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# MATHEMATICAL MODEL OF PV MODULE WITH PULSE WIDTH MODULATION CONTROL

*The paper describes a mathematical model of solar photovoltaic power plant with a PWM (Pulse Width Modulation) regulator matching between the PV arrays and loads. For controling of regukator which retains the power plant performance at the point of maximum output, there is used an adaptive algorithm based on the combination of methods of the direct measurement and of the constant coefficient of short-circuit current. The advantages of this control algorithm is a relatively simplicity and good efficiency. Mathematical model of the characteristics of photovoltaic power module is made on the basis of educational bench in school laboratory in the Department of space technology and alternative energy sources, «KhAI».*

***Keywords:*** *Photovoltaic Module, pulse width modulation control, Simulink model.*

## Introduction

Recent years have witnessed a technological revolution in the use of renewable energy sources, the energy due to the photovoltaic (PV) effect can be considered the most essential and prerequisite sustainable resource because of the ubiquity, abundance, and sustainability of solar radiant energy. Regardless of the intermittency of sunlight, solar energy is widely available and is free. The photovoltaic system has been remarkably used prior to the other renewable electric power generations It can generate direct current electricity without environmental impact and contamination when exposed to solar radiation. A photovoltaic system converts sunlight into electricity. The basic device of a photovoltaic system is the photovoltaic cell. Cells may be grouped to form panels or modules. Panels can be grouped to form large photovoltaic arrays. The term array is usually employed to describe a photovoltaic panel (with several cells connected in series and/or parallel) or a group of panels. Most of time one is interested in modeling photovoltaic panels, which are the commercial photovoltaic devices.

The mathematical model of the photovoltaic array may be useful in the study of the dynamic analysis of converters, in the study of maximum power point tracking (MPPT) algorithms and mainly to simulate the photovoltaic system and its components using simulators. This text presents in details the equations that form the I-V model and the method used to obtain the parameters of the equation. The aim of this paper is to provide the reader with all necessary information to develop photovoltaic array models, and circuits that can be used in the simulation of power converters for photovoltaic applications [1, 2].

The relationship between the current and voltage of the photovoltaic cell is highly nonlinear and it can be observed that there is a unique maximum power point (MPP) at a particular environment, and this peak power point keeps changing with solar irradiation and ambient temperature. So it is important to match the PV source and load impedance properly for any weather conditions, to obtain maximum power generation. Therefore, PV power generation system a maximum power point tracker (MPPT) [2, 3].

## System description

The overall block diagram of the proposed PV system is visualized as in Fig. 1.

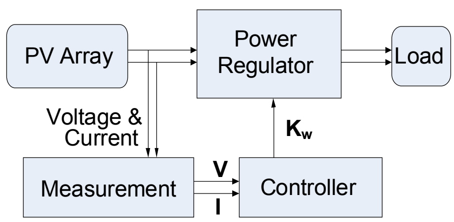


Fig. 1. Block Diagram of Power Conditioning PV System

As mentioned above, the simulation model consists of different elements like solar PV array, Pulse Width Modulation, Controller, Power Regulator, Voltage &Current Measurement, and the load. The system is explained in detail. The structure of model is given in the Fig. 2.

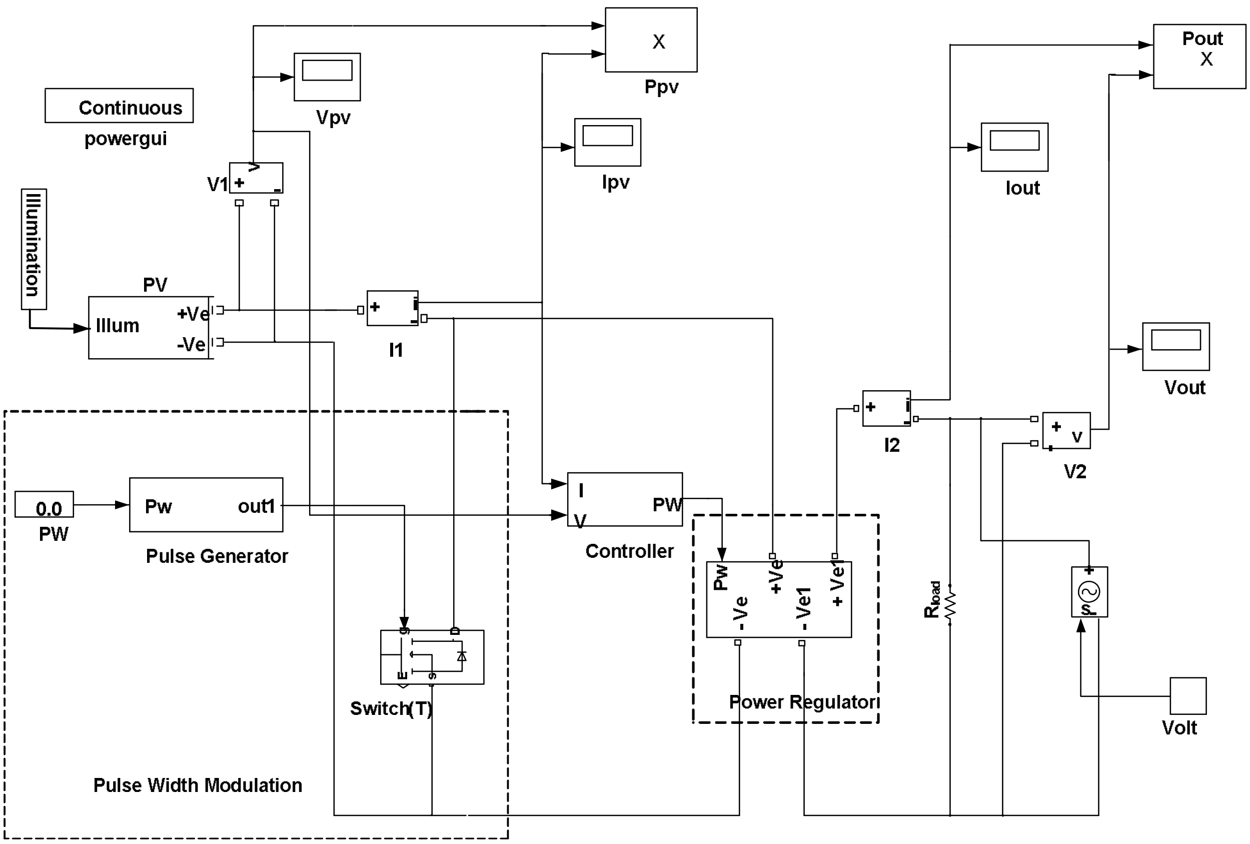


Fig. 2. The structure of model

## PV Cell Model

A photovoltaic cell can be modeled as a current source connected in parallel with a diode. Current source produces a constant current. This current is proportional to the intensity of the Light falling upon the cell. Photovoltaic systems affected weather conditions and solar radiation by directly. The yield of the photovoltaic system and the price related to external working conditions and the variable conditions the operation of the system components at the best spot. Therefore, solar Energy applications, while the photovoltaic system under different and varying conditions Accurately assesses the performance of each element are important. This situation also affects the System design and cause electrical parameters sudden by changing the network changes set over Time in the event of certain.

The performance of a solar cell is needed to understand the correlation Between current and voltage of the cell [4, 5]. There is a current source (solar cell), a parallel diode and a resistance ($R_{sh}$). The shunt resistance $R_{sh}$, as a representative of all losses verified inside the cell, As an effect of the parasitic currents. And a resistance is connected in series to them ($R_s$) at single-diode model equivalent circuit. A series $R_S$ resistance that represents the parasitic resistance of the cell and which includes the resistance of two constituent layers of the cell as well as the resistance of the contacts, this circuit is given in the Fig. 3.

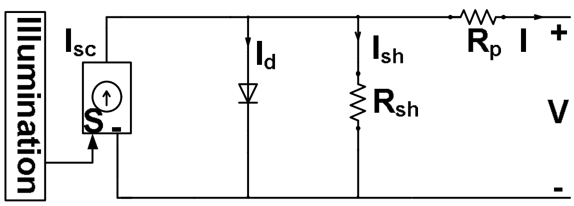


Fig. 3. Equivalent Circuit of a PV Cell

According to the model of a solar cell, the relationship between the cell`s current and voltage, and by applying Kirchoff's laws, we can determine the voltage-ampere dependency of the photovoltaic cell.

### A. Mathematical Model of PV Panel

A PV cell is represented by a simplified equivalent circuit model. The PV battery output current I [4].

$$I = I_{ph} - I_d - I_{Sh}\,, \qquad (1)$$

where $I_{ph}$ – Photovoltaic Current;

$I_d$ – diode current;

$I_{Sh}$ – Short circuit current;

All this is the total current through to the p-n junction, is the total currents generated by the electrons $I_e$ and holes $I_h$ activated by photons mathematical.

States of electrons current in the conduction band $I_{eo}$ and space current in the valence band $I_{ho}$. Boltzmann distribution net flows and the space electron current flows:

$$I_e = I_{eo} \cdot \left[ e^{\frac{qV_D}{K_B bT}} - 1 \right], \qquad (2)$$

where $V_D$ – Diode Voltage;

$K_B$ – Boltzmann constant ($1.381 \times 10^{-23}$ J/K);

T – absolute temperature in Kelvin;

b – Constant Semiconductor.

$$I_h = I_{ho} \cdot \left[ e^{\frac{qV_D}{K_B bT}} - 1 \right], \qquad (3)$$

$$I_d = I_e + I_h = I_S \cdot \left[ e^{\frac{qV_D}{K_B bT}} - 1 \right]. \qquad (4)$$

Varies the absolute temperature of the diode, voltage and as a function of the current drawn by the load. At equation 4, $I_S$ is the diode saturation current, the q electron charge ($1.602 \times 10^{-19}$ C), the potential difference between the ends of the diode $V_D$ [6].

$$I_d = I_S \cdot \left[ e^{\frac{qV_D}{NK_B T}} - 1 \right], \qquad (5)$$

where N – the ideality factor ;

$$I_d = I_S \cdot \left[ e^{\frac{q(V_{PV} + I \cdot R_S)}{NK_B T}} - 1 \right], \qquad (6)$$

where $V_{PV}$ – Panel voltage ;

$$I_{sh} == \frac{V_D}{R_{sh}} = \frac{(V_{pv} + I \cdot R_s)}{R_{sh}}. \qquad (7)$$

The I-V characteristics of the ideal cell with single-diode, shunt resistance $R_{sh}$ and series resistance $R_s$:

$$I = I_{ph} - I_S \cdot \left[ e^{\frac{q(V_{PV} + I \cdot R_S)}{NK_B T}} - 1 \right] - \frac{(V_{pv} + I \cdot R_s)}{R_{sh}}. \qquad (8)$$

Equation (8) is used for the simulation in the computer for the extract of P-V and V-I characteristics of a PV cell as shown in Fig. 3.

A solar cell can at least be characterized by the short circuit current $I_{sc}$.

For V = 0, the $I_{sc}$ current is called the current of the short circuit, and it is the maximum current that the cell can produce cells for $R_s << R_{sh}$.

$$I_{sc} = I_L \cdot \left[ \frac{NK_B T}{K_B NT + qI_O R_s} \right]. \qquad (9)$$

In open circuit conditions, the potential difference between the edges of the photovoltaic cell is marked with the $V_{OC}$. Its analytical expression can be derived from that of the currents I, by equating it to zero and neglecting the resistance of the $R_s$ and $R_{sh}$. The Voltage Temperature Coefficient $V_T$, The photo current $I_T$ and $I_o$ Reverse saturation current.

$$V_T = K_B \cdot \frac{T}{q}, \qquad (10)$$

$$V_{oc} = N \cdot V_T \cdot \ln \left[ \frac{I_L + I_O}{I_O} \right]. \qquad (11)$$

The output power P is given by:

$$P = V \cdot I \qquad (12)$$

$$P = V \cdot \left[ I_{ph} - I_S \cdot \left( e^{\frac{q(V_{PV} + I \cdot R_S)}{NK_B T}} - 1 \right) - \frac{(V_{pv} + I \cdot R_s)}{R_{sh}} \right]. \qquad (13)$$

## Determination of Series and Shunt Resistances

As initial data for Model there were used experimental data from solar panel educational bench in school laboratory in National Airspace University «KhAI», the Department of space technology and alternative energy sources with Si PV cell manufactured by Siemens Corp. - Fig.4.

Common structure of PV Panel Simulation Model is represented at Fig. 5.

List of main model parameters is represented at Table 1.

Once the first three unknown parameters in (8) have been obtained according to the mentioned methodologies, the values of $R_s$ and $R_{sh}$ are required to complete the simulation model. Although in general, it is assumed that $R_{sh} >> R_s$, it should be considered that the simulation model should reflect the real characteristics of a solar module/array. Thus an accurate determination of the $R_s$ and $R_{sh}$ is required. Various methods have been proposed by many researchers to obtain the values of these unknown parameters. Extracting the values from I-V characteristics provided in the manufacturer`s data sheet or analytical and iteration based calculation methods have been subjects of many researches in the literature [7]. However the unavailability of information on I-V characteristics, computational complexity, uncertainties, etc. are the main issues by utilizing these methods.

Fig. 4. Solar panel educational bench

The values of $R_s$ and $R_{sh}$ are obtained using the methods introduced by [8] due to their simplicity and reliable results. According to $R_{sh}$ and $R_s$ can be obtained using (14) and (15) and [6] defines (16) to take. The effects of irradiance variations on the value of $R_{sh}$ into account.

$$R_{sh} > 10\frac{V_{oc}}{I_{sc}}, \qquad (14)$$

where $V_{OC}$ – Nominal Open Circuit Voltage.

$$R_s < 0.1\frac{V_{oc}}{I_{sc}}; \qquad (15)$$

Table 1

Photovoltaic Model Parameters

| Para-metr | Description |
|---|---|
| $I_{ph}$ | Photovoltaic Current (A) |
| $I_S$ | Diode Saturation Current (A) |
| q | Electron Charge(1.60217646 × $10^{-19}$C) |
| N | Coefficient without dimension, |
| $k_B$ | Boltzmann Constant (1.3806503 × $10^{-23}$ J/K) |
| T | PV cell temperature (K) |
| $R_s$ | Series Resistance (Ω) |
| $R_{sh}$ | Shunt Resistance (Ω) |
| $I_d$ | Diode current |
| $1_o$ | Reverse saturation current |
| $V_{pv}$ | Panel voltage |
| $V_T$ | Voltage Temperature Coefficient |
| G | Irradiance on the Surface of the Cell (Wh/m$^2$) |
| I | PV Battery Output Current (A) |
| $V_D$ | Diode Voltage |
| b | Constant Semiconductor |
| $I_e$ | Electron Current |
| $I_{eo}$ | Electrons current in the conduction band |
| $I_h$ | Hole Current |
| $I_{ho}$ | Space current in the valence band |
| $I_L$ | The photo current (A) |
| P | The output power |
| $I_{SC}$ | Nominal Short-Circuit Current |
| $V_{oc}$ | Nominal Open Circuit Voltage |
| PV | Photovoltaic |
| $I_{sh}$ | Short circuit current |
| $I_{pv}$ | Current through Panel |
| $G_{STC}$ | Standard Irradiance 1000 Wh/m$^2$ |
| $R_{sh,\ STC}$ | Shunt Standard Resistance |
| $I_{mp}$ | Current at the maximum power point |

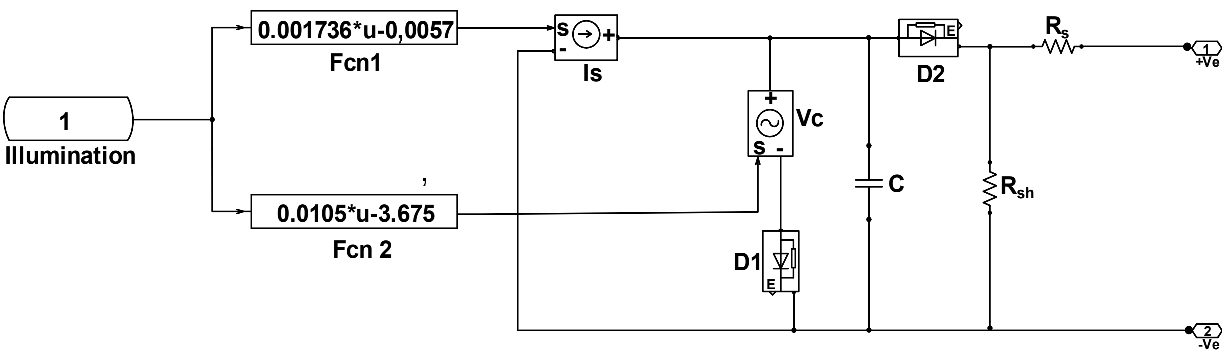


Fig. 5. PV Panel Simulation Model

$$\frac{R_{sh}}{R_{sh,\,STC}} = \frac{G}{G_{STC}}, \quad (16)$$

where G – Irradiance on the Surface of the Cell;

$G_{STC}$ – Standard Irradiance;

$R_{sh,\,STC}$ – Standard Shunt Resistance.

## Irradiance Effects

The effects of variations of the amounts of the received solar irradiance on module`s I-V and P-V characteristics are shown in Fig. 6. The module output characteristics are experimental and modeling under four different irradiance levels, namely being 460 Wh/m2, 360 Wh/m2, 260 Wh/m2 and 160 Wh/m2. It is obvious that reductions in the amount of solar insolation received by the module have direct effects on module short-circuit current value while Voc is also subjected to small reductions at the same time. It is observed that reductions in solar irradiance cause significant amounts of power loss by the module.

## Pulse Width Modulation

Pulse width modulation (PWM) is a powerful technique for controlling analog circuits with a processor's digital outputs. The applications of PWM are wide variety used like ranging from measurement and communications to power control and conversion. PWM provides a way to decrease the Total Harmonic Distortion (THD) of load current. The (THD) requirement can be met more easily when the output of PWM inverter is filtering [6]. The unfiltered PWM output will have a relatively high THD, but the harmonic will be at the much higher frequency than for a square wave, making filtering easily.

The Pulse Width Modulation technique is used to control the closing and opening switches. The switching scheme applied is unipolar. The PWM signal is used to control ON/OFF switching state of the IGBTs will function in driver model that created to control the switching scheme. The duty cycle of a square wave is modulated to encode a specific analog signal level by using a higher resolution counter. The PWM signal is still digital because, at any given instant of time, the full DC supply is either fully on or fully off. The voltage or current source is supplied to the analog load by means of a repeating series of on and off pulses. The on time is the time during which the DC supply is applied to the load, and the off-time is the period during the supply is switched off. Given a sufficient bandwidth, any analog value can be encoded with PWM [7].

The benefit of choosing the PWM over analog control increases noise immunity, which the PWM is sometimes used for communication. Switching from an analog signal to PWM can increase the length of a communications channel dramatically.

At the receiving end, a suitable $R_C$ (resistor-capacitor) or $L_C$ (inductor capacitor) network can remove the modulating high frequency square wave and return.

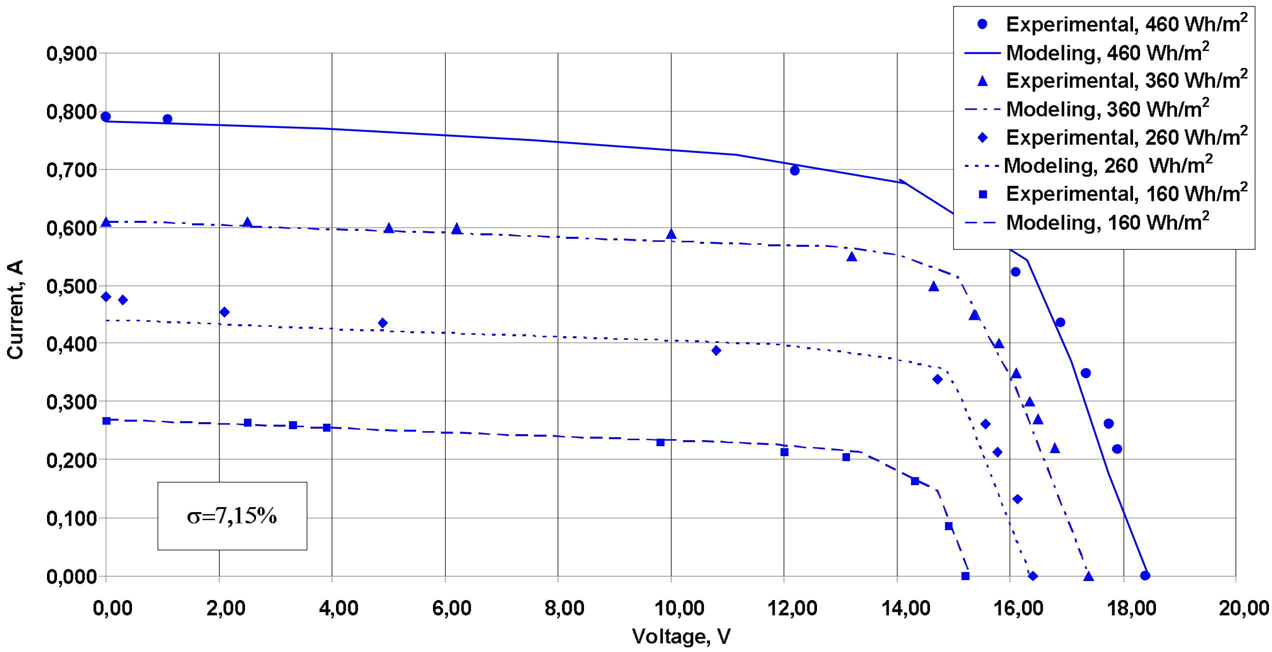


Fig. 6. Effects of Irradiance Variations on Experimental Short Circuit Current &Open Voltage Circuit $I_{sc}$-$V_{oc}$ Characteristics

The switching element, (T), is operated by a modulator stage which consists mainly of a triangle wave generator that generates the switching frequency of interest and a comparator that compares the high frequency carrier wave with a certain reference voltage to produce the desired Pulse Width Modulated (PWM) signal which will then be sent to the switching gate.

The input voltage is chopped by the switching (T). When T is on, Since the input voltage is greater than the load voltage, energy is transferred from the dc voltage supply input voltage to L, C, and the load R. When T is turned off, stored energy in L is transferred via the diode to C and the load R. Consider inductor operation is considered to be continuous in our analysis, i.e. T turns on before the current in L reaches zero, so a continuous current flows through L [5, 8]. Pulse generator is showed at Fig. 7.

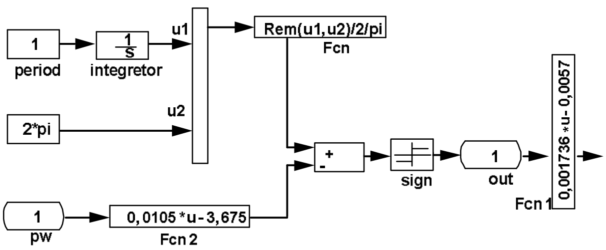


Fig. 7. Pulse generator

Power regulator is showed at Fig. 8.

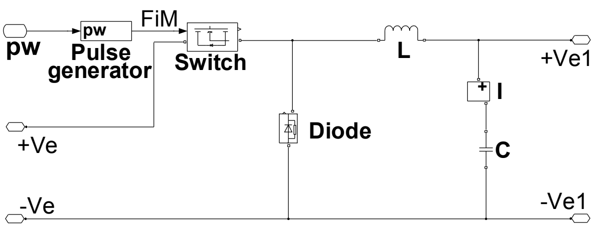


Fig. 8. Power Regulator

## Controller

The algorithm used is showed at Fig. 9.

Periodically, each 0.5...1 minute there are scanning of $k_w$ values in the range of 0...0.5 with step 0.01 for 0.5 seconds. Value that provides the maximum output power of the module is kept in memory and retained till the next scanning.

## Conclusions

Using Pulse Width Modulation Control produced a decent efficiency of the system. System response, hence the tracking ability is fast and dynamic. The output power has a high ripple content. The proposed PV system is simple, robust and prolific. Employing a synchronous Pulse Width Modulation would lead to a better efficiency. The pragmatism of this simulation largely depends on the value of $k_w$ where $k_w$ is a constant in range 0,0...0,4.

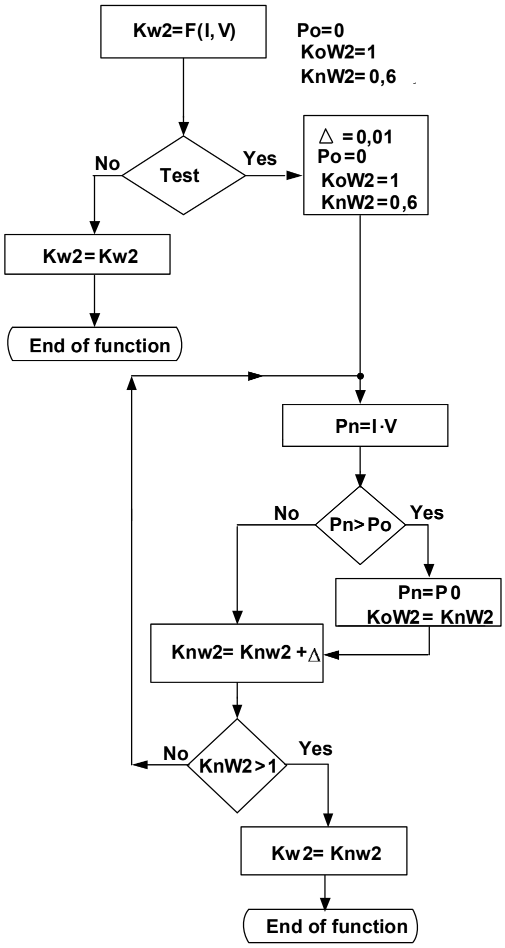


Fig. 9. Flow chart of the Controller algorithm

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## МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ФОТОЭЛЕКТРИЧЕСКОГО МОДУЛЯ С ШИМ-УПРАВЛЕНИЕМ

***Али. М. Джасим, Ю. А. Шепетов***

В статье представлено описание математической модели солнечной фотоэлектрической энергоустановки с ШИМ-регулятором согласования генератора и нагрузки. Для управления регулятором, который осуществляет удержание рабочих характеристик энергоустановки в точке максимальной выходной мощности, используется адаптивный алгоритм на базе комбинации методов прямого измерения и постоянного коэффициента по току короткого замыкания. Достоинствами данного алгоритма управления являются сравнительная простота и хорошая эффективность. Математическая модель фотоэнергетических характеристик энергоустановки выполнена на базе учебной солнечной энергоустановки кафедры космической техники и нетрадиционных источников энергии ХАИ.

**Ключевые слова:** фотоэлектрический модуль, ШИМ-управление, Simulink-модель.

## МАТЕМАТИЧНА МОДЕЛЬ ФОТОЕЛЕКТРИЧНОГО МОДУЛЯ З ШІМ-УПРАВЛІННЯМ

***Алі. М. Джасім, Ю. О. Шепетов***

У статті представлено опис математичної моделі сонячної фотоелектричної енергоустановки з ШІМ-регулятором погодження генератора і навантаження. Для управління регулятором, який здійснює утримання робочих характеристик енергоустановки в точці максимальної вихідної потужності, використовується адаптивний алгоритм на базі комбінації методів прямого виміру і постійного коефіцієнта по струму короткого замикання. Перевагами даного алгоритму управління є порівняльна простота і гарна ефективність. Математичну модель фотоенергетичних характеристик енергоустановки виконано на базі навчальної сонячної енергоустановки кафедри космічної техніки та нетрадиційних джерел енергії ХАІ.

**Ключові слова:** фотоелектричний модуль, ШІМ-управління, Simulink-модель.

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