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CRACKED STRUCTURE RESPONSE ON EXTERNAL HARMONIC EXCITATION

Contact analysis was performed to demonstrate crack breathing process with contact inside of crack as the response on external harmonic excitations. The two approaches of the system order reduction to proceed high time consumable nonlinear contact analysis are considered for the structure with crack: the fixed-interface method and the free-interface method.

system order, degree of freedom, fixed-interface, free-interface, eigenmodes, contact, crack, crack breathing, penalty method, Lagrange multipliers

Introduction

The main goal of the research presented in the article is to elaborate appropriate and efficient methods for crack detection in aircraft engine turbine blade on the base of vibration characteristics. Vibration-based inspection of structures could bring an effective set of methods and approaches for non-destructive testing. The analysis of the dynamic response of a structure on external excitation and detection of structure state changes taking place during its lifetime could be used as on-line faults assessment/detection technique in operation [5].

To reach this it is first necessity to create simplified model of the 2d or 3d cracked structure. Then, the second stage is time integration analysis of the structure dynamic response to external loading taking into account non-linear effect from contact inside of the crack.

Among these problems another one exists – how to reduce computation time expenses which are extremely high during nonlinear analysis. To solve it, two approaches of the system matrices reduction (whether mass or stiffness matrices) can be used. They are based on classical free-interface and fixed-interface methods [3, 9]. In this case, there is only one structure with auto-contact at crack interface.

The majority of the damage-identification methods rely on linear structural models. The dynamic behavior of the damage structure may be influenced by non-linear effects: the opening and closing of the crack (crack breathing) during cyclic loading or in operational situation. The main results obtained through simulations or experimental studies were that the observed decrease in the natural frequencies of the cracked structure can not be described by a model of crack, which is always open. A breathing crack gives rise to natural frequencies falling between those corresponding to the open and closed cases.

Many researches used periodically varying stiffness during crack opening-closing process or bilinear model with periodical response on external periodical loading [6,7]. The non-linear effects of vibrating cracked structures can be understood with the help of contact modeling inside of the crack (between crack sides).

1. System reduction methods

Contact at the cracked interface is modelled by node-to-node contact. The matrix equation of the motion can be partitioned into interface relative (node-to-node) displacements and other DOFs:

$$[\mathbf{u}] = \begin{bmatrix} \mathbf{u}_m \\ \mathbf{u}_s \end{bmatrix}, \quad (1)$$

$$[\mathbf{M}] = \begin{bmatrix} \mathbf{M}_{ms} \mathbf{M}_{mm} \\ \mathbf{M}_{sm} \mathbf{M}_{ss} \end{bmatrix}, [\mathbf{K}] = \begin{bmatrix} \mathbf{K}_{ms} \mathbf{K}_{mm} \\ \mathbf{K}_{sm} \mathbf{K}_{ss} \end{bmatrix}. \quad (2)$$

where subscripts m and s refer to: m – master DOFs defined only on interface nodes; s – all DOFs that are not master DOFs.

1.1. Fixed-interface method. In order to project physical coordinates on the set of generalized coordinates the transformation matrix has to be constructed. For the fixed interface method this matrix is:

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{G}_{sm}\mathbf{\Phi}_{ss} \end{bmatrix}, \qquad (3)$$

where $[\mathbf{G}_{sm}] = -[\mathbf{K}_{ss}]^{-1}[\mathbf{K}_{sm}]$ – redundant static constraint modes and $\mathbf{\Phi}_s$ – fixed-interface normal modes (eigenvectors obtained with interface nodes coupled).

1.2. Free-interface method. For the free-interface method transformation matrix can be derived as follows:

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}_{sm}\mathbf{\Phi}^*_{sr}\mathbf{\Phi}^*_{ss} \end{bmatrix}, \qquad (4)$$

where $\mathbf{\Phi}^*_{sr}$ – matrix of inertia relief modes, which is zero in our case; $\mathbf{\Phi}^*_{ss} = [\mathbf{\Phi}_s - \mathbf{G}_{sm}\mathbf{\Phi}_m]$, $\mathbf{\Phi}_m$ – matrix of the master DOF partition of the free interface normal modes (eigenvectors obtained with interface nodes free) and $\mathbf{\Phi}_s$ – matrix of the slave DOF partition of the free-interface normal modes.

Final stage is same for both methods: reduction of the mass and stiffness matrices and solving of the eigenvalues problem:

$$[\mathbf{M}]^* = [\mathbf{T}]^T[\mathbf{M}][\mathbf{T}], [\mathbf{K}]^* = [\mathbf{T}]^T[\mathbf{K}][\mathbf{T}]. \qquad (5)$$

To summarise both methods we can outline that the system reduction with fixed-interface method (Craig-Bampton method) suppose as [9]:

1. Master DOFs – relative displacements of the contact pairs.
2. Constrained modes – eigenmodes for uncracked structure.

System reduction process with free-interface method (McNeal method) retains as [3]:

1. Master DOFs – relative displacements of the contact pairs.
2. Constrained modes – free modes (eigenmodes of the linear cracked structure).

2. Simplified cracked plate 2d model

We started with 2d model of cracked rectangular plate as with the simplest one. Its FE model (fig. 1) was created in ANSYS and then both mass and stiffness matrices were transfered to MATLAB for subsequent processing.

Fig. 1. Simplified 2d model of the cracked structure

The relative displacements were introduced in crack tips (coinciding nodes) to control and simulate in future analyses contact interaction of the crack sides.

2.1. Reduced systems results with full system. For reduced analysis as master two relative displacements of the crack tips and three additional modal DOFs were retained. In this case total size of system of 14 DOFs was reduced to 5 DOFs.

To proceed the comparisons we used three structure state (table 1): plate with fully opened crack, plate with closed crack and not sliding edge (relative displacements supposed to be zero), plate with closed crack and sliding edge (vertical relative displacement supposed to be zero) [8].

3. Modeling of the crack breathing process

Previously contact modeling was performed in ANSYS for the 2d cracked plate (fig. 1). External harmonic excitation $F = 100\sin\varpi t$ was applied to the top right node of the plate (fig. 2).

Table 1

Comparison of the eigenvalues problem solving results of the full and reduced systems

System state	Opened crack			Closed crack (not sliding edge)			Closed crack (sliding edge)		
System model	First mode, Hz	Second mode, Hz	Third mode, Hz	First mode, Hz	Second mode, Hz	Third mode, Hz	First mode, Hz	Second mode, Hz	Third mode, Hz
Full system	2342	4879	9076	3524	8274	10325	3441	8240	10000
Fixed-interface	2342	4883	9083	3524	8274	10325	3441	8241	10016
Free-interface	2342	4879	9076	3524	8281	10369	3441	8249	10039

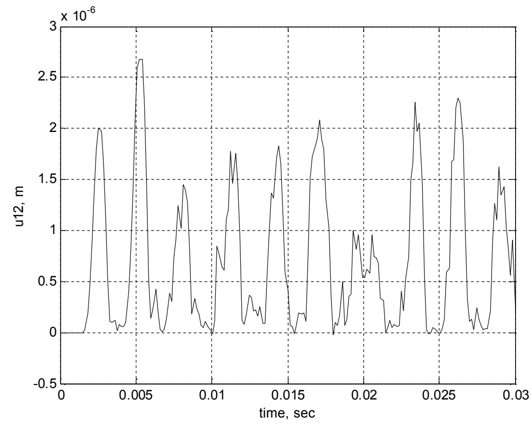

Fig. 2. Time-history of the relative vertical displacement between crack tips in ANSYS

3.1. Contact modeling with penalty method. This method is widely used in many FEM software because of its simplicity and quite efficient results. It was used during time integration of the equation of the motion of the system by adding additional stiffness to the diagonal element of the stiffness matrix for normal relative displacement at crack interface in case of interference (fig. 3). In our case equation of motion without damping matrix was used to simulate the structure dynamic behavior:

$$\mathbf{M}\ddot{u} + \mathbf{K}\dot{u} = \mathbf{F}\sin\omega t. \quad (5)$$

The time-integration in MATLAB consisted in reduction of the system to the first order and utilization of one of the solvers available for the stiff systems. The Gear's method based variable-order solver using the numerical differentiation formulas was employed to solve equation of motion. But to control the solution during analysis the central differences method was then used:

$$\mathbf{u}^{t+\Delta t} = 2\mathbf{u}^t - \mathbf{u}^{t-\Delta t} - \Delta t^2\mathbf{M}^{-1}\mathbf{K}\mathbf{u}^t + \Delta t^2\mathbf{F}\sin\omega t. \quad (6)$$

This method showed the same results with a smaller computational time expenses and was implemented for all following calculations.

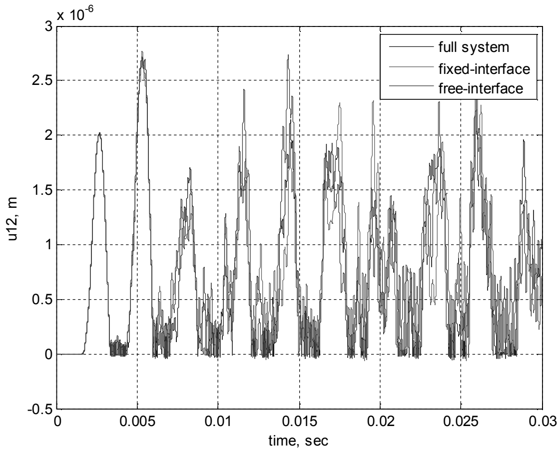

Fig. 3. Time-history of the vertical relative displacements with penalty method

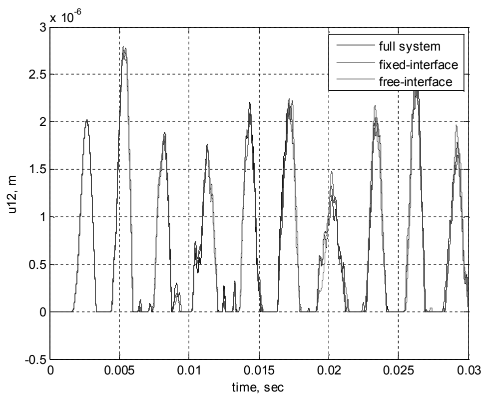

Fig. 4. Time-history of the vertical relative displacement with Lagrange multiplier method

3.2. Contact modeling with Lagrange multipliers method. When constraints equations (contact) are active lagrange multiplier represents reaction force between contact nodes. With the assumption of zero vertical displacement between contact nodes, we can derive, from the equation of motion, reaction force vector that defines the contact status. If contact force is negative or zero then the contact status is supposed to be closed. Otherwise positive contact force refers to a separation between the contact nodes and open contact status.

3.3. Contact modeling with taking into account of damping. When doing non-linear analysis without damping it is quite difficult to obtain steady solution that is necessary for prospect signal processing analysis, e.g. Fourier transformation. A light Rayleigh damping gives an apparent steady periodic solution after a few periods of excitation.

For central differences method the new iterative equation for displacements vector calculation is:

$$\mathbf{u}^{t+\Delta t} = (2\mathbf{M} + \Delta t\mathbf{C})^{-1}[2\Delta t^2\mathbf{F}\sin\omega t - 2\Delta t^2\mathbf{K}\mathbf{u}^t + 4\mathbf{M}\mathbf{u}^t - 2\mathbf{M}\mathbf{u}^{t-\Delta t} + \Delta t\mathbf{C}\mathbf{u}^{t-\Delta t}]. \quad (7)$$

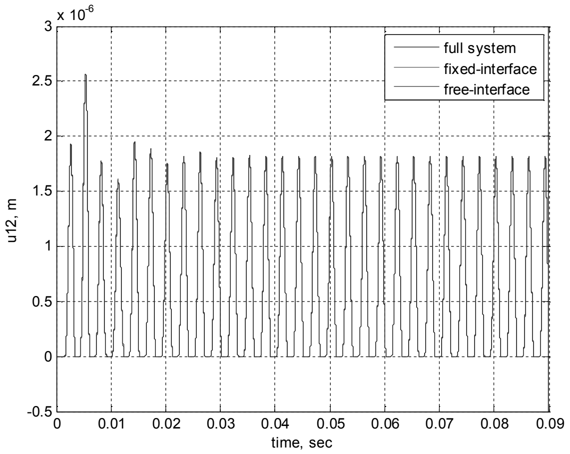

Fig. 5. Time-history of the relative displacement with Lagrange multipliers method with damping

Conclusions

Proposed original approaches for the system order reduction gave very good results (compared with full system solution) and computational time expense reduction. For example, total analysis with contact of the full system in ANSYS takes about 1,5 hour then the same analysis in MATLAB (reduced model) takes about 2 min. Now there are no so important differences between time expenses for full and reduced systems due to quite small size of the system. But in future analysis of realistic 3d cracked structures with the complex geometry it would be more evident. Moreover 3d model will give us opportunity to employ a friction model inside of crack. Signal processing of the response data may help to define typical behavior of a cracked blade that could help us in crack detection.

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