УДК 629.7.02 : 620.191.3.001.57

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HARMONIC BALANCE METHOD IMPLEMENTATION FOR CRACK BREATHING PROCESS SIMULATION

The harmonic balance method was applied to simulate nonlinear behaviour of the cracked structure subjected to the external periodic excitation. Such nonlinearity occurs then crack's sides come to the contact and it is known as crack breathing process. Contact interaction was simulated by penalty method and expression for contact force approximation was proposed.

contact, crack, crack breathing, harmonic balance, degree of freedom, eigenmode, penalty stiffness

Introduction

The motivation for this study comes from inside crack contact process inducing nonlinear behavior of the cracked rotating structures like turbine or compressor blades subjected to periodical external excitation. The main goal is to create simple mathematical model able to simulate such nonlinearity and the main ideas of that would be projected on more complex models for their subsequent utilization in methods for blade-in crack presence identification, like blade-tip timing method [8].

Generally analytical solutions of such dynamical systems are practically impossible to obtain and thus researchers and engineers turn to numerical techniques. The systems usually are first discretized as a set of non-linear ordinary differential equations with high dimension. Traditional direct time integration solution techniques are then applied. However, this process is extremely time-consuming. Therefore, it is a way to examine new and more efficient techniques to reduce the computational costs. One such technique is related with harmonic balance (HB) method.

There are several ways to implement the harmonic balance method, among which three approaches are particularly attractive: classical harmonic balance method, the high-dimensional harmonic balance method and incremental harmonic balance method. The HB method gives accurate enough results but it is difficult to implement it for high-dimensional systems or for systems with complex or non-smooth nonlinearities. On the other hand, the high-dimensional HB method [5] is easy to implement for high-dimensional systems regardless of the nonlinearities complexity, but it may produce spurious solutions in addition to the physically meaningful solutions. Also the incremental HB method is capable of dealing with strongly non-linear systems to any desired accuracy [3, 6].

As for our first attempt to implement such technique to crack breathing caused nonlinearities the classical HB method was decided to employ.

1. Harmonic balance method

Harmonic balance method is one of the most widely used methods for nonlinear dynamic analysis. The basic physics is to transform the problem under consideration into a set of nonlinear algebraic equations by truncated Fourier series. Next, the equations are expanded, and the terms associated with each harmonic are then balanced. This method gives us a possibility to obtain periodical solution (response) of the nonlinear system on an external periodical excitation. The response is supposed to be of the same period with excitation frequency.

In our case cracked structure is presented by flat plate with dimensions 0,1*0,1 m and restrained at bottom line (fig. 1). Then it was meshed to create finite-elements model. Crack presence in the structure was simulated by introducing additional node creating the contact pair. The external load was applied to the top right corner of the plate as the point force with amplitude 100N and excitation frequency 2000 rad/sec that is close to the first eigenfrequency of the cracked plate model (2300 rad/sec). Such loading mode was chosen to be a bit close to the simulation of the gas-turbine engine turbine or compressor blade excited oscillation. As material properties the next data were accepted: Young's modulus $2\cdot10^9$N, Poisson's ratio – 0,3 and density –7800 kg/m^3.

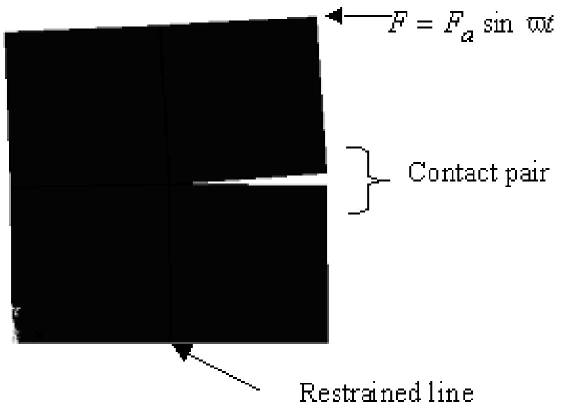

Fig. 1. Simplified two-dimensional model of the cracked structure

Firstly, all loading and contact interaction processes simulations were fulfilled by finite-elements method in ANSYS finite-elements software. Plate was discretized by plane elements plane42 and the contact one – contac12. Cracked interface interaction was modelled by node-to-node contact element with the using of penalty method. Normal penalty level was accepted as 10^{11}N/m. The level of the modal damping ratio – ξ=0.001. On the fig. 2 the time-history of the finite-elements simulation results is shown.

Then both mass and stiffness matrices of the finite-elements model were transferred to MATLAB engineering software for subsequent processing. Then the system matrices were transformed to rearrange absolute displacement between contact nodes to the relative ones [1].

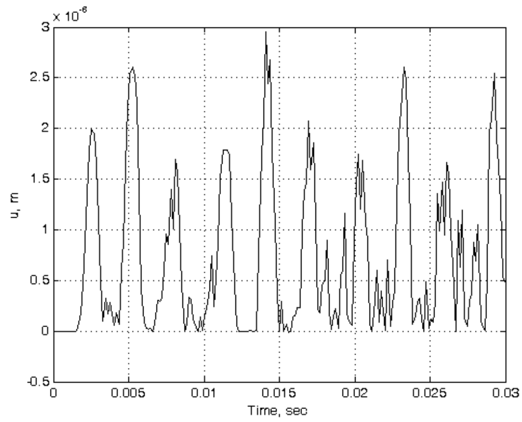

Fig. 2. Time-history of the relative vertical displacement between crack tips in ANSYS

As it was mentioned before the HB method gives us the possibility to obtain periodical solution passing to the frequency domain. So the solution is represented in the following form:

$$u(t) = a_0 + \sum_{k=0}^{\infty} a_k \cos k\omega t + \sum_{k=0}^{\infty} b_k \sin k\omega t, \quad (1)$$

where a_0, a_k, b_k – Fourier series coefficients, ω – excitation frequency.

In general case system motion equation is expressed by:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_\xi \dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{F}_{nl}(u) = \mathbf{F}(t), \quad (2)$$

where **M** – mass matrix; **K** – stiffness matrix; $\mathbf{C}_\zeta$ – damping matrix; u – displacements vector; $\mathbf{F}_{nl}$ – nonlinear force vector; **F** – external excitation time-varying periodic force vector.

Damping matrix was calculated on the base of modal damping ratio ξ from the next relation:

$$\phi^T \mathbf{C}_\zeta \phi = 2\zeta\omega_c; \;\; \mathbf{C}_\zeta = (\phi^T)^{-1} 2\zeta\omega_c (\phi)^{-1},$$

where ϕ – eigenmodes vector; ω_c – diagonal matrix of eigenfrequencies.

Then we are searching for the $u(t)$ in the form of the truncates trigonometric series of $k = 1, \ldots, N$ harmonics:

$$u(t) = a_0 + \sum_{k=0}^{N} a_k \cos k\omega t + \sum_{k=0}^{N} b_k \sin k\omega t. \quad (3)$$

The j-th nonlinear degree of freedom could be expressed in the same way as (1)

$$u_{nl}^{j}=a_{nl0}+\sum_{k=0}^{N}a_{nlk}\cos k\omega t+\sum_{k=0}^{N}b_{nlk}\sin k\omega t. \quad (4)$$

If we put equation (1), (4) to (2) the last would be changed to:

$$\mathbf{A}\tilde{\mathbf{u}}+\mathbf{b}(\tilde{\mathbf{u}})=\mathbf{C}, \quad (5)$$

where **A** is diagonally symmetric in block matrix:

$$\mathbf{A}=\begin{bmatrix}\mathbf{L}_0 & 0 & \dots & 0\\ 0 & \mathbf{L}_1 & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & \mathbf{L}_N\end{bmatrix}_{N_{dof}(2k+1)\times N_{dof}(2k+1)}; \quad (6)$$

$$\mathbf{L}_0=\mathbf{K};\ \mathbf{L}_k=\begin{bmatrix}\mathbf{K}-(k\omega)^2\mathbf{M} & k\omega\mathbf{C}\\ -k\omega\mathbf{C} & \mathbf{K}-(k\omega)^2\mathbf{M}\end{bmatrix};$$

b – nonlinear member:

$$\mathbf{b}(\tilde{\mathbf{u}})=\begin{bmatrix}\frac{1}{T_s}\int_0^{T_s}F_{nl}dt\\ \begin{Bmatrix}\frac{2}{T_s}\int_0^{T_s}F_{nl}\cos(k\omega t)dt\\ \frac{2}{T_s}\int_0^{T_s}F_{nl}\sin(k\omega t)dt\end{Bmatrix}\\ \dots\end{bmatrix}_{N_{dof}(2k+1)\times 1}, \quad (7)$$

where $T_s=2\pi/\omega$ – excitation period;

C – external excitation force member:

$$\mathbf{C}=\begin{bmatrix}\frac{1}{T_s}\int_0^{T_s}Fdt\\ \begin{Bmatrix}\frac{2}{T_s}\int_0^{T_s}F\cos(k\omega t)dt\\ \frac{2}{T_s}\int_0^{T_s}F\sin(k\omega t)dt\end{Bmatrix}\\ \dots\end{bmatrix}_{N_{dof}(2k+1)\times 1}; \quad (8)$$

$\tilde{\mathrm{u}}$ – vector of Fourier series coefficients:

$$\tilde{\mathbf{u}}=\begin{bmatrix}a_0 & a_1 & b_1 & \dots & a_N & b_N\end{bmatrix}^T. \quad (9)$$

Any conventional numerical integration techniques could be applied for integral calculation in (7) and (8), in our case the trapezoidal method was used.

1.2. Linear solution. The linear problem was solved without taking into account nonlinear member (7) in the equation (5). $k = 5$ harmonics were used for solution:

$$\tilde{\mathrm{u}}=\mathrm{A}^{-1}\mathrm{C}. \quad (10)$$

Then using (1) the coefficients obtained by (10) are collected in the vector form (9) and used to calculate nodal displacements (inverse Fourier transformation). The solution is shown on fig. 3. There are two curves. One is subjected to the relative vertical displacement between contact nodes "contact point" and another one – to the horizontal displacements at the tip point (node of excitation force application) "force point".

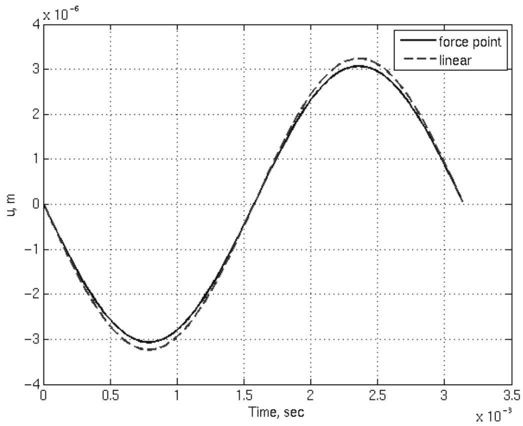

Fig. 3. Linear solution by HB method

It is seen also from fig. 3 that there are very small difference between the solutions at two different points due to the absence of nonlinearity and supposition that crack is always opened.

1.3. Nonlinear force representation. The nonlinear force representation is one of the most important tasks in any nonlinear analysis, as well as for the harmonic balance method. In our case we have only one nonlinear degree of freedom (relative vertical displacement between contact nodes).

For the contact force determination the Lagrange multipliers or penalty methods could be utilized. The easiest way is to use the penalty method to approximate this force by the following expression:

$$F_{nl}=k_{nl}\cdot\left(\frac{u_{nl}+|u_{nl}|}{2}\right), \quad (11)$$

where k_{nl} – penalty stiffness and u_{nl} – nonlinear dof displacement. Penalty stiffness value should be chosen to provide minimum penetration in the contact zone.

It should be mentioned the disadvantage of such nonlinear force approximation when it cross the zero. So then $u_{nl} = 0$, $\partial F_{nl} / \partial u_{nl} \to \infty$.

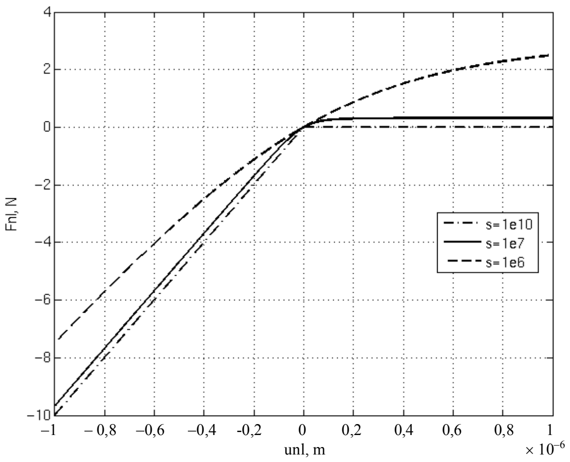

Fig. 4. Nonlinear force representation

Thus the calculation of the Jacobean will be theoretically unattainable. In order to pass up such problem the smoothing function should be applied. In the work [2] tangent function was used for smoothing.

We applied with some modification and have gotten the next expression:

$$F_{nl} = \frac{1}{\pi} k_{nl} \left(\arctan(s u_{nl}) - \frac{\pi}{2} \right) u_{nl}, \qquad (12)$$

where s – coefficient, the sufficiently high level of that is required to accurately represent force-displacement relationship smoothing (fig. 4).

1.4. Nonlinear solution. Nonlinear solution requires taking into account system nonlinearity, in our case – nonlinear contact force between two nodes (12).

The governing equation (5) of HB method represents by itself the system of nonlinear equations to that some linear transformation could be applied in the following way for a Newton-type iterative solver of nonlinear algebraic equations system implementation:

$$\tilde{\mathbf{u}}_{n+1} = \tilde{\mathbf{u}}_n + \Delta\tilde{\mathbf{u}};$$

$$\mathbf{A}(\tilde{\mathbf{u}}_n + \Delta\tilde{\mathbf{u}}) + \mathbf{b}(\tilde{\mathbf{u}}_n + \Delta\tilde{\mathbf{u}}) = \mathbf{C};$$

$$\mathbf{A}\tilde{\mathbf{u}}_n + \mathbf{A}\Delta\tilde{\mathbf{u}} + \mathbf{b}(\tilde{\mathbf{u}}_n) + J\mathbf{b}(\tilde{\mathbf{u}}_n) \cdot \Delta\tilde{\mathbf{u}} - \mathbf{C} = 0, \qquad (13)$$

where n – iteration number.

(13) is local linear transformation of (5) and $J\mathbf{b}(\tilde{\mathbf{u}}_n)$ is the Jacobean of nonlinear member :

$$J\mathbf{b}(\tilde{\mathbf{u}}) = \begin{bmatrix} \frac{1}{T_s}\int_0^{T_s} \frac{\partial F_{nl}}{\partial u}[\mathbf{I}_{N_{dof}}, \mathbf{I}_{N_{dof}}\cos(k\omega t), \mathbf{I}_{N_{dof}}\sin(k\omega t), ...]dt \\ \left\{ \begin{matrix} \frac{2}{T_s}\int_0^{T_s} \frac{\partial F_{nl}}{\partial u}[\mathbf{I}_{N_{dof}}, \mathbf{I}_{N_{dof}}\cos(k\omega t), \mathbf{I}_{N_{dof}}\sin(k\omega t), ...]\cos(k\omega t)dt \\ \frac{2}{T_s}\int_0^{T_s} \frac{\partial F_{nl}}{\partial u}[\mathbf{I}_{N_{dof}}, \mathbf{I}_{N_{dof}}\cos(k\omega t), \mathbf{I}_{N_{dof}}\sin(k\omega t), ...]\sin(k\omega t)dt \end{matrix} \right\} \\ ... \end{bmatrix}_{N_{dof}(2k+1) \times N_{dof}(2k+1)}, \qquad (14)$$

where $\mathbf{I}_{Ndof}$ – unity matrix of dimension $N_{dof} \times N_{dof}$., N_{dof} – number of degrees of freedom.

The solution vector update is calculated from the expression:

$$\Delta\tilde{\mathbf{u}}_n = \mathbf{J}^{-1} \cdot \mathbf{R}_n, \qquad (15)$$

where

$$\mathbf{J} = \mathbf{A} + J\mathbf{b}(\tilde{\mathbf{u}}_n), R_n = \mathbf{A}\tilde{\mathbf{u}}_n + \mathbf{b}(\tilde{\mathbf{u}}_n) - \mathbf{C}.$$

The nonlinear solution and as the linear one are shown for two degree of freedom: relative vertical displacement between two conceding contact nodes – “crack point” (fig. 5) and excitation force application node – “force point” (fig. 6).

From the results of contact interaction simulations we can see the accurate representation of contact force presence (fig. 5). The set value of penalty stiffness is enough to avoid penetration and nonlinear force approximation (12) allows us to precisely enough simulate system nonlinearity.

Taking into account contact has sufficient impact on plate tip node response (fig. 6). It becomes unsymmetrical and has lower maximum amplitude value in comparison with the linear solution. On the fig. 6 the solution of the uncracked plate model also is shown.

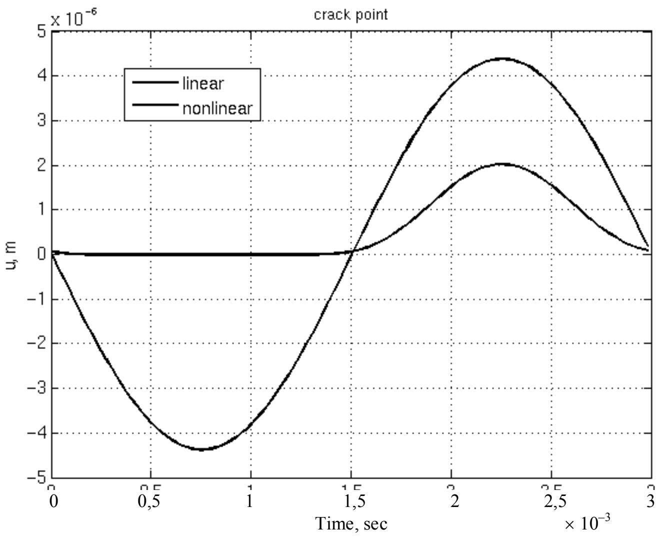

Fig. 5. Nonlinear solution by HB method at the “crack pint”

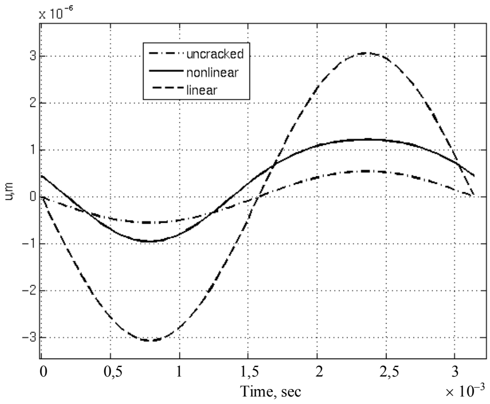

Fig. 6. Nonlinear solution by HB method at the “force pint”

2. Direct integration of the system motion equation

At the work [1] the solution of the same system by the direct integration method was submitted. It dealt with full and two reduced systems. For contact simulation two methods were applied: Lagrange multipliers method and penalty method. In the present work for comparison with HB method penalty method was used.

It is necessary to point out that for getting steady-state solution by direct integration of the system motion equation the time interval of the integration is sufficiently big. Also time step Δt should be small enough to obtain accurate solution results.

For central differences method the new iterative equation for displacements vector calculation is:

$$\mathbf{u}^{t+\Delta t} = (2\mathbf{M} + \Delta t\mathbf{C})^{-1}[2\Delta t^2 \mathbf{F}\sin\omega t - -2\Delta t^2 \mathbf{K}\mathbf{u}^t + 4\mathbf{M}\mathbf{u}^t - 2\mathbf{M}\mathbf{u}^{t-\Delta t} + \Delta t\mathbf{C}\mathbf{u}^{t-\Delta t}].$$

3. Comparison of solutions by harmonic balance method and direct integration

On fig. 7, 8 the system motion equation solutions by both HB and central difference direct integration methods are presented. They confirm declared above that HB method is more effective in the means of computational time expenses, because it is enough to have time interval equal to the excitation period then for a direct integration approach it is much more bigger.

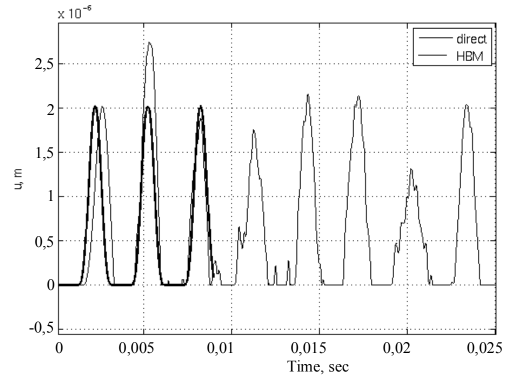

Fig. 7. Time-history of the relative vertical displacement between crack tips by direct integration and HB method

In reality the response period (in our case crack breathing period) of the nonlinear cracked structure isn’t coincided with the excitation force period and it’s slightly different that is seen also from the simulation comparison results.

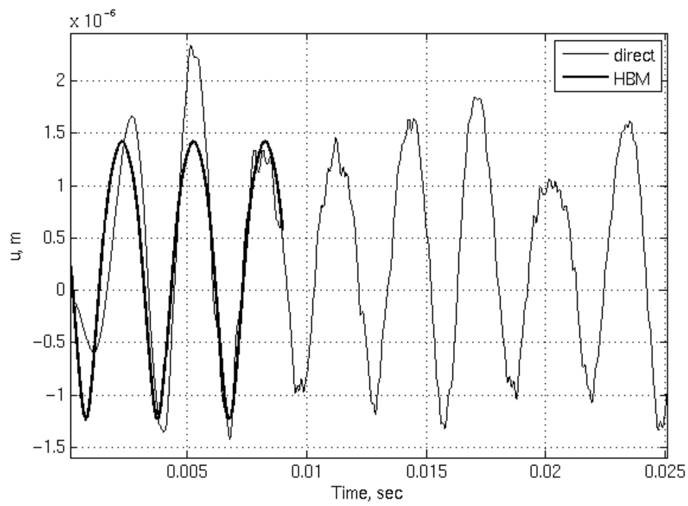

Fig. 8. Time-history of "force point" displacement by direct integration and HB method

Conclusions

The nonlinear model of the cracked structure dynamic response on external periodic excitation was developed and could be used for the following utilization in the simulation of the crack identification non-destructive methods, like blade tip-timing method. It describes accurately enough the contact interaction between crack sides. For the last one it is very important to have a precise blade dynamic behavior model due to very small difference of the cracked blade response with the relation to the unckacked one, even when passing through the resonance [8].

Therefore, the frequency domain analysis should be accomplished for the diapason that covers minimum three first eigenfrequenciesy of vibration, which are often induced in gas-turbine engines. Due to high nonlinearity some methods must be applied to accurately follow the resonance picks, like arc-length continuation technique [4, 7].

The one of the key questions for the future model improvement is subjected to the more accurate and correct nonlinear force approximation expression formulation.

References

1. Laine J.-P., Kharyton V. Cracked structure response on external harmonic excitation // Aerospace technics and technology. – 2006. – Vol. 61. – P. 1057-1074.

2. Chengwu Duan, Rajendra Singh. Dynamic analysis of preload nonlinearity in a mechanical oscillator // Journal of Sound and Vibration. – 2007. – Vol. 301. – P. 963-978.

3. Yongjun Shena, Shaopu Yanga, Xiandong Liu. Nonlinear dynamics of a spur gear pair with time-varying stiffness and backlash based on incremental harmonic balance method // International Journal of Mechanical Sciences. – 2006. – Vol. 48. – P. 1256-1263.

4. Von Groll G., Ewins D. The harmonic balance method with arc-length continuation in rotor/stator contact problems // Journal of Sound and Vibration. – 2001. – Vol. 241 (2). – P. 221-233.

5. A comparison of classical and high dimensional harmonic balance approaches for a Duffing oscillator / L. Liu, J.P. Thomas, E.H. Dowell, P. Attar, K.C. Hall // Journal of Computational Physics. – 2006.– Vol. 215. – P. 298-320.

6. Zhou J.X., Zhang L. Incremental harmonic balance method for predicting amplitudes of a multi-d.o.f. non-linear wheel shimmy system with combined Coulomb and quadratic damping // Journal of Sound and Vibration. – 2005.– Vol. 279. – P. 403-416.

7. Meyer S., Link M. Modelling and updating of local non-linearities using frequency response residuals // Mechanical Systems and Signal Processing. – 2003. – Vol. 17 (1). – P. 219-226.

8. Кучер О.Г., Харитон В.В., Лане Ж.-П., Туврез Ф. Вібраційне діагностування лопаток робочого колеса ГТД безконтактним методом (blade tip-timing method) // Матеріали 8-ї міжнародної науково-технічної конференції АВІА-2007, 25-27 квітня 2007 року. – К.: НАУ, 2007. – Т. 2. – С. 31.64-31.69.

Поступила в редакцию 25.05.2007

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