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CRACKED BLADE DYNAMIC RESPONSE MODEL

Simulation of the cracked blade dynamic response on periodically varying load is presented. Crack induced nonlinearity is taken into account by contact simulation between crack sides. Such approach allows simulating crack breathing process in accurate way. The problem of nonlinearity is solved by harmonic balance method application. As nonlinear degrees of freedoms the relative displacements between crack sides are accepted that sufficiently decrease time expenses during nonlinear solution process.

cracked blade, eigenfrequency, contact, harmonic balance method, frequency response function

Introduction

The aim of the presented study is to offer the model of the cracked blade behaviour which contains as much as possible information to describe realistically its identification process.

Cracked blade identification is very important problem and can be divided in the following tasks: (I) understanding of the crack nature appearance and its subsequent propagation; (II) precision of the crack most probable location depending on particular external conditions; (III) defining crack probable critical size that allows or forbids consequent operation of the damaged assembly; (IV) development of the appropriate cracked blade model; (V) application of the blade amplitude measurement methods with goal of cracked blade identification [3].

The cracked blade model was created and involved introducing of the relative DOF between coinciding nodes of the crack location region. It was necessary to implement a reduction procedure to the cracked blade model for computational expenses decreasing, which are high at full solution, moreover, in the case of nonlinear solution methods requiring iteration procedures. The reduction approach was applied using crack location as interface between two blade model parts. They were considered as sub-structures for subsequent fixed-interface method application [10].

The questions of crack nonlinear behaviour under periodically varying load were considered and the solution on the base of harmonic balance method application was proposed. Application of the nonlinear solution procedure depends on centrifugal forces forming initial gap. Such gap can result in always open crack case and thus in useless of crack nonlinearity simulation.

1. Linear crack case

The linear presentation of the cracked blade consists in simulation of the crack presence supposing the crack to be always open. Generally, crack influence on dynamic response is simulated by stiffness reduction when solving eigenvalues problem. The crack models used in these analyses are divided into two categories:

– open crack models – linear statement;

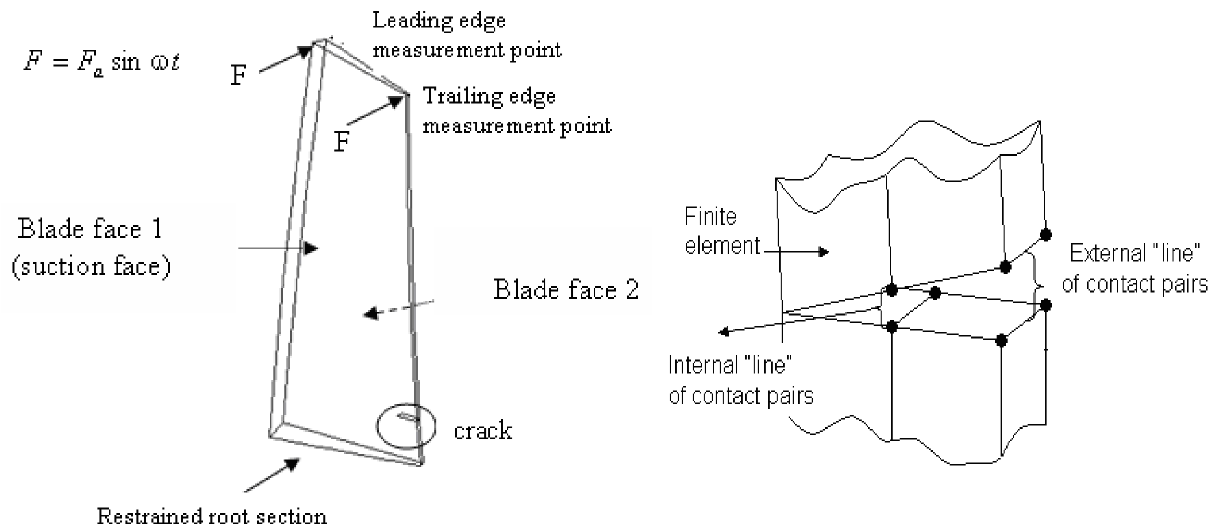
– opening and closing or breathing crack models – nonlinear statement [1, 2, 11].

Most researchers use always open crack models in their studies and have claimed that the change in natural frequencies might be a parameter for crack presence detection. We see that eigenfrequencies magnitudes for all modes become smaller in comparison to uncracked blade case (Table 1).

Let the blade dynamic behaviour be described by the equations of motion

$$\mathbf{M}\ddot{\mathbf{u}} + \mathrm{C}_{\xi}\dot{\mathbf{u}} + \mathrm{K}\mathbf{u} = \mathbf{F}e^{i\omega t}, \qquad (1)$$

where **M**, $\mathbf{C}_\zeta$, and **K** are the symmetric mass, damping, and stiffness matrices of the disk model; **F** – amplitudes vector of external excitation force. Damping matrix was calculated on the base of structural damping ratio ξ and stiffness matrix $\mathbf{C}_\zeta = \xi \mathbf{K}$.



a b

Fig. 1: a – cracked blade model subjected to dynamic response analysis;
b – cracked zone finite-elements representation

Table 1

Change of cracked blade eigenfrequencies due to crack presence

Crack location **hc**, mm	Crack size **a**, mm	Deviation from uncracked blade model, %							
		Trailing edge				Leading edge			
		1st mode	2nd mode	3rd mode	4th mode	1st mode	2nd mode	3rd mode	4th mode
10	2	0,0035	0,0203	0,0297	0,3186	0,3664	0,2729	0,1949	0,8465
	4	0,0247	0,1503	0,1781	2,1491	2,3357	1,7400	1,1844	5,1722
	6	0,0776	0,4550	0,4649	5,4869	5,8197	4,4378	3,2373	12,8839

Then, in the linear case we assume that system response is steady-state and has the form $X(t) = xe^{i\omega t}$ that yields to the set of algebraic equations

$$\mathbf{H}x = \mathbf{F}, \qquad (2)$$

where $\mathbf{H} = \mathbf{K} - \omega^2\mathbf{M} + i\omega\mathbf{C}_\xi$ is the impedance matrix at the excitation frequency ω.

Application of external excitation forces is shown on fig. 2. Forces are applied in points of blade tip: at leading and trailing edges.

2. Nonlinear crack case

The assumption that crack is always open in vibration is not realistic because compressive loads may close the crack. The main results obtained through simulations or experimental studies were that the observed decrease in the natural frequencies is not sufficient to be described by a model of open crack [12]. So, the real changes in resonances can be calculated only on the base of nonlinear cracked blade dynamic model. Two approaches of crack breathing process simulation might be used:

– periodical varying stiffness introduction [11];

– contact simulation between crack sides in the moment of crack closing [2].

In most cases analytical solutions of such dynamical systems are practically impossible to obtain. Thus researchers and engineers turn to numerical techniques. Firstly, systems are discretized as a set of nonlinear ordinary differential equations with high dimension.

Then traditional direct time integration solution techniques are then applied. However, this process is extremely time-consuming. Therefore, it seems to be necessary to examine more efficient techniques to reduce the computational costs. One such technique is harmonic balance (HB) method [5 – 9]. Earlier a simple mathematical model able to simulate such nonlinearity was created [2]. In such way it was possible to prove correctness of the method formulation by comparing its results with direct integration. Now this approach can be projected on more complex 3 dimensional cracked blade model.

In nonlinear case system motion equation (1) is expressed by:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathrm{C}_{\xi}\dot{\mathbf{u}} + \mathrm{K}\mathbf{u} + \mathbf{F}_{nl}(u) = \mathbf{F}(t), \qquad (3)$$

where $\mathbf{F}_{nl}$ – nonlinear force vector.

Then we are searching for the $u(t)$ in the form of the truncates trigonometric series of $k = 1, \ldots, N$ harmonics:

$$u(t) = a_0 + \sum_{k=1}^{N} a_k \cos k\omega t + \sum_{k=1}^{N} b_k \sin k\omega t, \qquad (4)$$

where a_0, a_k, b_k – Fourier series coefficients, ω – excitation frequency.

If we put equation (4) to (3) the last would be changed to:

$$\mathbf{A}\tilde{\mathbf{u}} + \mathbf{b}(\tilde{\mathbf{u}}) = \mathbf{C}, \qquad (5)$$

where **A** is diagonally symmetric in block matrix:

$$\mathbf{A} = \begin{bmatrix} \mathbf{L}_0 & 0 & \ldots & 0 \\ 0 & \mathbf{L}_1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mathbf{L}_N \end{bmatrix}_{N_{dof}(2k+1) \times N_{dof}(2k+1)};$$

$$\mathbf{L}_0 = \mathbf{K};\ \mathbf{L}_k = \begin{bmatrix} \mathbf{K} - (k\omega)^2 \mathbf{M} & k\omega \mathbf{C}_{\xi} \\ -k\omega \mathbf{C}_{\xi} & \mathbf{K} - (k\omega)^2 \mathbf{M} \end{bmatrix}, \qquad (6)$$

where **b** – nonlinear member; **C** – external excitation force vector and $\tilde{\mathbf{u}}$ – vector of Fourier series coefficients.

Nonlinear solution requires taking into account system nonlinearity, in our case – nonlinear contact force between two nodes. The governing equation (5) of HB method represents by itself the system of nonlinear equations to that some linear transformation could be applied for a Newton-type iterative solver of nonlinear algebraic equations system implementation [2].

Nonlinear forces are calculated only for degrees of freedom (DOF) accepted as nonlinear. In our case they are relative vertical displacements between contact nodes. For the contact force determination the Lagrange multipliers or penalty methods could be utilized [1]. The easiest way is to use the penalty method to approximate these forces by the following expression:

$$F_{nl} = k_{nl} \cdot \left(\frac{u_{nl} + |u_{nl}|}{2} \right), \qquad (7)$$

where k_{nl} – penalty stiffness and u_{nl} – nonlinear DOF displacement. Penalty stiffness value should be chosen to provide minimum penetration in the contact zone.

It should be mentioned the disadvantage of such nonlinear force approximation when it crosses the zero. So then $u_{nl} = 0$, $\partial F_{nl} / \partial u_{nl} \to \infty$. In order to pass up such problem the smoothing function should be applied. In the work [4] tangent function was used for smoothing. We applied it with some modification and have gotten the next expression:

$$F_{nl} = \frac{1}{\pi} k_{nl} \left(\arctan(s u_{nl}) - \frac{\pi}{2} \right) u_{nl}, \qquad (8)$$

where s – coefficient, the sufficiently high level of that is required to accurately represent force-displacement relationship smoothing.

3. Results of harmonic balance method application to cracked blade model

The presented above nonlinear model formulation of the cracked blade describes accurately enough the contact interaction between crack sides. For the last one it is very important to have precise dynamic behavior model due to very small difference of the cracked blade response with the relation to the unckacked one, even when passing through the resonance. Therefore, the frequency domain analysis should be accomplished for the diapason that covers minimum three or four first eigenfrequenciesy of vibration, which are often induced in gas-turbine engines.

Due to model size and its higher level of flexibility in comparison with 2d model the penalty contact stiffness value was accepted to be the same for all contact pairs and equal to $k_{nl} = 10^9 N/m$. For 3d model the number of contact pairs depends on crack length. With the purpose to be able to fill a difference of the eigenfrequencies changes with crack length and at the same time to ensure the number of finite elements (DOF) to be processed we accepted level of meshing as 1 line of contact pairs on each 2mm of crack length. And each line of contact pairs consisted of 2 contact pairs: blade face 1 contact pair and blade face 2 contact pair (fig. 1 b). Consequently, for 2mm crack we had one contact pair and for 4mm crack – 2 contact pairs correspondingly.

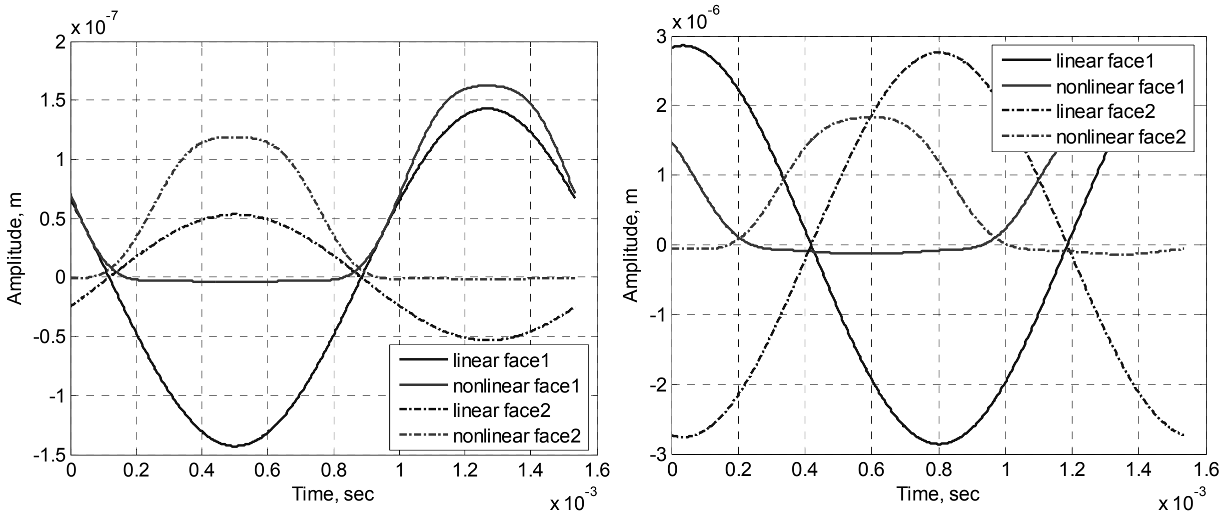


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Fig. 2. System response in time domain at "crack" point (a = 2 mm, ω = 3500 rad/sec):
a – trailing edge crack; b – leading edge crack

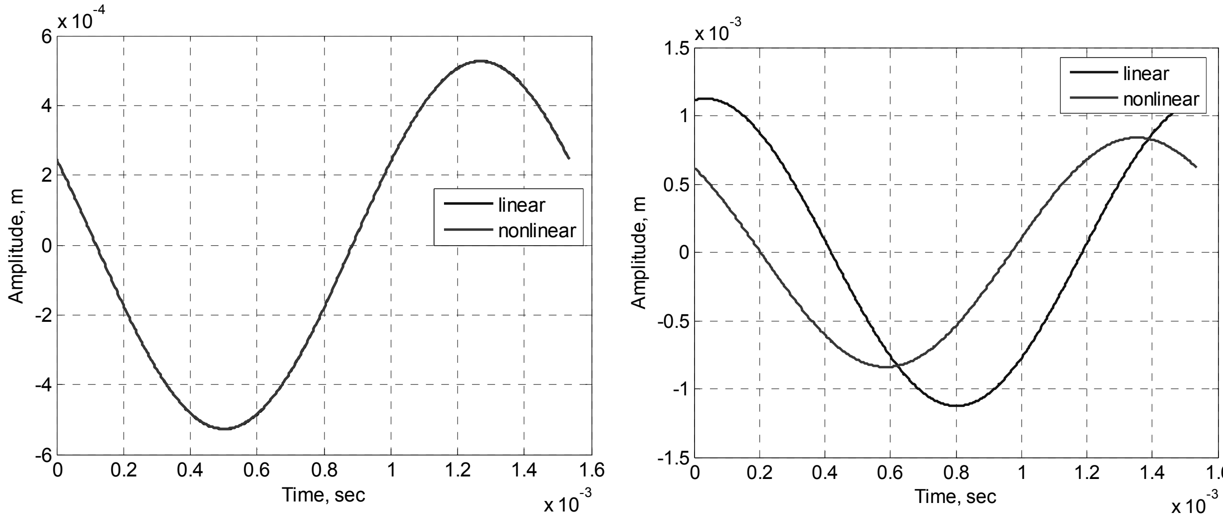

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Fig. 3. System response in time domain at "force" point (a = 2 mm, ω = 3550 rad/sec):
a – trailing edge crack; b – leading edge crack

The scheme of blade model loading is presented on Fig. 1, a. Two point forces with amplitude 0,2N are applied at two tip nodes normally to blade tip section. One force is applied at leading edge tip node and another at trailing edge tip node. Next these two nodes are accepted as measurement points of blade tip deflection and their data will be used for frequency response function construction. The solutions of cracked blade linear and nonlinear models formulations in the time domain are reconstructed by inverse Fourier transformation on the base of equation (5) solution. Then they are shown for: relative vertical displacement between two coinciding contact nodes – "crack point" (fig. 2 and 4) and excitation force application nodes horizontal displacement – "tip point" (fig. 3 and 5).

From the results of cracked blade model solution in time domain with introduced crack of 2 mm length on trailing edge it is seen that the crack presence influence on blade tip response is almost invisible. It is also grounded by modal analysis results where such case has the minimum shift of the first eigenmode frequency (Table 1). For the case when crack is located on leading edge due to higher level of stiffness reduction we have visually detectable difference between both linear-nonlinear solutions.

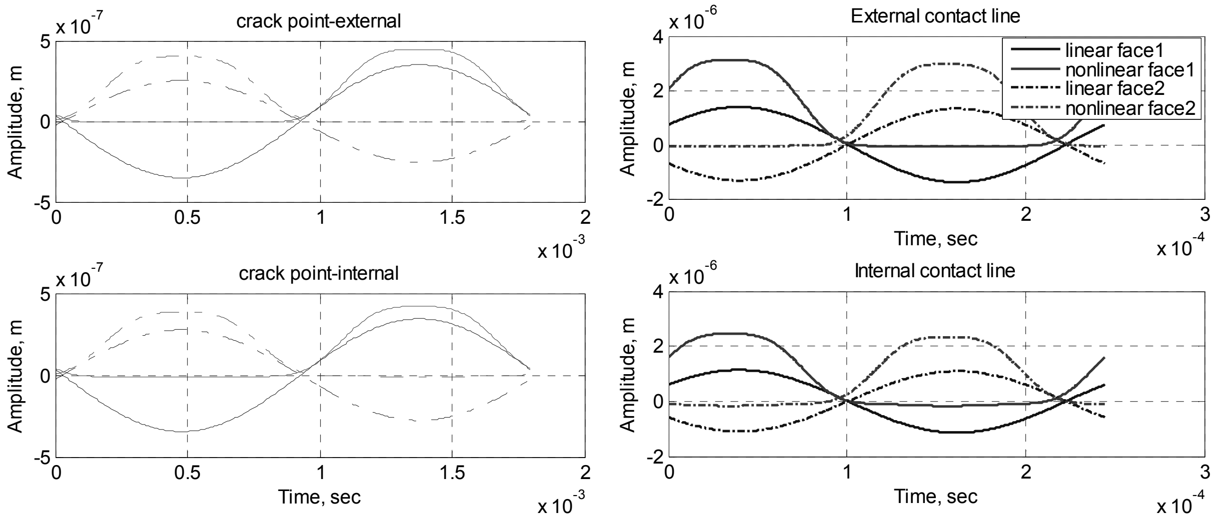

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Fig. 4. System response in time domain at "crack" point (*a* = 4 mm, ω = 3500 rad/sec):
a – trailing edge crack; b – leading edge crack

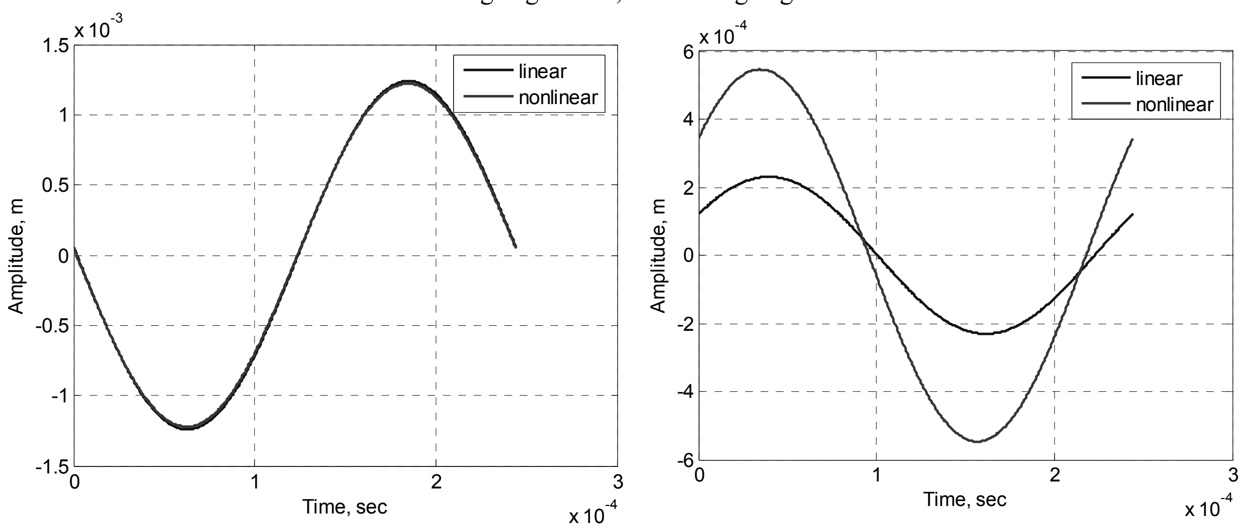

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Fig. 5. System response in time domain at "force" point (*a* = 4 mm, ω = 3550 rad/sec):
a – trailing edge crack; b – leading edge crack

Analyzing results of cracked blade model response in time domain with the cracks of 4 mm it is evident that the influence of crack is stronger if to compare with 2mm crack even for crack location at the trailing edge. The number of contact pairs through the crack thickness is 2 (external and internal). This allows more accurate describing of displacements between crack sides in dynamics. For 4mm crack relative vertical displacements between contact nodes of external and internal pairs are nearly equal for trailing edge crack option whereas for leading edge crack the internal displacements are approximately in 2 times less then external contact pair displacements (fig. 4).

Also it is necessary to emphasize considerable difference between nonlinear model solution and linear model solution for some crack location cases does not reflect real difference in amplitudes. Generally it is resulted by resonance shift because excitation frequency is not far from first eigenmode frequency of the linear model or nonlinear model. The contraposition in time-history of relative displacements between crack sides for two blade faces is caused by excited natural mode. The frequency point of 3550 rad/sec is located in the range of the first blade flexural mode inducing such contrast difference.

For all simulations in this chapter number of harmonics retained for nonlinear analysis was equated to 5. In order to accelerate the analysis at frequency response function construction 2 harmonics were used as the y shown almost the same results at both "crack" and "tip" points.

4. Frequency response function of the cracked blade model

The frequency response function construction of the cracked blade model was performed for the same cases as system response simulation in time domain. Comparison of the frequency response was fulfilled between linear cracked, nonlinear cracked and linear uncracked blade models. Frequency range covered first three eigenmodes. As we did not know exactly the eigenfrequencies of the cracked blade model taking into account crack induced nonlinearity, the frequency discretisation was done around eigenmodes frequencies of the linear cracked and uncracked models. Due to this sometimes the resonances picks of the nonlinear model seem to be not smooth and with amplitude a bit lower as it would be.

In the previous chapter we dealt with system solution in particular frequency point (in our case – 3550 rad/sec) having initialized the nonlinear procedure by linear approximation. Such approach is suitable in the frequency diapasons situated far from a resonance, where difference between linear and nonlinear solutions is almost barely visible. Whereas in the resonance area such initialization can lead to longer convergence process or even to its unconvergence. To tackle this problem during frequency response analysis as initialization for particular frequency point the nonlinear solution obtained at the previous point was used. Another measure for problems of solution unconvergence avoiding deals with frequency continuation approaches [9] where next frequency point is seeked by prediction on the base of a polynomial approximation.

Frequency response for the cracked blade with crack of 2mm reflects said in the previous chapter: such crack size can have visible influence only for leading crack case. When crack is located on trailing edge its effect became more or less observable for forth higher order eigenmodes (fig. 6). The results of such crack size can be nominated as the minimum crack size around which it will be almost impossible to identify crack presence in the blade. Such crack detectability could be possibly more affected when crack blade will be considered within frameworks of the bladed disk model.

In the 4mm crack case the crack influence is evident at all modes for both leading and trailing edge crack locations. Blade tip amplitude of cracked blade shows sufficiently high difference with uncracked one even at trailing edge crack case. When crack is located at leading edge the crack presence detectability grows due to resonance peaks shift. Such crack size as the most representative will be used in following studies in conjunction with tip-timing method simulation [3] and development of the bladed disk model containing cracked blade.

Conclusions

Frequency response function construction was the final stage of the cracked blade nonlinear dynamical model development. This model allows us cracked behavior describing at any loading amplitude and excitation frequency. We can more or less accurately see crack influence on blade model dynamic behavior and we can derive from the developed model some very important factors which can be used for crack presence detection comparing them with uncracked blade:

– eigenmode frequency reduction;
– increase of the tip response amplitude;

All these factors are very important and can be accepted as diagnostics signs and used in tip-timing method simulation. By this method it is possible to reconstruct approximately a blade frequency response passing through engine rotational frequency range. But in reality it will not be so easy task, because we will meet all this parameters random spread caused by inequalities of individual blades under investigation and at the same time spread of different external factors which imply on each blade dynamic behavior. The next stage of the study will deal with bladed disk dynamic model development containing one or some cracked blade, able to take into account mistuning effect, excitation force frequency lag and cracked blade dynamic localization.

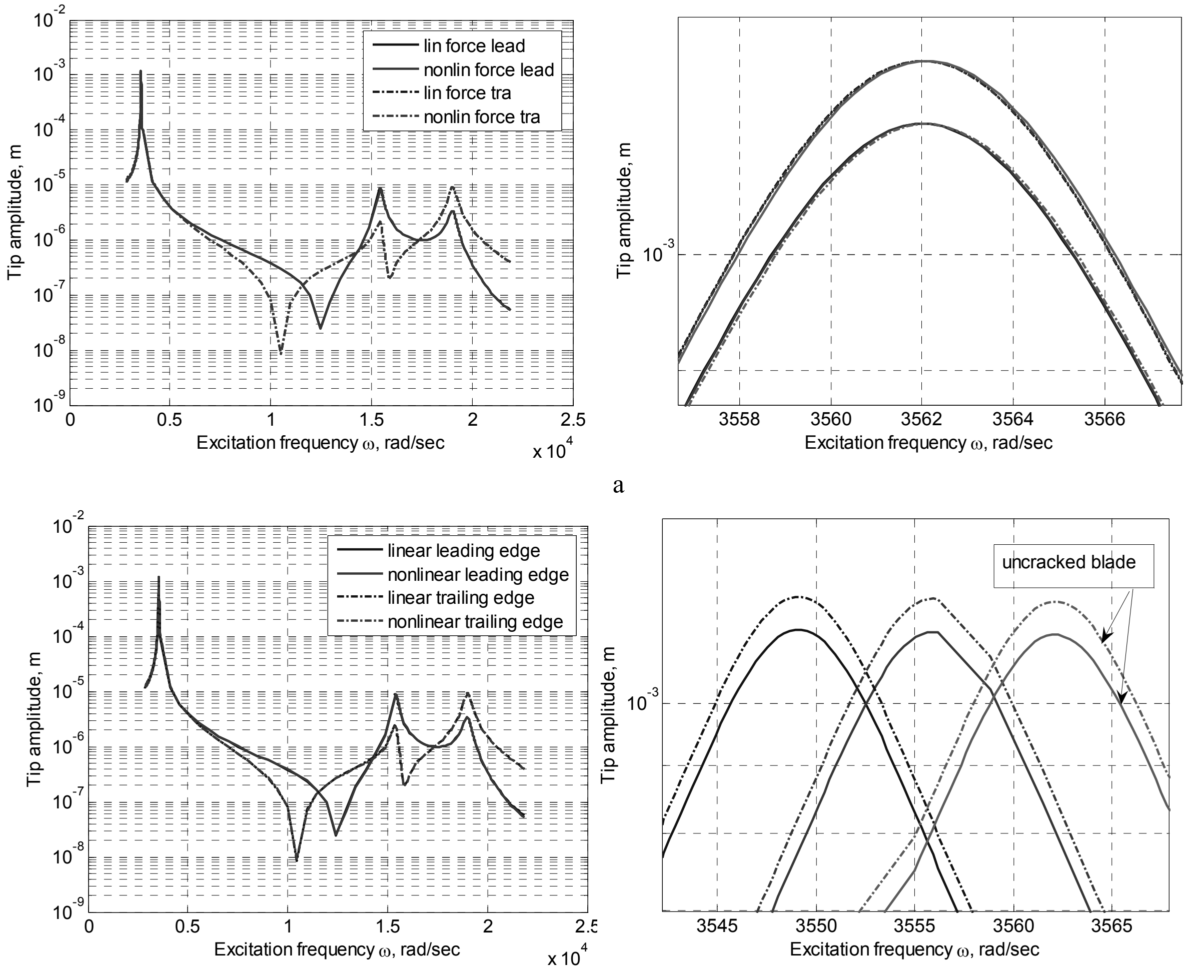

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Fig. 6. Blade frequency response function (*a* = 2 mm): a – trailing edge crack; b – leading edge crack

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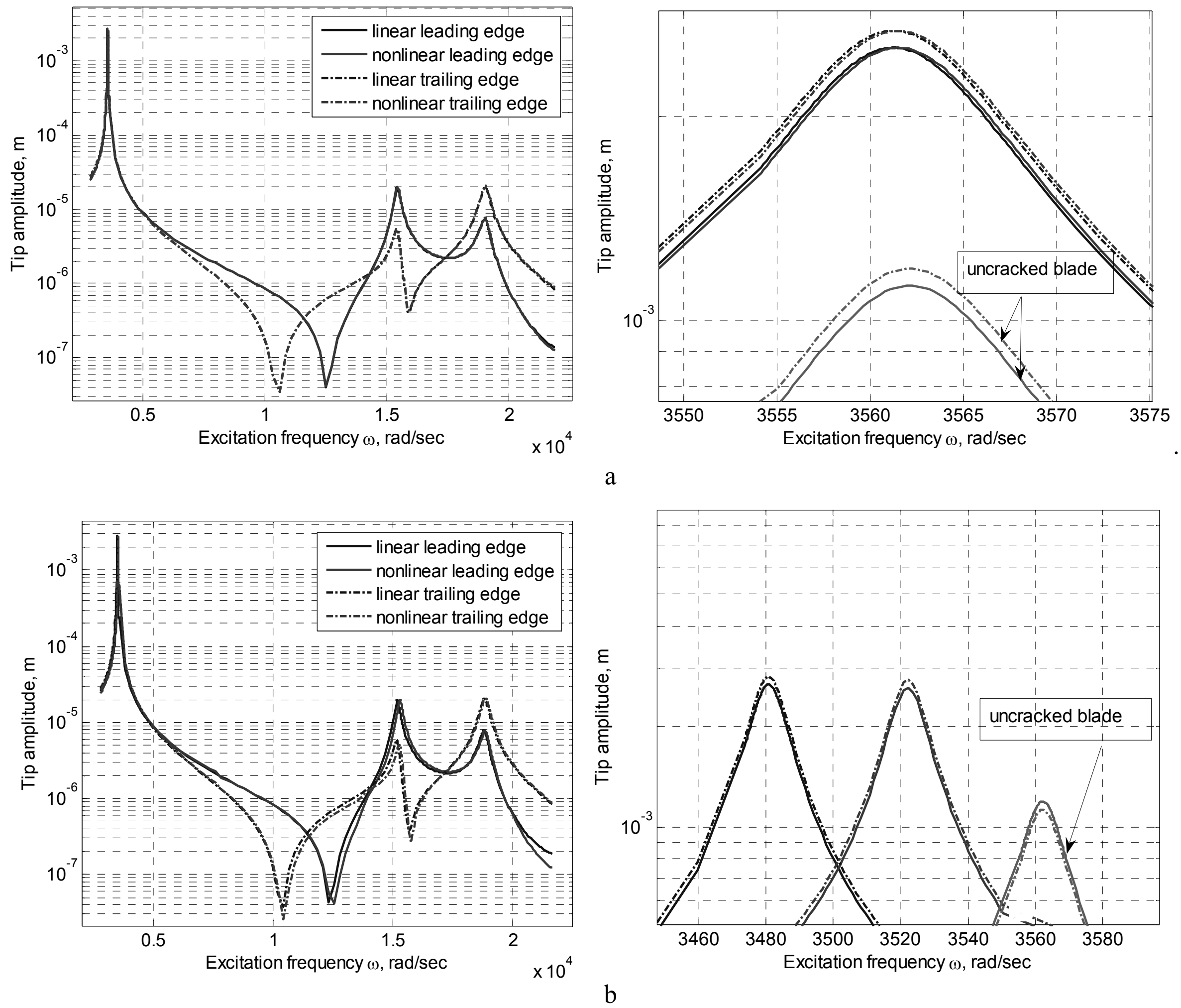

Fig. 7. Blade frequency response function (a = 4mm): a – trailing edge crack; b – leading edge crack

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