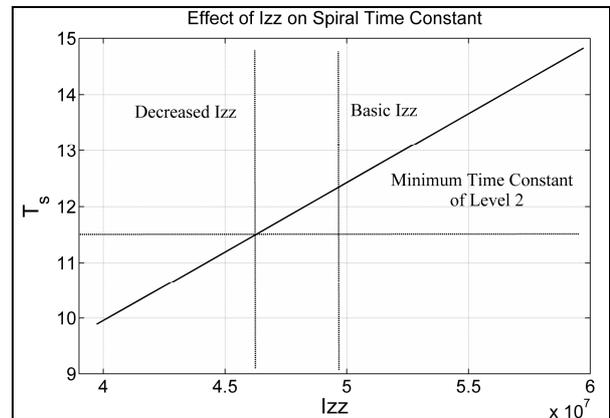


Fig. 4. Effect of moment of inertia change on Spiral time constant

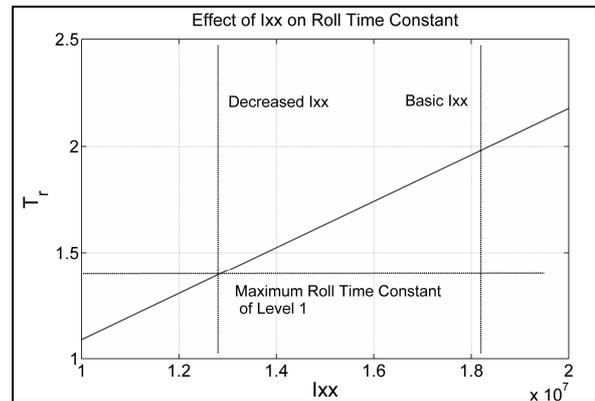


Fig. 5. Effect of moment of inertia change on response of angle of attack

### Conclusion

Confidence about obtaining reliable data from flight test is one of the challenges of FDSM design and development. Usually the complete similarity of mass-inertial characteristics cannot be ensured, and the constructed model must be evaluated as a research tool. In the present paper, the existing methods for evaluation of

FDSM competence were studied. The relationship between flying quality characteristics of the full-scale and scaled model is developed. Furthermore, the approach of flying quality requirements analysis for comparison of ideal and actual FDSM is proposed. An example is solved on B-747 transport airplane. Analyses included the determination level of flying qualities in longitudinal and lateral-directional modes and sensitivity analysis for changes in MI. Results showed that the decrease of  $I_{xx}$  by 7%, improved the flying quality level from 2 to 1 in Roll mode. Similarly, the changes of  $I_{zz}$  and  $I_{xx}$ , altered the flying quality level of Spiral and Short-period modes respectively. The proposed approach gives a better understanding of effect of MI deviations in FDSM compared with previous methods. Additionally, it gives the specific range for allowable variation of MI. Moreover, the results of study indicated the possibility of FDSM flight testing to predict the flying qualities characteristics of the full-scale airplane. However, there is still a need for further studies in this area. Especially it is recommended to carry out the study on spin characteristics that are more sensitive to mass distribution of the airplane.

This approach has some advantages in comparison to previous works. Unlike the preceding papers, this method can guarantee that the two models are similar or not. Moreover, in the present work, the dynamic system can be evaluated in both cases of the closed-loop and open-loop controller.

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## ОЦЕНКА ВЛИЯНИЯ НЕПОЛНОГО ПОДОБИЯ СВОБОДНОЛЕТАЮЩЕЙ МОДЕЛИ НА РЕЗУЛЬТАТЫ ЛЕТНЫХ ИССЛЕДОВАНИЙ

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Рассмотрен новый подход в численном моделировании функциональных возможностей свободнолетающих динамически подобных моделей самолетов. Оценку степени подобия на стадии подготовки к модельным летным исследованиям предложено проводить с использованием пилотажных характеристик, для которых выполнен анализ чувствительности к изменению моментов инерции. Анализ результатов на Боинг-747 показал, что изменение моментов инерции относительно продольной и поперечной осей на 7% и 13% соответственно может изменить уровень пилотажных характеристик базирующихся на стандарте MIL-8785C. Такой подход позволил дать заключение о диапазоне возможных отклонений массово-инерционных параметров при принятии конкретной свободнолетающей модели в качестве инструмента летных исследований.

**Ключевые слова:** неполное подобие, свободнолетающая динамически подобная модель, численное моделирование, пилотажные характеристики, моменты инерции, летные исследования.

## ОЦІНКА ВПЛИВУ НЕПОВНОЇ ПОДІБНОСТІ ВІЛЬНОЛІТАЮЧОЇ МОДЕЛІ НА РЕЗУЛЬТАТИ ЛЬОТНИХ ДОСЛІДЖЕНЬ

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Розглянуто новий підхід у чисельному моделюванні функціональних можливостей вільнолітаючих динамічно подібних моделей літаків. Оцінку ступеня подібності на стадії підготовки до модельних льотних досліджень запропоновано проводити з використанням пилотажних характеристик, для яких виконано аналіз чутливості до змінення моментів інерції. Аналіз результатів на Боїнг-747 показав, що змінення моментів інерції відносно повздовжньої та поперечної осей на 7% та 13% відповідно може змінити рівень пилотажних характеристик, які базуються на стандарті MIL-8785C. Такий підхід дозволив дати висновок про діапазон можливих відхилень масово-інерційних параметрів при прийнятті конкретної вільнолітаючої моделі в якості інструменту льотних досліджень.

**Ключові слова:** неповна подібність, вільнолітаюча динамічно подібна модель, чисельне моделювання, пилотажні характеристики, моменти інерції, льотні дослідження.

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